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Effect of Radius Ratio on the Stability of Co- and Counter-Taylor Couette Flows

Nabila Ait Moussa
National Computer Sciences Engineering School,
BP 68M Oued Smar, 16309, Algiers, Algeria
n_ait_moussa@esi.dz

Sébastien Poncet
Aix-Marseille Université, CNRS, Ecole Centrale,
Laboratoire M2P2 UMR 7340, Marseille, France
sebastien.poncet@univ-amu.fr

Abdelrahmane Ghezal
Faculty of Physics, University of Science and Technology Houari Boumediene,
Algiers, Algeria
abdghezal@yahoo.fr

Mourad Lounis
Department of Physics, University of Khemis Meliana,
Algeria
loumou2000@yahoo.fr

Abstract – Taylor-Couette flows in the annular region between rotating concentric cylinders are studied numerically to determine the combined effects of the co - and counter-rotation of the outer cylinder and the radius ratio on the system response. The computational procedure is based on a finite volume method using staggered grids. The axisymmetric conservative governing equations are solved using the SIMPLER algorithm. One considers the flow confined in a finite cavity with radius ratios η = 0.25, 0.5, 0.8 and 0.97. One has determined the critical points and properties for the bifurcation to the Taylor Vortex Flow (TVF) state that bifurcate from the basic circular Couette flow (CCF). Indeed, the results are presented in terms of the critical Reynolds number Re_i of the inner cylinder that depends on Re_o and η. To show the capability of the present code, excellent quantitative agreement has been obtained between the calculations and previous experimental measurements for a wide range of radius ratios and rotation rates.

Keywords: Taylor-Couette flow, Co- and counter rotating cylinders, instability, Finite volume method.

1. Introduction

In recent years, the Taylor vortex flow pattern has been applied intensively to enhance thermal exchange in food processing industry or mixing in bio-industry and medical field such as catalytic chemical reactors, dynamic filtration devices and cell culture bioreactors. This flow is induced by the force balance between the centrifugal force and the pressure gradient in the radial direction within the gap of two concentric rotating cylinders**.** If the outer cylinder is held stationary and the inner one rotates at low angular velocities, the flow is steady and purely azimuthal (circular Couette flow CCF). Taylor (1923) showed that when the angular velocity of the inner cylinder is increased above a certain threshold, CCF becomes unstable and is replaced by a series of axisymmetric counter-rotating toroidal vortices known as Taylor Vortex Flow (TVF). A further increase in the rotation rate of the inner cylinder gives rise to series of fluid transitions with following flow modes, Wavy Vortex Flow (WVF), Modulated Wavy Vortex Flow (MWVF) and ending with turbulence.

In most of the cases, rotation is not limited to the inner cylinder. In fact, many investigations have been carried out where both cylinders rotate. Andereck et *al.* (1986) have well examined this problem in the small gap size and shown experimentally that the simplest flow CCF can bifurcate out to the three flow modes, Taylor vortex flow (TVF), spiral vortices (SPI) and interpenetrating spirals (IPS) in the case of counter-rotating cylinders. On the other side, when the cylinders rotates in the same direction the more complex flow patterns appear only for high values of angular velocities. The wide gap case was addressed in detail experimentally by Schulz et *al.* (2000, 2003) and numerically by Hoffmann et *al.* (2000, 2005). For a radius ratio η = 0.5, they have determined the spatio-temporal properties and the bifurcation behaviour of TVF and of SPI states that bifurcate out of CCF. Recently, Khali et *al.* (2013) have studied this problem in the case of non Newtonian fluids using the Lattice Boltzmann Method (LBM).

The present paper deals with the numerical examination of the structure and the dynamic properties of the Taylor vortex flow (TVF) that bifurcates out of the unstructured base state of circular Couette flow (CCF). The gap width effects on this transition are also discussed.

2. Physical Problem and Geometry

One considers the flow confined between two concentric cylinders of radii R_i and R_o respectively and height h. The working fluid is assumed to be incompressible, isothermal and Newtonian of mass density ρ and kinematic viscosity ν. Both cylinders can rotate independently around their common axis Z at the angular velocities Ω_i and Ω_o, respectively, while the top and bottom end-walls are stationary.

The system is characterized by two geometric parameters: the radius ratio $\eta=R_i/R_o$ and aspect ratio $\Gamma=h/d$, where $d=R_o-R_i$ is the gap width. Four values of η have been here investigated: η = 0.25 and 0.5 (large gap) and η = 0.8 and 0.97 (small gap). In order to minimize the influence of the Ekman vortices, a large aspect ratio of $\Gamma=20$ was chosen. Thus, the flow physical parameters can be expressed in terms of the inner and outer Reynolds numbers, $Re_i=\Omega_i R_i d/\nu$ and $Re_o=\Omega_o R_o d/\nu$ respectively.

