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Inverse Heat Conduction by the Calibration Integral Equation Method

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Abstract **-** This paper describes a transformative surface heat flux and temperature calibration method for resolving inverse heat conduction problems. In hypersonic flight, high temperatures and heat fluxes ensue on both the external portion of the flight vehicle and inside the combustor. These locations require specialized and often non-receding materials for their thermal protection systems. A new calibration methodology is presently under development for estimating surface heat flux and temperature based on in-depth temperature measurements applicable to coupon or plug geometries composed of multi-regions and/or orthotropic materials. The resulting mathematical system produces Volterra integral equations of the first kind where the unknown is either the net surface heat flux or temperature. Regularization is required in order to extract a stable and accurate prediction. Uncertainties associated with probe(s) location, thermophysical properties, and sensor properties are removed from the analysis. Hence, a substantial reduction in systematic errors takes place when resolving inverse heat conduction problems in this manner.

Keywords**:** Inverse heat conduction, First kind Volterra integral equation, Calibration, Transfer functions.

1. Introduction

Ground testing plays an important preparatory role for flight tests of high-speed vehicles. At hypersonic speeds, high temperatures and heat fluxes are produced on the flight vehicle's external structure. Well-designed thermal protection systems (TPS's) are required to maintain flight integrity by managing thermal penetration into the vehicle. Inverse heat conduction analysis (Beck et al., 1985, Kurpisz and Nowak, 1995) provides a useful tool for resolving surface thermal conditions when surface instrumentation is impossible due to the hostile thermal environment. A calibration-based methodology devised from a series of observations developed from the frequency domain is presently under development at the University of Tennessee, Knoxville (UTK) for estimating surface heat fluxes and temperatures based on in-depth temperature measurements (Frankel et al. 2013) in hostile environments. Alternatively, a system identification approach (Löhle et al. 2006, 2007, 2008) involving fractional derivatives has also been demonstrated in the context of linear analysis for estimating surface heat fluxes based on in-depth sensors. The Non-Integer System Identification (NISI) method develops an impulse response function based on the calibration run. During this stage, numerous parameters must be estimated that serve as coefficients to a finite series expansion involving fractional derivatives of the measured temperature and known (net) surface heat flux. Once these coefficients are determined, the impulse response function is constructed and used for later estimations of the surface heat flux.

The UTK calibration approach involves input-output variables and is applicable to both in-depth and surface analyses (e.g., null-point calorimetry, Frankel and Keyhani, 2013). That is, the analytical transfer function is expressed in terms of calibration data. The resulting Volterra integral equation of the first kind (Kress, 1989) is solely composed of discrete data and the unknown variable. Being ill-posed, regularization is required for stability. The present numerical implementation is based on a local-future information method (Lamm, 2000). The optimal regularization parameter is estimated based on the study of the phase plane (Frankel and Keyhani, 2014b) involving the predicted heat flux (W/cm^2) and predicted heat flux rate (W/(s-cm^2)). Using this phase plane concept and cross-correlation, the optimal regularization parameter can be extracted.

2. Concept Development

For simplicity and mere demonstration purposes, consider the transient, linear heat equation governing the geometry displayed in Fig. 1 as

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}(x,t) = \frac{\partial^2 T}{\partial x^2}(x,t), \quad x \in [0,w], \;\; t > 0, \tag{1a}$$

subject to the boundary and initial conditions (T=temperature, x=position, t=time, ↦=thermal diffusivity, and w=depth of plate)

$$q_s''(t) = -k\frac{\partial T}{\partial x}(0,t) = q''(0,t) = ?, \quad \mathit{and} \quad q''(w,t) = 0, \quad t \geq 0,$$

$$T(x,0) = T_o, \qquad x \in [0,w], \tag{1b-d}$$

where q''=heat flux, q_s''=heating source, T_o=initial condition, and k=thermal conductivity. Unlike the forward problem where the boundary conditions are specified, the inverse problem involves resolving the surface heat flux, q''(0,t) using information obtained from an internal measurement, say T(b,t). One can derive an input-output framework producing a Volterra integral equation of the first kind (Frankel et al., 2013). Here, we develop the calibration integral equation based on the measured thermocouple temperature (discrete and noisy) at x=b and assume that an adiabatic back surface boundary condition is maintained between the calibration and reconstruction tests. During the calibration run, suppose that we know the net heat flux, q_c''(0,t) and measure the response temperature T_c(b,t). During the reconstruction phase, we must predict q_r''(0,t) based on measured T_r(b,t). The resulting calibration equation for surface heat flux becomes

$$\int_{u=0}^{t} q_r''(0,u)(T_{tc,c}(b,t-u) - T_{tc,oc})du = \int_{u=0}^{t} q_c''(0,u)(T_{tc,r}(b,t-u) - T_{tc,or})du, \quad t \geq 0. \tag{2a}$$

Here, the subscript "tc" represents the thermocouple temperature which is not normally the positional temperature (T_{tc}(b,t)   T(b,t)) required by the heat equation (see Frankel et al., 2013 for further details). Likewise, it is possible to derive the corresponding calibration integral equation for the surface temperature as

$$\int_{u=0}^{t} (T_r(0,u) - T_{or})(T_{tc,c}(b,t-u) - T_{tc,oc})du = \int_{u=0}^{t} (T_c(0,u) - T_{oc})(T_{tc,r}(b,t-u) - T_{tc,or})du, \quad t \geq 0, \tag{2b}$$

where it is assumed that we can measure the surface temperature during the calibration process. The subscript "c" represents calibration while the subscript "r" represents reconstruction. Calibration can be performed by carefully designed experimental processes. This case is restrictive owing to the constant property assumption and requiring that the back boundary condition remain fixed during the calibration and reconstruction tests. Fixed implies that the effective heat transfer coefficient remains constant and the environment on the passive side is maintained at the initial condition which could be different between test runs. However, these restrictions can be removed (Frankel and Keyhani, 2014c). The calibration integral equation method (CIEM) has broad application for heat conduction theory based on thermocouples.

