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Modified Relative Permeability Functions for a Non-communicating Stratified Reservoir

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***Abstract* -** In paper a method for constructing pseudorelative phase permeability functions for two-phase immiscible flow in a stratified oil reservoir with impermeable barriers between layers is considered. A method is based on a direct modeling of two-phase flow with neglect of capillary and gravity forces as well as of compressibility of the reservoir and its saturating fluids. An analytical form of the modified functions of relative phase permeability is proposed. Successful application of the proposed upscaling method is demonstrated at test problem of the oil reservoir development.

***Keywords*:** Two-phase flow in porous media, Relative phase permeability, Upscaling, Pseudofunctions, Stratified oil reservoir.

1. Introduction

Optimization of oil field development requires a large number of multivariant calculations of the multi-phase flow problem. The computing costs can be reduced via the coarsening of the computational grid. Mazo et al. (2013) proposed a super element modeling method for oil reservoir development, using a computational grid wherein the target sizes of the cells correspond to the average distance between the wells, and the vertical ones coincide with the thickness of the geological packs or the whole reservoir. To preserve computational accuracy, upscaling of the two-phase flow properties of each element is needed. This work deals with the method of constructing modified pseudofunctions of relative phase permeability of a layered reservoir with neglect of the crossflows between the layers as well as of capillary and gravity forces. The results of using pseudofunctions in case of reducing dimensionality of the two-phase flow problem are illustrated with test calculations.

2. Upscaling Problem Solution

The process of plane-parallel flow in a horizontal layered reservoir is described by non-dimensional equations of two-phase flow (1) with initial and boundary conditions (2):

$$\frac{\partial}{\partial x}\left(k\varphi(S)\frac{\partial p}{\partial x}\right)=0,\quad m\frac{\partial S}{\partial t}-\frac{\partial}{\partial x}\left(k\,k_w(S)\frac{\partial p}{\partial x}\right)=0,\quad x\in(0,1),\quad y\in(0,1) \tag{1}$$

$$t=0: S=0;\quad x=0: p=1,\ S=1;\quad x=1: p=0. \tag{2}$$

Here x is the horizontal coordinate counted from the injection well gallery along the reservoir and normalized to the length of the model section; y is the vertical coordinate counted upwards from the bottom of the reservoir and normalized to the reservoir thickness; $k(m(y))$ is the non-dimensional absolute permeability of the reservoir, which is a given porosity function $m(y)$, which in turn depending only on the vertical coordinate; t is the non-dimensional time; $S=(s-s_*)\big/(s^*-s_*)$ is the effective water saturation; s_*, s^* are the irreducible and maximum values of water saturation; p is the dimensionless pressure in the two-phase mixture. The total mobility function $\varphi(S)=k_w(S)+K_\mu k_o(S)$ is expressed via the relation $K_\mu=\mu_w/\mu_o$ (of the water and oil dynamic viscosity values) and the relative water and oil permeability functions $k_w(S), k_o(S)$ taken in the form of quadratic dependences without limitation of generality of the upscaling technique

$$k_w(S)=S^2,\ k_o(S)=(1-S)^2 .$$

Introduction of the thickness-averaged parameters

$$\bar{m}=\int_0^1 m(y)dy,\quad \bar{S}(t,x)=\int_0^1 m(y)S(t,x,y)dy\Big/\int_0^1 m(y)dy,\quad \bar{k}=\int_0^1 k(y)dy\ ,$$
$$\bar{p}(t,x)=\int_0^1 k(y)\,p(t,x,y)dy\Big/\bar{k}\ ,\quad \bar{\varphi}(t,x)=\int_0^1 k(y)\varphi(S)\nabla_x p(t,x,y)dy\Big/\left(\bar{k}\,\nabla_x\bar{p}\right),$$
$$\bar{k}_l(t,x)=\int_0^1 k(y)k_l(S)\nabla_x p(t,x,y)dy\Big/\left(\bar{k}\,\nabla_x\bar{p}\right),\quad (l=w,o) \tag{3}$$

makes it possible to obtain from equations (1), (2) the thickness-averaged fully equivalent equations of two-phase flow in a homogeneous reservoir:

$$\frac{\partial}{\partial x}\left(\bar{k}\,\bar{\varphi}(t,x)\frac{\partial\,\bar{p}(t,x)}{\partial x}\right)=0,\quad \bar{m}\frac{\partial\bar{S}(t,x)}{\partial t}-\frac{\partial}{\partial x}\left(\bar{k}\,\bar{k}_w(t,x)\frac{\partial\,\bar{p}(t,x)}{\partial x}\right)=0 \tag{4}$$

$$t=0:\bar{S}=0;\qquad x=0:\bar{p}=1,\ \bar{S}=1;\qquad x=1:\bar{p}=0 . \tag{5}$$

The average relative permeability values (3) are time-and-space-variant, so we deal with the so-called dynamic phase permeabilities (Kyte et al., 1975, Stone, 1991). To use relative permeability in the form of the functions only of medium saturation when solving the problem of two-phase flow in a homogenized reservoir, let us formulate the method of constructing relative permeability modified functions $K_w(\bar{S})$ and $K_o(\bar{S})$.

