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A Numerical Study of Effect of Inner and External Cylinder Rotating on Heat Transfer of a Non Newtonian Fluid

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Abstract- In this paper, we present a numerical study of the flow characteristics and heat transfer mechanism of a non-Newtonian fluid in an annular space between two coaxial rotating cylinders. The Carreau stress-strain relation was adopted to model the rheological fluid behaviour. We consider here two cases, the case where the inner cylinder is rotated and the outer cylinder is at rest, and the case where the external cylinder is rotated. The horizontal endplates are assumed adiabatic. The effects of inner and outer cylinder rotation on heat transfer are examined.

Keywords**:** Heat transfer, Non Newtonian fluid, Finite element, Carreau fluid, two rotating cylinders

1. Introduction

The laminar flow and the heat transfer of a non-Newtonian fluid between rotating concentric annulus are encountered in a large number of industrial processes as the catalytic chemical reactors (Cohen 1983), the filtration devices (Holeschovsky 1991), the blood plasmaphosresis devices (Beaudoin 1989), the plant cell bioreactors (Janes 1987) and the liquid-liquid extractors (Davis 1960).

The convective heat transfer mechanisms of the non-Newtonian fluids are the subject of considerable works and are well understood today. The mixed convection between two concentric horizontal cylinders is reported in references (Nieckele 1985, Nguyen 1983 and Naozo 1979). A survey of laminar flow of non-Newtonian fluids in a rotating concentric annulus has been reported by Batra and Eissa (1994). Flow of a Casson and Robertson-Stiff fluids between two rotating cylinder has been investigated by Batra (1992) and Eissa (1999). Kouitat *et al*. (1990) investigated theoretically and numerically the laminar Couette at the start-up stage of the fluid motion within a coaxial cylinder viscosimeter. For a Carreau model, Khellaf and Lauriat (2000) studied numerically the heat transfer between two rotating concentric vertical cylinders. A great deal of theoretical and numerical works dealing with flow and associated heat transfer characteristics of natural and mixed convection in annuli between two isothermal concentric cylinders are reported in the cited literature (Khellaf 2000, Grecov 2005, Baloch 2003, Gandjalikhan 2003, Nouri 1997 and Gwynllyw 1996).

In this study, we present a numerical study of the flow characteristics and heat transfer mechanism of a non-Newtonian fluid in an annular space between two coaxial rotating cylinders. The Carreau stress-strain relation was adopted to model the rheological fluid behavior. In comparaison with the Newtonian case, this model invoves four additional parameters, namely the zero-and infinite-shear rate viscosities ($\mu_0$and μ_∞, respectively), the relaxation time of the fluid, λ, which describes the transition to a constant viscosity in the limit of zero shear rate, and the index of structure, n, which is a measure of the degree of a non-Newtonian behavior. We consider here two cases, the case where the inner cylinder is rotated and the outer cylinder is at rest, and the case where the external cylinder is rotated. The effects of inner and outer cylinder rotation on heat transfer are examined. A computational code applied to the fluid mechanics and the heat transfer by using the finite elements method is developed. This code is validated by comparison with results reported in the literature. This computational code takes into account the non-Newtonian effects.

2. Problem Formulation

The geometry under investigation is shown in Fig. 1. We consider two coaxial cylinders with a finite length H. The inner cylinder, of radius r_i, is maintained at a hot uniform temperature T_h. The outer cylinder of radius r_e is at a cold temperature T_c.

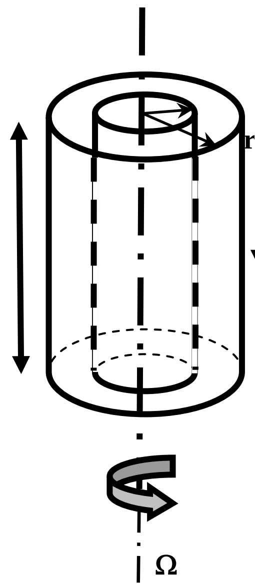


Fig.1. Geometry of the problem.