Frankel and Keyhani (2014a) extended the one-probe formulation to include temperature dependent thermophysical properties in such a manner that the temperature varying properties can be accounted during a second calibration study. The resulting Volterra integral equation for surface heat flux was developed based on property linearization and given as

$$\begin{aligned}&\int_{u=0}^{t} q_r^{''}(0,u)\left(a_0+T_c(b,t-u)+a_2T_c^2(b,t-u)+\ldots-T_{oc}\right)du=\\&\int_{u=0}^{t} q_c^{''}(0,u)\left(a_0+T_r(b,t-u)+a_2T_r^2(b,t-u)+\ldots-T_{or}\right)du,\quad t\geq 0\end{aligned}\tag{3}$$

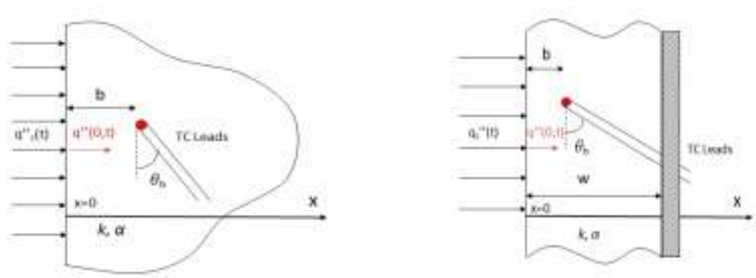
Here, the unknown expansion coefficients $\{a_k\}$, k=0,2,3,...are determined through calibration. More recently, Frankel and Bottlaender (2014) proposed a new theory based on an alternative sensor arrangement. This arrangement involves placing an ultrasonic transducer/receiver on the backside of the sample (x=w in Fig. 1b). In this case, a pulse-echo arrangement is proposed for capturing the time-of-flight (T.o.F). The T.o.F. measurement can be calibrated to the integral of the temperature over the spatial domain based on the temperature dependency of the speed of sound. For linear theory, the calibration integral equations for surface heat flux (net) and temperature are

$$\begin{aligned}&\int_{u=0}^{t} q_r^{''}(0,u)\left(G_c(t-u)-G_o\right)du=\int_{u=0}^{t} q_c^{''}(0,u)\left(G_r(t-u)-G_o\right)du,\quad t\geq 0\\&\int_{u=0}^{t}(T_r\ (0,u)-T_o)\left(G_c(t-u)-G_o\right)du=\int_{u=0}^{t}(T_c\ (0,u)-T_o)\left(G_r(t-u)-G_o\right)du,\quad t\geq 0\end{aligned}\tag{4a,b}$$

respectively, where the T.o.F. function, G(t) is explicitly

$$G(t)-G_o\approx\frac{2}{c(T_o)}\left(\beta-\frac{1}{c(T_o)}\left.\frac{dc}{dT}\right|_{T_o}\right)\int_{x=0}^{w}(T(x,t)-T_o)dx\tag{4c}$$

where β=linear expansion coefficient, c(T)=speed of sound, G_o=G(T_o) and T_o= initial and reference condition. The above formulation assumes that the initial condition (heat equation) and reference temperature (speed of sound) are identical for all test runs. This restriction can be removed (Frankel and Bottlaender, 2014).



(a) (b)

Fig. 1. Schematic of simplified geometry for concept demonstration (a) semi-infinite geometry and (b) finite width geometry.

Finally, Chen et al. (2014) expanded the calibration integral equation method to two-dimensional linear systems where the total heat transfer (W) is sought during the testing program. This is a practical issue since often total energy considerations are more important than the local heat flux. To reiterate, this approach is unlike conventional methods where property data and probe locations are required before forming the prediction. In essence, the proposed calibration framework and its accompanying mathematics express the system transfer function in terms of calibration data instead of mathematical functions.

3. Results

Equations (2-4) are highly unstable and small errors in the data will produce highly unreliable predictions for $q_r^{''}(0,t)$. To stabilize the system, a local future-time method is introduced for estimating $q_{r,\gamma}^{''}(0,t) \approx q_r^{''}(0,t)$ where • is the future-time regularization parameter. Alternative methods such as Tikhonov's or singular-value decomposition (SVD) have also been demonstrated. However, the future time method retains causality. A new concept (Frankel and Keyhani, 2014b) has been demonstrated for extracting the optimal regularization parameter, • through phase-plane analysis and cross-correlation in the •-spectrum of heat flux and heat flux rate predictions. Extraction of the optimal regularization parameter is demonstrated using experimental data collected at the UTK electrical heating sandwich (Elkins et al., 2013) and 500 W Class 4 (0.91 micron) laser facilities. The UTK Class 4, 500W (0.91 micron) facility allows for higher temperatures and heat fluxes than the sandwich facility though quantification of the surface (net) heat flux and temperature require additional analysis, configuration design and instrumentation. This is on-going research for acquiring high accuracy calibration data. Similar results have been produced indicating the merit and accuracy of the approach; and, independence of the experimental heating source.

4. Conclusion

This paper illustrates a highly effective alternative methodology for investigating inverse problems based on the calibration integral equation approach. The concept involves expressing the analytic transfer function in terms of calibration data. The approach is versatile in terms of geometry, nonlinearities, and instrumentation. Additionally, the results obtained by this approach are highly accurate and stable. However, significant care must be taken during the calibration tests to insure good predictive results during later tests.

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