3. Numerical Method

The fluid flow is described by the Navier-stokes and continuity equations for the velocity field, written in cylindrical coordinate system (r,θ,z). The variables are dimensionless using the scales h, $h\Omega_i$, Ω_i^{-1} and $\rho\,(h\Omega_i)^2$ for length, velocities, time and pressure respectively. No-slip boundary conditions are applied on the inner and outer cylinders.

To solve this problem numerically, one uses an in-house axisymmetric code based on the finite volume method using staggered grids in a $(r\text{-}z)$ plane fully described by Elena (1994). The numerical procedure is based on the SIMPLER algorithm to solve the velocity-pressure coupling. A (40×200) mesh in the (r,z) frame has proved to be sufficient to get grid independent solutions for both configurations (Re_o=0 and $Re_o\neq 0$). For this grid, the size of the thinner mesh is $\Delta_1 r = 6.2\times10^{-4}h$ and $\Delta_1 z = 1.85\times10^{-3}h$ in the radial and axial directions respectively. It will be used for all cases considered in the following. About 3.10^4 iterations are necessary to obtain the numerical convergence of the calculations.

4. Results

The first state to be computed was that of circular Couette flow (CCF) in which the velocity field is given by the well known exact solution

$$V(r)=\frac{R_i R_o}{R_o^2-R_i^2}\left[\left(\frac{R_o}{r}-\frac{r}{R_o}\right)\mathrm{Re}_i+\left(\frac{r}{R_i}-\frac{R_i}{r}\right)\mathrm{Re}_o\right] \tag{1}$$

Appropriate tests to validate our code are comparing the numerical results against the analytical solutions (Eq.1). In this sense, Figure 1a shows the tangential velocity profile along the radius for Re_i=50, Re_o=0 and η = 0.5. An excellent agreement has been obtained between the calculated solution and the analytical one. In the case of stationary outer cylinder and for the radius ratios considered here, the critical Reynolds number, Re_{ic}, for the onset of Taylor vortices is compared with values given by the linear stability of Gebhardt and Grossmann (1993) on Figure 1b. One can note that these values agree well with our computational data.

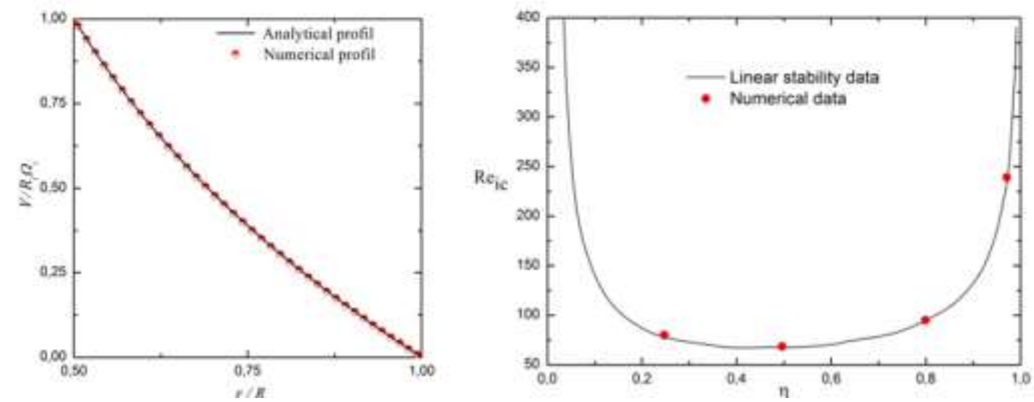

Fig. 1. (a) Comparison of the azimuthal velocity with analytical solution for z = 0,1h, Re_o=0 and η =0,5; (b) Linear stability theory for the transition boundary CCF and TVF (Re_o=0). The solid line represents the values given by the linear stability analysis of Gebhardt and Grossmann (1993) and the symbols represent the present calculations.

