Proceedings of the International Conference on Heat Transfer and Fluid Flow
Prague, Czech Republic, August 11-12, 2014
Paper No. 202

The Effect of Plate Thermal Conductivity on the Optimal Porosity in Plate Heat Exchangers

Hossein Shokouhmand, Seyed Mostafa Saadat
University of Tehran, College of Engineering, School of Mechanical Engineering
North kargar, Tehran, Iran
hshokoh@ut.ac.ir; sm.saadat@ut.ac.ir

***Abstract* -** In this paper, the laminar flow in plate heat exchanger (PHE) with sinusoidal plates has been modelled. In this regard, the PHE has been considered as a porous medium. Porosity in a porous medium is defined as the total fluid volume divided by the total volume occupied by the solid and the fluid. A two-dimensional array of parallel sinusoidal plates with laminar flow and different plate thicknesses and plate conductivities has been considered. Energy and momentum equations are numerically solved and the effect of plate waviness with zero angle of inclination, Reynolds number, Porosity and the thermal conductivity of the solid phase has been investigated. It is shown that there is an optimal porosity in which the efficiency of heat transfer is up to 4% more than the time when the porosity is near one. According to the results of this study, the optimal porosity increases when the aspect ratio increases, but its changes with Reynolds numbers is up to the thermal conductivity of the plate.

***Keywords*:** Plate heat exchanger, Optimal porosity, Efficiency, Aspect ratio, Thermal conductivity.

1. Introduction

Heat exchangers were initially developed to use smooth heat transfer surfaces. An Enhanced heat transfer surface has a special surface geometry that provides a higher thermal performance, per unit base surface area than a plain surface. Studies on heat transfer in laminar flows show that complex and enhanced duct geometries are important for designing compact heat exchangers (Webb and Kim, 1994). Plate heat exchangers, also known as PHE, are one of the main parts of process and power industries. Although this type of heat exchangers were specially designed for sanitary uses such as dairy, pharmaceuticals and food processing industries, it has also found place in the modern power and process industries because of its advantages such as flexibility, high heat transfer capability, simplicity of maintenance, compactness, and less sediment. PHEs have a major advantage over other types of heat exchangers in that the fluids are exposed to a much larger surface area because the fluids spread out over the plates and can recover heat with small temperature difference (Shaji and Das, 2010).

The most prevalent plate-surface pattern in a PHE is chevron type corrugations with sinusoidal profile (Focke et al., 1985) as shown in Fig. 1(a). β is the angle of inclination and could vary in the rang $0^\circ < \beta < 90^\circ$. The case in which $\beta = 0^\circ$ yields a set of parallel sinusoidal ducts (Manglik and Ding, 1997) as shown in Fig. 1(b). According to Fig. 1(b), *a*, is the amplitude of waviness and, 2λ, is the wavelength. The distance between the plates is considered to be *2a*, which is much smaller than the plate width, so the flow can be modeled as two-dimensional. Because of different values of wavelength, 2λ, and amplitude of waviness, *a*, a dimensionless parameter called aspect ratio, γ, defined by (Metwally and Manglik, 2004) to obtain the possibility of comparison.

$$\gamma = \frac{2a}{\lambda} \tag{1}$$

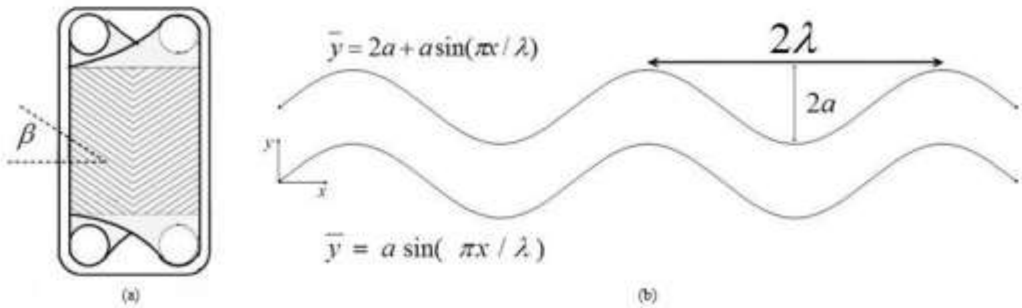

Fig. 1. (a) Chevron type corrugations with sinusoidal profile, (b) Sinusoidal profile and coordinate system.

