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A Mathematical Model for Analysing the Thermal Characteristics of a Planar Heat Pipe for Space Application

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Abstract - Planar heat pipe radiator with non-condensable gas is a potential solution to satisfy heat rejection requirements for future NASA exploration and discovery missions. In order to investigate the feasibility of variable conductance planar heat pipe (VCPHP) operation in space environment, a simple one-dimensional mathematical model predicting hydrodynamic and thermal characteristics of VCPHP is developed. In this model, the planar heat pipe with triangular groove is divided in to three regions, (vapour, liquid wick and wall). Under several assumptions, three regions with different governing equation are solved separately and combined together with boundary conditions. Liquid height variation in liquid wick region is considered by solving Young-Laplace equation with triangular geometry relation. Under different liquid filling ratio, thermal load and cooling rate, the vapour velocity, liquid velocity, liquid height and wall temperature profiles can be calculated. Theoretical results are validated with a bench-top brass triangular flat heat pipe experimental data.

Keywords: Planar heat pipe, Mathematical model, Effective thermal conductivity, Triangular grooves.

1. Introduction

Due to the variation of solar position, freezing in the coolant loop is the most significant challenge when designing the radiator for space application. For example, the maximum temperature of lunar surface is approximately 400K while the minimum temperature is less than 100K. (Stephan, 2010). In order to meet heat rejection requirements for future NASA exploration and discovery missions, it is proposed that research should be conducted into integral planar variable conductance heat pipe technology. This represents a novel, low technology readiness level heat rejection technology that, when developed, will operate efficiently and reliably across a wide range of thermal environments. The concept consists of a planar heat pipe with groove structure whose evaporator acquires the excess thermal energy from the thermal control system and rejects it at its condenser whose outer surface acting as a radiating surface. The VCPHP contains a non-condensable gas (air) to vary the active radiator surface depending on the heat load. In this study, a simple mathematical model is introduced to capture the flow and thermal characteristic of the VCPHP.

During the last decade, the research on flat heat pipe is mainly focus on miniature size. K.H Do introduces the mathematical model for micro flat heat pipe with rectangular groove structure. In his model, the axial variations of wall temperature and non-uniform phase change are included. As the result, the assumption for uniform evaporation and condensation only valid for large scale heat pipe which axial wall heat conduction effect can be neglected. (Do et al., 2008) An analytic model describing flat heat pipe heat transfer is presented by M. Aghvami and A. Faghri. In their model, constant thermal conductivity in the wick structure is considered. However, for groove heat pipe, liquid thickness is changing along axial direction, which will vary the effective thermal conductivity at liquid-wick region. (Aghvami and Faghri, 2011)

In this proposed model, the planar heat pipe is divided into three regions, which are vapour flow region, liquid-wick region and solid wall region. In each section, several assumptions are made to simplify the calculation. The calculation result will be compare with the bench-top brass planar heat pipe (Figure 1a.) experiment conducted by our research team.

2. Mathematical Model

To capture the steady-state operation characteristic of planar heat pipe, it is necessary to separate the planar heat pipe into three different regions, which are vapour duct, liquid-wick region and wall region and (Figure 1b). Three regions are solved separately, and coupled by assigning proper boundary conditions.