For analytical representation of modified relative permeability functions, we suggest using power-law dependences with the power exponents being dependent on the average water saturation:

$$K_w(\bar{S})=\bar{S}^{A_1\bar{S}^2+A_2\bar{S}+A_3},\quad K_o(\bar{S})=(1-\bar{S})^{B_1\bar{S}^2+B_2\bar{S}+B_3} . \tag{6}$$

Here A_i, B_i coefficients are determined from the condition of minimum mean-square deviation with approximation of relative permeability table values, obtained according to (3) when solving problem (1)–(2), by the dependences (6). The constructed functions (6) are used to write the equations (4) in a homogenized reservoir.

3. Results

In test calculations, the porosity and thickness of the reservoir layers were set by the normal law of distribution. The viscosity ratio was set as $K_{\mu} = 0.1$. The normalized reservoir permeability was calculated by the Kozeny formula written in terms of porosity (Daigle et al., 2009):

$$k = m^3 \big/ (1-m)^2 \ .$$

The calculations were carried out for two different reservoirs: 1) 10-layer reservoir, using table values K_w, K_o and 2) 100-layer reservoir, with the construction of pseudofunctions (6). Three computation variants were compared: 1) solution of the problem (1)–(2) for an initial layered reservoir (Scheme I); 2) solution of the problem (4) for an averaged reservoir with initial relative permeability functions (Scheme II); 3) solution of the problem (4)-(5) for a homogeneous reservoir, using the technique of relative permeability upscaling (Scheme III). Fig. 1 shows the relative permeability initial functions, the results of their upscaling for 10-layer and 100-layer reservoirs, and the results of compute the thickness-averaged water-saturation distribution at time moment $t = 10$ and oil recovery for each reservoir according to all three schemes.

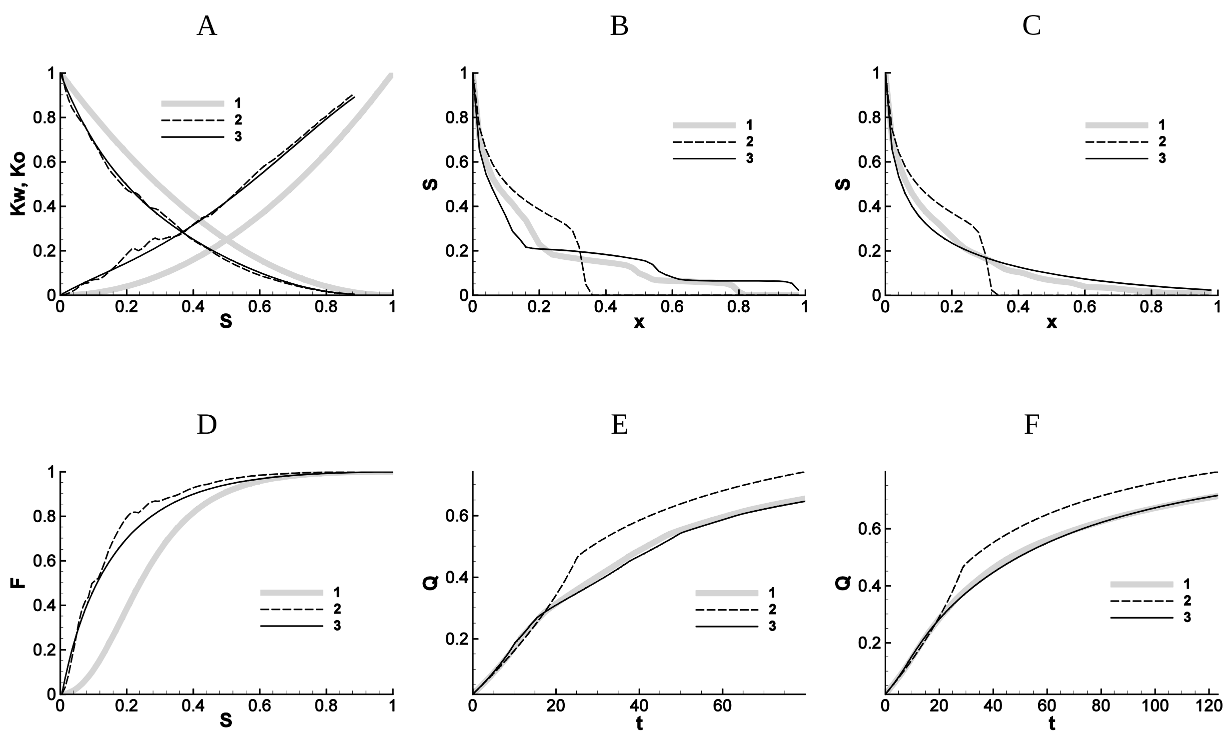

Fig. 1. Upscaling results and test calculations:
A – relative permeability curves: initial (1) and upscaled for 10-layer (2) and 100-layer (3) reservoirs;
B, C – average saturation distribution at t = 10 for 10 (B) and 100 (C) layers by three (1, 2, 3) schemes;
D – content of water in the flow: initial (1) and rescaled for 10-layer (2) and 100-layer (3) beds;
E, F – dynamics of oil recovery for 10 (E) and 100 (F) layers by three (1, 2, 3) schemes

4. Conclusions

The comparison of the results from the calculations using test problems and real oil reservoirs allows to make a conclusion that the use of the proposed modified functions of relative permeability for two-phase flow indeed makes it possible to reduce the dimensionality of the stratified reservoir flooding problem without significant loss of accuracy when calculating the reservoir thickness-related total phase flows and average saturation. In addition, analytical form of modified functions of relative permeability allows construction of the dependences of their coefficients on the reservoir parameters, the power exponents of the initial relative permeability functions, and the ratios of the fluid viscosities.

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