Heat transfer enhancement has been studied for several geometries by several methods. Few studies have investigated laminar flow heat transfer in sinusoidal parallel-plate channels of the type shown in Fig.1(b). Rush et al. (1999) and Nishimura et al. (1985), have suggested that if the distance of parallel plates is comparable to the plate width or the aspect ratio is more than one ($\gamma > 1$), transition to turbulence might occur at lower Reynolds numbers ($\mathrm{Re} < 1000$). Otherwise Gschwind et al. (1995) suggested that when the inter-plate spacing is small in comparison to the plates width and the aspect ratio is lower than one ($\gamma < 1$), the flow region would be laminar in Reynolds numbers less than 1000. Mettwally and Manglik (2004) studied the effect of aspect ratio variation in laminar flows in sinusoidal corrugated-plate channels. They considered that the wall thickness is zero and the duct walls were maintained at constant temperature. They realized that the best heat transfer enhancement occur where the aspect ratio is in range $0.3 \le \gamma \le 0.6$.

Dispersion occurs in flow through disordered or periodic structures is based on the same interactions between the velocity and temperature nonuniformities (Kaviany, 1995). Indeed thermal dispersion is the convection-diffusion phenomena in a porous medium (Moyne et al., 2000). Taylor (1953) studied dispersion of soluble matter through a tube. Jiaying et al. (2010) analyzed thermal dispersion in an array of parallel plates with fully-developed laminar flow and realized that the effective thermal dispersivity depends upon the thermal setting and structure properties. Khaled and Vafai (2005) showed that porosity caused by either nano particles or structure increases the efficiency of heat transfer.

Porosity, ϕ, in a porous medium as in Eq. (2) is defined as the total fluid volume, V_f, divided by the total volume occupied by the solid, V_s, and fluid.

$$\phi = \frac{V_f}{V_f + V_s} \tag{2}$$

The objective of this study is to investigate the two-dimensional conduction heat transfer in a homogenous solid phase and convective heat transfer in laminar flows in an array of sinusoidal parallel plates. The array of flows in the PHE is counter and the flows heat capacities are equal. The geometry modeled and numerical solutions are done using COMSOL Multiphysics. Different wall thicknesses caused different porosities. The efficiency of heat transfer has the maximum available value in a porosity called the optimal porosity. To find this value, conjugate heat transfer is studied in the array of parallel sinusoidal plates for variations in Reynolds number, fluid and solid thermal conductivity ratio, porosity and aspect ratio.

2. Problem Definition

To model the plate heat exchanger, the two-dimensional array of sinusoidal plates as shown in Fig. 2 is considered where, t, is the wall thickness and the inter-spacing of plates is $2a$. It is considered that the flow velocity and heat capacities in channels are equal. Constant property, periodically developed, steady laminar flows of viscous Newtonian fluids with heat transfer are also considered. The flows Prandtl number is one (*Pr=1*). Reference coordinate, Plate's profile, the amplitude of the sinusoidal plate (*a*), and the length of waviness (2λ) are clearly shown in Fig. 1(a). For two-dimensional, steady-state conditions with no generation and constant thermal conductivity, the conduction heat transfer equation in the solid phase is defined in (Incropera, Lavine et al. 2011) as Eq. (3) in which, T_s is the temperature of the solid phase.