The transition from circular Couette flow (CCF) to Taylor vortex flow (TVF) is located by fixing Re_o and slowly increasing Re_i. In Figure 2a, one examines the effect of the outer cylinder rotation in a Taylor-Couette apparatus of large (η =0.25 and 0.5) and small (η =0.8 and 0.97) radius ratios. The base flow is unstable to time-independent axisymmetric Taylor vortex flow. Indeed, for a given radius ratio, one can see that the rotation of the outer cylinder in opposite direction is at first weakly destabilizing, and becomes then stabilizing. As an example, for a radius ratio η = 0.5, the minimal value of the transition point is for an outer cylinder Reynolds number Re_o = −15 and Re_i = 66.3, which is close to the point found by Schulz et *al.* (2000) (Re_{omin} =-15.26, Re_i =66.05). Beyond this value, the inner Reynolds number Re_i increases monotonically as Re_o increases. However, the co-rotation stabilizes the symmetric flow and the vortices appear for relatively high values of angular velocities of the inner cylinder and the delay of the transition is more marked. It can be seen that the rotation of the outer cylinder has the same qualitative effect as the imposed axial through flow. In fact, an axial flow stabilizes the circular Couette flow and delays the transition to TVF (Lueptow, 2000).

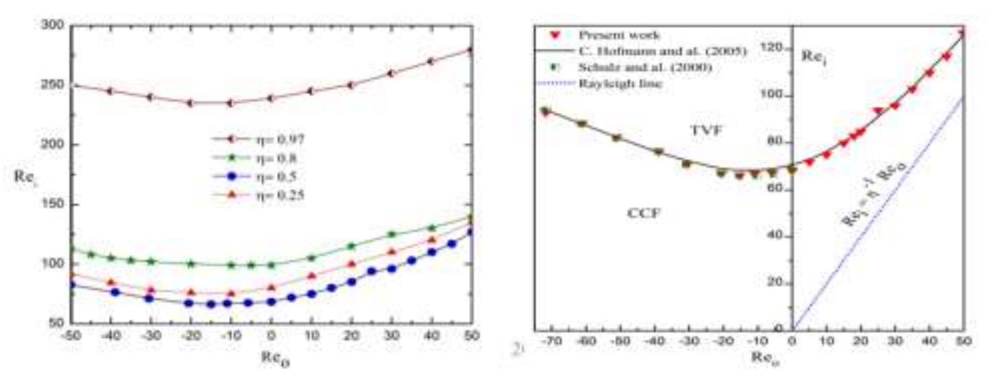

Fig. 2. (a) Stability diagram for the primary instability: Re_i versus Re_o for different values of η; (b) Comparison of the critical inner Reynolds number for η =0.5 with different works.

One can also see that the transition behaviour is found to depend strongly on the gap size. Indeed, for wide gap $\eta < 0.5$, increasing the radius ratio has a destabilizing effect on the Couette flow. Beyond this value, as the annular gap decreases, the critical inner Reynolds number increases and the flow gets stable. Thus, It is preferable that the bifurcation threshold be examined by the Taylor number under the form Ta $\sim Re_i\ \delta^{0.5}$, where $\delta = R_i/d$ is the clearance characterizing the gap size. This non-dimensional number appeared in the pioneering analytical studies of the small gap approximation $\delta \rightarrow 0$.

To check the accuracy of our results (Fig.2b), the critical inner Reynolds number is compared with published results, the measurements of Schulz et *al.* (2000) and the analytical data of Hoffmann et *al.* (2005) for a similar radius ratio of $\eta=0.5$. One can observe a good agreement concerning the onset of the first instability for outer Reynolds numbers $-77.5 \leq Re_o \leq 0$. In this plane, the Rayleigh criterion corresponds to the straight-line ($Re_i = 2Re_o$) asymptote of the stability threshold of Couette flow at large and positive Re_o values.

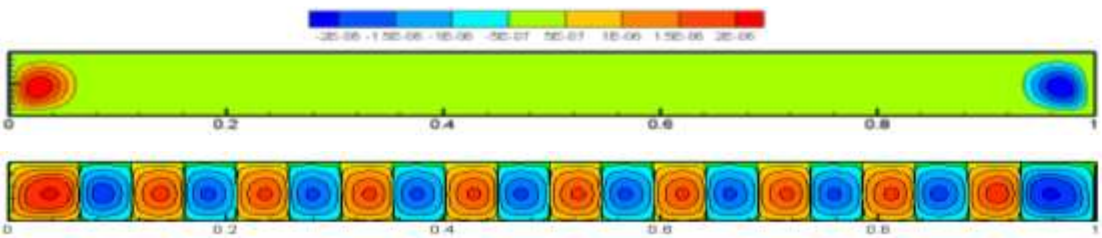

Fig. 3. Streamlines for $Re_O = 0$ (Γ=20, η=0.5)