The flow is assumed to be laminar, incompressible and axisymmetric. Non- Newtonian effects are considered for fluids obeying the Carreau constitutive relationship

$$\eta = \frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \left(1 + \lambda^2 \dot{\gamma}^2\right)^{\frac{n-1}{2}} \quad (1)$$

where μ_0 is the viscosity at low shear rate, μ_∞ is the viscosity at high shear rate, λ is the time constant, n is the power law index, and $\dot{\gamma}$ is the shear rate. It is given by:

$$\dot{\gamma} = 2\left[\left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{v}{r}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] + \left[r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right]^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial r}\right)^2 \quad (2)$$

In the following, it is assumed that the parameters of this constitutive equation do not vary with temperature. For many concentrated polymer solutions and melts, it can be assumed that $\mu_\infty << \mu_0$ (Bird 1987). So, μ_∞ is neglected here. η represents the dimensionless apparent viscosity. The fluid is Newtonian for $n=1$, and the shear thinning behaviour becomes more significant where n becoming smaller.

The dimensionless form of the governing equations can be obtained by use of dimensionless variables defined as:

$$Z=\frac{z}{r_e-r_i},\quad R=\frac{r}{r_e-r_i},\quad U=\frac{u}{\Omega\, r_i},\quad V=\frac{v}{\Omega\, r_i},\quad W=\frac{w}{\Omega\, r_i},\quad \theta=\frac{T-T_c}{T_h-T_c} \tag{3}$$

Variables u, v and w are the velocity components in the z, r direction and azimutal velocity. T is the temperature. The dimensionless deformation rate is the ratio $\frac{\Omega r_i}{r_e-r_i}$.

In dimensionless form, the relationship (1) is written as follow:

$$\eta=\left(1+W_e{}^2\,\dot{\gamma}\right)^{\frac{n-1}{2}} \tag{4}$$

where the flow index n and the Weissenberg number W_e, describe the rheological property of the fluid.

On the basis of the dimensionless variables defined in Eq.(3), the non-dimensional form of the conservation of mass, momentum and energy equations are:

$$\frac{\partial V}{\partial R}+\frac{V}{R}+\frac{\partial U}{\partial Z}=0 \tag{5}$$

$$\left(V\frac{\partial V}{\partial R}+U\frac{\partial V}{\partial Z}-\frac{W^2}{R}\right)=-\frac{\partial P}{\partial R}+\frac{1}{\mathrm{Re}}\left\{\frac{1}{R}\left[\frac{\partial\left(2R\eta\frac{\partial V}{\partial R}\right)}{\partial R}\right]-\frac{\eta V}{R^2}+\frac{\partial}{\partial Z}\left[\eta\left(\frac{\partial U}{\partial R}+\frac{\partial V}{\partial Z}\right)\right]\right\} \tag{6}$$

$$\left(V\frac{\partial U}{\partial R}+U\frac{\partial U}{\partial Z}\right)=-\frac{\partial P}{\partial Z}+\frac{1}{\mathrm{Re}}\left\{\frac{1}{\mathrm{R}}\frac{\partial}{\partial\mathrm{R}}\left[R\eta\left(\frac{\partial U}{\partial R}+\frac{\partial V}{\partial Z}\right)\right]+\frac{\partial}{\partial Z}\left(2\eta\frac{\partial U}{\partial Z}\right)\right\}+\frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta \tag{7}$$

$$\left(V\frac{\partial W}{\partial R}+U\frac{\partial W}{\partial Z}+\frac{WV}{R}\right)=\frac{1}{\mathrm{Re}}\left\{\frac{1}{R}\frac{\partial}{\partial R}\left(R\eta\frac{\partial W}{\partial R}\right)+\frac{\partial}{\partial Z}\left(\eta\frac{\partial W}{\partial Z}\right)-\eta\frac{W}{R}\right\} \tag{8}$$

$$\left(V\frac{\partial\theta}{\partial R}+U\frac{\partial\theta}{\partial Z}\right)=\frac{1}{\mathrm{Re}\;\mathrm{Pr}}\left\{\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)+\frac{\partial\theta}{\partial Z}\right\} \tag{9}$$

The dimensionless boundary conditions in this case of this geometry are:

W = 1, U = V = 0, θ_1 at the inner cylinder

W = 0, U = V = 0, θ_2 at the outer cylinder

W = U = V = 0 and $\frac{\partial\theta}{\partial Z}=0$ at the horizontal endwalls.