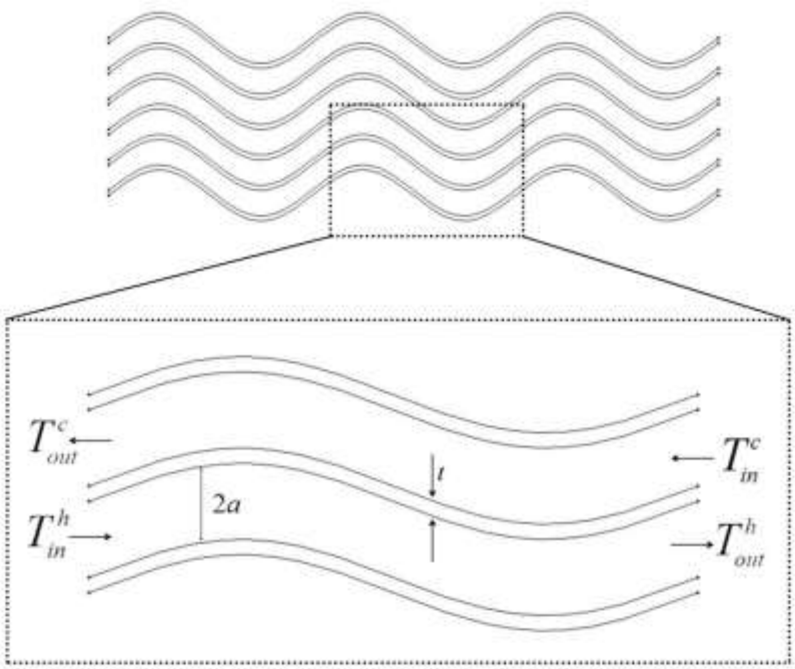

Fig. 2 Array of parallel sinusoidal plates and flows.

$$\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0 \tag{3}$$

For two-dimensional convection, steady-state flow with no generation and constant property, the governing equations for continuity, *x and y* direction momentum, and energy conservation are mentioned in Eqs. (4)-(7), respectively (Weigand et al., 2004).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{5}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{6}$$

$$u\frac{\partial T_f}{\partial x} + v\frac{\partial T_f}{\partial y} = \alpha\left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2}\right) \tag{7}$$

where u, v, p, υ, α and T_f are the velocity components in x, y directions, fluid pressure, kinematic viscosity, thermal diffusivity and fluid temperature, respectively. No slip, temperature and heat flux boundary conditions at all of the sinusoidal lines are given in (8-10), respectively, in which n is the normal vector of the plate.

$$u = v = 0 \tag{8}$$

$$T_f = T_s \tag{9}$$

$$-k_s \frac{\partial T_s}{\partial n} = -k_f \frac{\partial T_f}{\partial n} \tag{10}$$

Equations (3-7) with boundary conditions mentioned in (8-10) were solved numerically by using Conjugate Heat Transfer Module in COMSOL Multiphysics. A refined boundary layer mesh for fluid and Quad extreme fine mesh as shown in Fig. 3 was used to solve the equations. The error of solution is less than 10^{-6}. The wall temperature and the average of inlet and outlet fluid temperature are derived from COMSOL. The integral of heat flux, q_T, on the walls calculated with 4th-order integration by COMSOL.

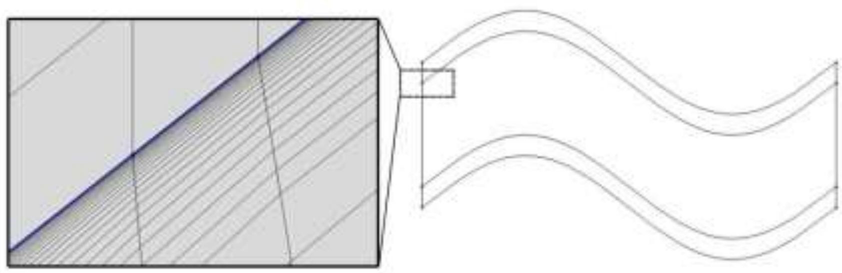

Fig. 3. The refined mesh that is used to solve the equations (3-7).