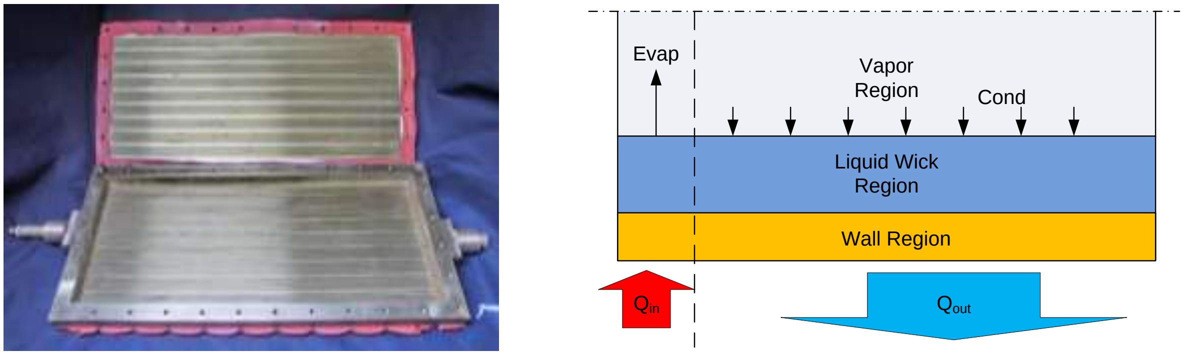

Fig. 1 (a) Brass planar heat pipe with triangular grooves. (b) Proposed planar heat pipe model.

2. 1. Vapour Flow Region

For the vapour flow region, the main assumptions are made to simplify the calculation:

(1) One-dimensional, steady-state, incompressible, laminar flow.
(2) Negligible convection in the liquid and vapour phase.
(3) Flat surface between liquid and vapour phase.
(4) Variation of cross section area of vapour chamber is negligible.
(5) Evaporation and condensation occur uniformly.
(6) Fluid properties remain constant along the vapour path.
(7) Vapour at saturated state.

The continuity equation and momentum conservation for vapour flow can be expressed as

$$V_{v,i}W_v - A_v\frac{du_v}{dx} = 0 \tag{1}$$

$$-2\rho_v A_v u_v\frac{du_v}{dx} - 2\tau_{v,w}D_v - \tau_{v,i}W_v - A_v\frac{dP_v}{dx} = 0 \tag{2}$$

Where $V_{v,i}$ denotes the averaged interfacial velocity. With the assumption of uniform evaporation and condensation, the interface average velocity can be expressed by heat rejection rate.

$$V_{v,i} = \begin{cases} \dfrac{\dot{Q}_{in}}{\rho_v \lambda W_v L_e}, & \text{Evaporator} \\ -\dfrac{\dot{Q}_{out}}{\rho_v \lambda W_v L_c}, & \text{Condenser} \end{cases} \tag{3}$$

Where $\tau_{v,w}$ and $\tau_{v,i}$ are the wall and interfacial shear stresses in the vapour region. Since the velocity of liquid is small compared with the vapour phase, the interfacial shear stress can be assuming the liquid to be stationary. The value of the wall and interfacial shear stresses for the rectangular channel flow can be expressed as

$$\tau_{v,i} = \tau_{v,w} = \frac{24\mu_v u_v}{\pi^2 W_v}\frac{\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^2}\tanh[\frac{(2n-1)\pi D_v}{2W_v}]}{[1-\frac{192}{\pi^5}\left(\frac{W_v}{D_v}\right)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}\tanh\left[\frac{(2n-1)\pi D_v}{2W_v}\right]]} \tag{4}$$

By solving the equations above, the pressure distribution and average velocity in axial direction can be calculated.

2. 2. Triangular Groove Liquid Flow

The flow in triangular groove is treated as one-dimensional, incompressible, low Reynolds number flow. Similar to vapour flow region, liquid velocity and pressure distribution can be calculated by continuity equation and Darcy law.

$$-2V_{l,i}w - A_l\frac{du_l}{dx} = 0 \tag{5}$$

$$\frac{dP_l}{dx} = -\frac{2\mu_l u_l}{D_{h,l}^2}(fRe)_l \tag{6}$$

For triangular groove, the cross section area and hydraulic diameter can be express as a function of radius of curvature which varies along the axial direction.

$$A_l = [\frac{1}{\tan\alpha} - \left(\frac{\pi}{2} - \alpha\right)] \times r_c^2 \tag{7}$$

$$D_h = 2\{\left[\frac{1}{\tan\alpha} - \left(\frac{\pi}{2} - \alpha\right)\right]\tan\alpha\} \times r_c \tag{8}$$

After the pressure distribution for both liquid and vapour flow are obtained. The radius of curvature variation can be determined by Young-Laplace equation.

$$\frac{dP_v}{dx} - \frac{dP_l}{dx} = -\frac{\sigma}{r_c^2}\frac{dr_c}{dx} \tag{9}$$

By assigning boundary conditions, the radius of curvature, liquid flow and liquid pressure can be evaluated by solving Eqns. (5)-(9) iteratively.