The problem is characterized by the following parameters of similarity; Prandtl number $\mathrm{Pr} = \frac{\mu_0 c_p}{\lambda}$, Reynolds number $\mathrm{Re} = \frac{\Omega r_i (r_e - r_i)\rho}{\mu_0}$, Weisseneberg $\mathrm{We} = \frac{\Omega r_i}{r_e - r_i}\lambda$, Grashof number $\mathrm{Gr} = \frac{g\beta(T_h - T_c)(r_e - r_i)^3}{\mu_0^2}$, (or Rayleigh number $Ra = Gr\,\mathrm{Pr}$)

3. Numerical Resolution

When simulating an incompressible fluid flow, we demand that divergence- free discrete velocities be attainable. Attempts to ensure that the fluid flow is everywhere and always incompressible have dominated the subject of computational fluid dynamics. We analyse equations (5-7) by using a mixed formulation rather than by using a segregated approach so that the mass and momentum conservations can be simultaneously coupled. The pressure in the incompressible Navier-Stokes equations serves as a Lagrangian multiplier. As a result, accurate predicted discrete solenoidal velocities may accompany a non-smooth pressure. Legitimate choice of finite element trial spaces for primitive variables is thus of importance because the mixed finite element method is subject to the LBB (Ladyzhenskaya- Babuska - Brezzi) stability condition. To retain a sufficiently smooth solution for the investigated elliptic system (5-7), we take an element free of the LBB stability constraint into consideration. By substituting the well-paired bilinear interpolation function for the pressure and the biquadratic interpolation function for the velocities into the weighted residual statement of (5-7), we can derive the following matrix equations along with bilinear test function for the mass conservation equation:

$$[C]\{U_n\}+[K]\{U_n\}=\{F\} \tag{10}$$

where

$$\{U_n\}=\begin{Bmatrix} U_j \\ V_j \\ P_j \end{Bmatrix}, \quad \{F\}=\begin{Bmatrix} F_u \\ F_v \\ 0 \end{Bmatrix} \tag{11a, b}$$

and

$$F_u = -\iint N_i \left(\frac{W^2}{R}\right) R\,dR\,dZ \quad , \quad F_v = -\iint \frac{Gr}{\mathrm{Re}} N_i(\theta)\,R\,dR\,dZ \tag{12a,b}$$

The matrixes (C) and (K) are organized in the following way:

$$[C]=\begin{bmatrix} C(u) & 0 & 0 \\ 0 & C(u) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \quad [K]=\begin{bmatrix} \bar{K}_{11} & K_{12} & -Q_1 \\ K_{21} & \bar{K}_{22} & -Q_2 \\ -Q_1^T & -Q_2^T & 0 \end{bmatrix} \tag{13a, b}$$

where

$$C(u) = \iint N_i \left(U \frac{\partial N_j}{\partial Z} + V \frac{\partial N_j}{\partial R} \right) R\, dR\, dZ \tag{14}$$

$$\overline{K}_{11} = 2K_{11} + K_{22}, \quad \overline{K}_{22} = K_{11} + 2K_{22} \tag{15a,b}$$

$$Q_1 = \iint M_i \frac{\partial N_j}{\partial Z} R\, dR\, dZ \quad , \quad Q_2 = \iint M_i \frac{\partial N_j}{\partial R} R\, dR\, dZ \tag{16a,b}$$

and

$$K_{11} = \iint \frac{1}{\text{Re}} \left(\frac{\partial N_i}{\partial Z} \frac{\partial N_j}{\partial Z} \right) R\, dR\, dZ \tag{17}$$

$$K_{11} = \iint \frac{1}{\text{Re}} \left(\frac{\partial N_i}{\partial R} \frac{\partial N_j}{\partial R} \right) R\, dR\, dZ \tag{18}$$