Nusselt number for both hot and cold sides, can then be calculated from

$$Nu = \frac{q_T\, 2a}{k_f \Delta T_{lm}} \tag{11}$$

Where

$$\Delta T_{lm} = \frac{(T_{in}^{w} - T_{in}) - (T_{out}^{w} - T_{out})}{\ln((T_{in}^{w} - T_{in})/(T_{out}^{w} - T_{out}))} \tag{12}$$

And T_{in}^{w}, T_{out}^{w}, T_{in}, T_{out} are wall inlet and outlet temperature, and the average of inlet and outlet flow temperature, respectively. The thermal efficiency of heat exchanger, ε, where the heat capacities of flows are equal is (Kuppan 2013):

$$\varepsilon = \frac{T_{in}^{h} - T_{out}^{h}}{T_{in}^{h} - T_{in}^{c}} = \frac{T_{out}^{c} - T_{in}^{c}}{T_{in}^{h} - T_{in}^{c}} \tag{13}$$

In which, T_{in}^{h}, T_{out}^{h}, T_{in}^{c}, T_{out}^{c} are the average of hot inlet and outlet flow temperature, and cold inlet and outlet flow temperature, respectively. In the classic ε-NTU method when the properties of flows are equal, Eq. (14) is used to calculate the total heat transfer coefficient in which h_1, h_2, t are the heat transfer coefficients and wall thickness, respectively.

$$\frac{1}{U}=\frac{1}{h_1}+\frac{t}{k_f}+\frac{1}{h_2} \tag{14}$$

According to (2), the porosity in the array of parallel plates shown in Fig. 3 is calculated from

$$\phi=\frac{2a}{2a+t} \tag{15}$$

The Reynolds number in this study is defined by Eq. (16) when u_m is the average inlet velocity which is defined by Eq. 17.

$$\mathrm{Re}=\frac{4au_m}{\upsilon} \tag{16}$$

$$u_m=\frac{1}{2a}\int_0^{2a} u\,dy \tag{17}$$

3. Results and Discussion

Heat transfer efficiency, ε, defined by Eq. (13), was calculated for variation in Reynolds numbers $(50<\mathrm{Re}<500)$, aspect ratios $(0.25<\gamma<1)$, porosities $(0.6<\phi<.99)$ and four different value of the fluid and solid thermal conductivity ratio (k_f/k_s). ε_0, is the efficiency of heat transfer whereas the porosity is near one. $\varepsilon/\varepsilon_0$, is the ratio of heat transfer efficiency to the efficiency whereas the porosity is near one. As seen in Fig.5, the efficiency of heat transfer increases when the porosity increases, but this behavior changes at a certain porosity after which the efficiency decreases. That certain porosity is the optimal porosity where the efficiency has the maximum value. Fig. 4 is plotted for Re = 200 and $\gamma=0.5$. In Fig. 5 the total Nusselt number for Re=200 and for variation of porosities is shown. The total Nusselt number has the same behavior as the efficiency.

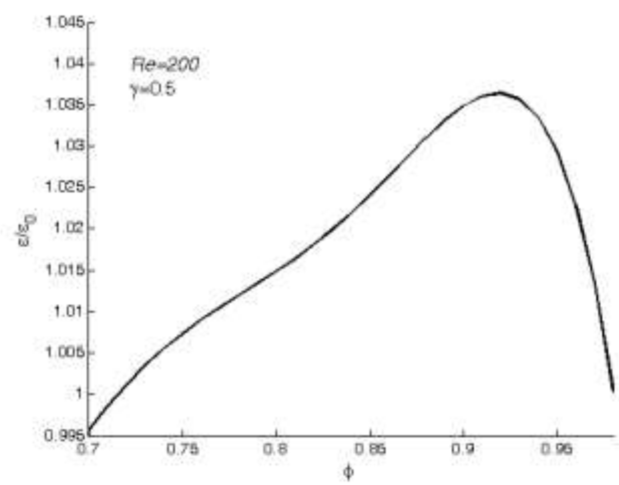

Fig. 4 Efficiency of heat transfer vs. variation of porosity for Re = 200, $\gamma=0.5$ and $\frac{k_f}{k_s}=\frac{1}{27}$