The liquid height variation can be easily determined by the geometry relationship for triangular grooves.

2. 3. Wall Region

Once the liquid height variation is obtained, the effective thermal conductivity for the wick/liquid region can be determined. By solving the two-dimensional heat conduction through planar wall with non-uniform thermal conductivity field with force convection (radiation) boundary conditions, wall temperature field can be determined.

3. Result and Discussion

In order to validate the model, the research team fabricates a bench top brass planar heat, and measures the wall temperature profiles under different heat load. Figure 2 depicts the variation of liquid thickness in axial direction under different heat loads. As heat pipe subjected to higher heat input, evaporation rate and condensation rate increase, more liquid will accumulate at the end of condenser .As more liquid occupy the grooves will decrease the thermal conductivity, and hindering the heat transfer from vapour phase to the environment. On the other evaporator end, the liquid level keeps decreasing. If the radius of curvature smaller than minimum radius of curvature (depends on contact angle and grooves shape), the heat pipe reaches the capillary limit.

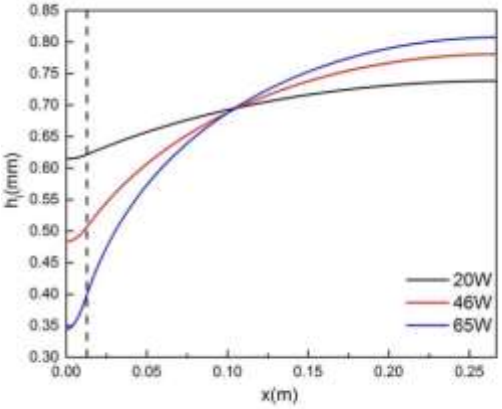

Fig. 2. Liquid thickness variation in axial direction. (Filling ratio = 0.7).

The predicted and measured wall temperature are compared and depicted in figure 3. The agreement between the model and experimental results is good, which suggest the proposed mathematical model can predict planar heat pipe thermal behaviour accurately.

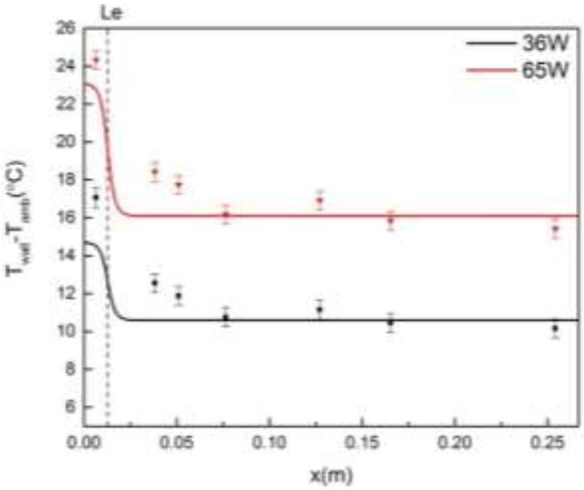

Fig. 3. Brass planar heat pipe wall temperature distribution under different heat loads.

4. Conclusion

In order to investigate the feasibility of variable conductance planar heat pipe (VCPHP) operates in space environment, a simple one dimensional mathematical model which is capable of predicting hydrodynamic and thermal characteristics of VCPHP is developed. The model is validated through comparison with brass planar heat pipe experiment and can provide quick and reliable evaluation of planar heat pipe design for future space applications.

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