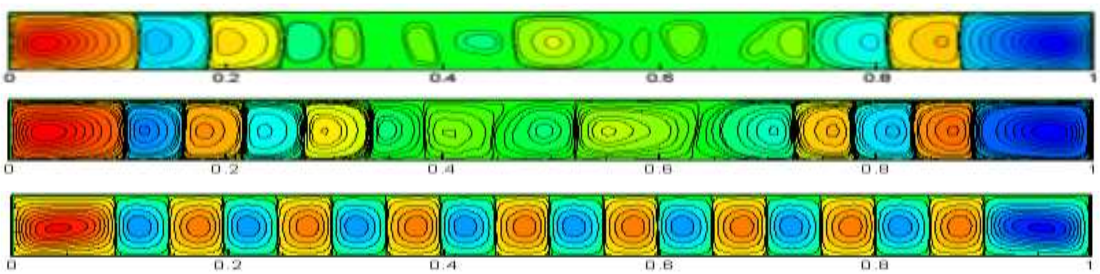

Fig. 4. Streamlines for $Re_O = +40$ (Γ=20, η=0.5)

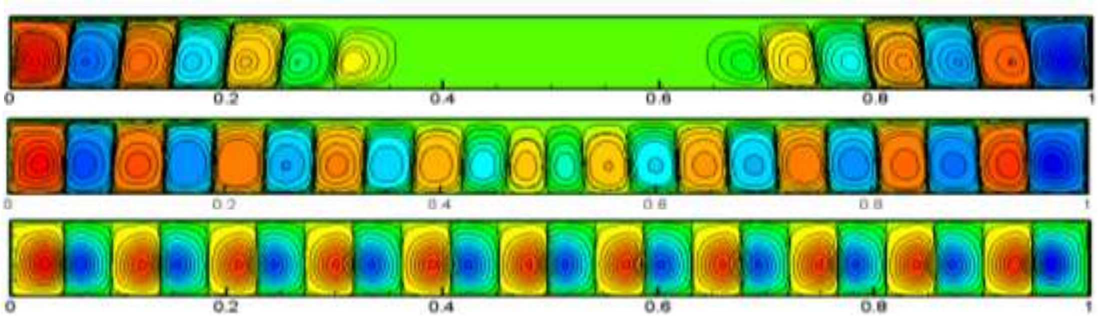

Fig. 5. Streamlines for $Re_o = -40$ (Γ= 20, η=0.5)

One has examined also the effects of the outer cylinder rotation rate on the development of the vortices within the gap. Figure 3 shows the numerical results contours of streamlines at $Re_o = 0$. The lower horizontal wall indicates the inner cylinder and the upper one the outer wall. It shows that there are two vortices in the upper and lower parts. These vortices known as Ekman vortices are due to the endwall effects. The meridional flow is due to the existence of the top and bottom stationary walls. In the vicinity of these walls, the centrifugal force is weaker than that in the middle part and therefore the opposing pressure gradient is dominant and two vortices are generated. By increasing the rotating speed, the Taylor vortices fill the whole gap between the cylinders. 20 vortices appear instead of two. This is due to the transition to an unstable state. In the case where the cylinders rotate in the same direction (Fig.4), the 18 vortices appear instead of 20 and the size of the Ekman vortices increases. This phenomenon can be explained by the great endwall influence. However, when the cylinders rotate in opposite directions (Fig.5), the number of vortices increases from 20 to 22. Also, the Ekman vortices size decreases. Similar observation has also been made by Khali et *al.* (2013). One can note that increasing the corotation has a more stabilizing effect compared to the counter rotation case.

4. Conclusion

The effects of the radius ratio and the inner Reynolds number on the transition between the circular Couette flow regime (CCF) and the axisymmetric Taylor vortex flow (TVF) has been investigated numerically. An excellent quantitative agreement has been obtained between finite-volume calculations with previous experimental and analytical studies for a wide range of radius ratios and rotation rates.

It has been found that the size of the annular gap plays an important role on the stability threshold of Couette flow. In the wide gap, as the radius ratio $\eta \leq 0.5$ is decreased, the critical inner Reynolds number increases contrary to the small gap case ($\eta > 0.5$), for which the critical inner Reynolds number is increased as the radius ratio is increased.

In addition, one can note that for a given radius ratio, the rotation of the outer cylinder in co- or counter direction delays the transition from the CCF to the TVF regimes. The flow in the co-rotating case is more stable than in the counter-rotating case. In the co-rotating case, the number of vortices is decreased from N to N-2. On the other hand, the number of vortices increases from N to N+2 in the case where the cylinders rotate in opposite directions.

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