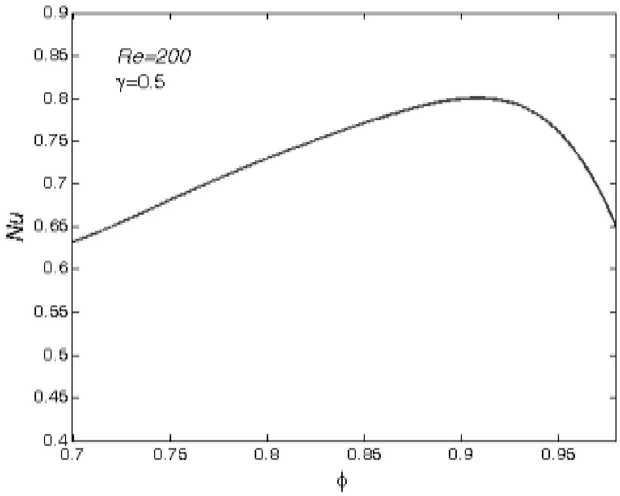

Fig. 5 The Nusselt number vs. porosity for Re =200, $\gamma = 0.5$ and $\frac{k_f}{k_s} = \frac{1}{27}$

In Fig.6 the optimal porosity for different values of plate waviness and the range of Reynolds numbers $(50 < \mathrm{Re} < 500)$ is shown for the $\frac{k_f}{k_s} = \frac{1}{27}$.

It shows that the optimal porosity in this case increases when either the Reynolds number or the plate waviness increases. It means that the axial conduction is less effective when the heat capacity of the flow increases. The structure of heat exchanger is more compact when the aspect ratio increases. In Fig. 7 variation of optimal porosity vs. Reynolds number for four different value of the fluid and solid thermal conductivities is plotted in aspect ratio $\gamma = 0.5$. It is shown that by decreasing conductivity ratio the behavior of the optimal porosity is changed and the optimal porosity decreases when the Reynolds number increases. The rate of changes increases by decreasing the conductivity ratio. Also, it can be figured out from Fig. 7 that by decreasing the conductivity ratio, the optimal porosity decreases and it means to have a higher efficiency, we need a thicker plate whereas the plate thermal conductivity increases.

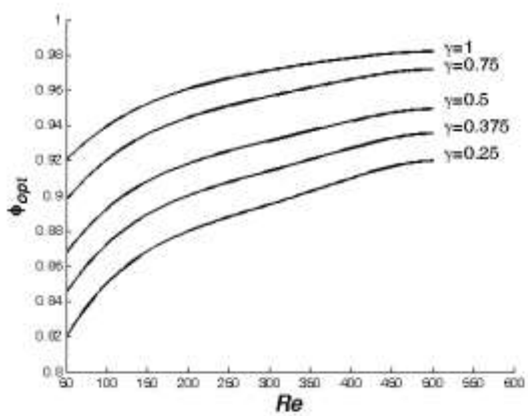

Fig. 6 The optimal porosity vs. Reynolds number for diferent values of aspect ratio and $\frac{k_f}{k_s} = \frac{1}{27}$

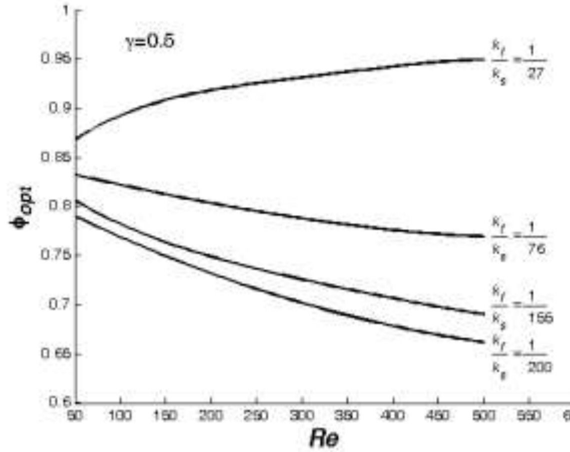

Fig. 7 The optimal porosity vs. Reynolds number for different values of conductivity ratio and $\gamma = 0.5$

4. Conclusion

In classic heat exchanger design, the total heat transfer coefficient is calculated from (14). As the wall thickness increases, the total heat transfer coefficient, *U*, decreases. But the thermal dispersion caused by structure feature can affect the efficiency of heat transfer. Axial conduction increases the efficiency of heat transfer in the certain range of porosity. The maximum value of efficiency belongs to the optimal porosity and it varies from 0.65 to 0.98. The efficiency of heat transfer in this optimal porosity is up to 4% more than the time when the porosity is near one. The thermal conductivity is an important parameter in such studies. It could influence the efficiency more than any parameter.

Although we studied the problem computationally, however, it seems to be necessary that our results should be compared with experimental studies investigating conjugate heat transfer in PHEs with different plate thicknesses and thermal conductivities. Considering different array of flows with non-equal heat capacities can also leads to better results to find the optimal porosities.

References

Focke W., Zachariades J., Olivier I. (1985). "The effect of the corrugation inclination angle on the thermohydraulic performance of plate heat exchangers." International Journal of Heat and Mass Transfer 28(8): 1469-1479.

Gschwind P., Regele A., Kottke V. (1995). "Sinusoidal wavy channels with Taylor-Goertler vortices." Experimental thermal and fluid science 11(3): 270-275.

Incropera F.P., Lavine A.S., DeWitt D.P. (2011). Fundamentals of heat and mass transfer. New York, NY, John Wiley & Sons.

Kaviany M. (1995). Principles of heat transfer in porous media. New York, NY, Springerverlag

Khaled A.R., Vafai K. (2005). "Heat transfer enhancement through control of thermal dispersion effects." International Journal of Heat and Mass Transfer 48(11): 2172-2185.

Kuppan T. (2013). Heat exchanger design handbook. New York, NY, Marcel Dekker.

Manglik R., Ding J. (1997). "Laminar flow heat transfer to viscous powerlaw fluids in double-sine ducts." International Journal of Heat and Mass Transfer 40(6): 1379-1390.

Metwally H., Manglik R. (2004). "Enhanced heat transfer due to curvature-induced lateral vortices in laminar flows in sinusoidal corrugated-plate channels." International Journal of Heat and Mass Transfer 47(10): 2283-2292.

Moyne C., Didierjean S., Amaral H. Souto, Da Silveira O. (2000). "Thermal dispersion in porous media: one-equation model." International journal of heat and mass transfer 43(20): 3853-3867.

Nishimura T., Ohori Y., Kajimoto Y., Kawamura Y. (1985). "Mass transfer characteristics in a channel with symmetric wavy wall for steady flow." Journal of chemical engineering of Japan 18(6): 550-555.

Rush T., Newell T., Jacobi A. (1999). "An experimental study of flow and heat transfer in sinusoidal wavy passages." International Journal of Heat and Mass Transfer 42(9): 1541-1553.

Shaji K., Das S.K. (2010). "The effect of flow maldistribution on the evaluation of axial dispersion and thermal performance during the single-blow testing of plate heat exchangers." International Journal of Heat and Mass Transfer 53(7): 1591-1602.

Taylor G. (1953). "Dispersion of soluble matter in solvent flowing slowly through a tube." Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences 219(1137): 186-203.

Webb R.L., Kim N.H. (1994). Principl of enhanced heat transfer. New York, NY, John Wiley and Sons.

Weigand B., Crawford M., Kays W. (2004). Convective Heat and Mass Transfer. New York, NY, McGraw-Hill Inc.

Xu J., Lu T.J., Hodson H.P., Fleck N.A. (2010). "Analysis of thermal dispersion in an array of parallel plates with fully-developed laminar flow." International Journal of Heat and Fluid Flow 31(1): 57-69.