Again the finite-element technique and Galerkin's principle can be used for solving the rotating velocity and the energy equation. The following matrix equation (19 and 23) can be obtained by adopting the same approach as outlinned previously:

$$N(W)\, W + K_W W = F \tag{19}$$

Where

W is the azimutal velocity

And

$$N(W) = \iint N_i \left(V \frac{\partial N_j}{\partial R} + U \frac{\partial N_j}{\partial Z} + \frac{V N_j}{R} \right) R dR dZ \tag{20}$$

$$K_W = \iint \frac{1}{\text{Re}} \left(\frac{\partial N_i}{\partial R} \frac{\partial N_j}{\partial R} + \frac{N_i N_j}{R^2} + \frac{\partial N_i}{\partial Z} \frac{\partial N_j}{\partial Z} \right) R\, dR\, dZ \tag{21}$$

$$F = 0 \tag{22}$$

$$N(\theta)\theta + K\theta = F \tag{23}$$

where

$$N(\theta) = \iint N_i \left(V \frac{\partial N_j}{\partial R} + U \frac{\partial N_j}{\partial Z} \right) R\, dR\, dZ \tag{24}$$

$$K = \frac{1}{\text{Re}\Pr} \iint \left(\frac{\partial N_i}{\partial R} \frac{\partial N_j}{\partial R} + \frac{\partial N_i}{\partial Z} \frac{\partial N_j}{\partial Z} \right) R\, dR\, dZ \tag{25}$$

and

N_i is the shape functions of the 9-node quadrilateral element and M_i is shape functions of the 4-node quadrilateral element.

Calculation of the stream function and Nusselt number: The quantities of interest in the present problem are also the stream function and the Nusselt number. These can be calculated a posteriori once the solution for the velocity and temperature fields has been obtained.

The distribution of the stream function ψ can be obtained via the velocity field by solving separately the Poisson equation subject to the boundary condition $\psi = 0$ on all walls:

$$\begin{cases} U = \dfrac{1}{R}\dfrac{\partial \psi}{\partial R} \\ V = -\dfrac{1}{R}\dfrac{\partial \psi}{\partial Z} \end{cases} \quad ; \quad \frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R}\frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} = -R\left(\frac{\partial V}{\partial Z} - \frac{\partial U}{\partial R}\right) \tag{26a,b}$$

The mean Nusselt number is defined as the ratio of heat flux crossed through the total cylindrical surface of radius r to the conduction heat flux:

$$Nu = \frac{r}{AR} \ln\frac{r_e}{r_i} \int_0^K \left(-\frac{\partial \theta}{\partial R} + U\theta\right) dR \tag{27}$$

where $AR = \dfrac{H}{r_e - r_i}$ *and* $K = \dfrac{r_e}{r_i}$

Although a Newton-Raphson iterative scheme would be recommended for the solution of nonlinear system of equations, here we have used a direct substitution scheme (Picard method), sometimes with underrelaxation to accelerate convergence. This avoids calculating the Jacobian which can be time consuming, and it also enjoys a wider convergence range. The solution process starts from the Newtonian field (n=1), which is used to obtain first approximation.

4. Results and Discussion

Calculations were performed with two grids, one having 400 elements, 1681 nodes, and 3803 unknown degrees of freedom and another denser having 900 elements, 3721 nodes, and 8403 degrees of freedom. The results were virtually identical and thus mesh independent.

The convergence criterion used here is that the maximum relative change in dependant variables between successive Newton iterations is less than 10^{-5}. All the calculations have been conducted at We = 10 and for a fluid with Pr=5, corresponding to the dilute solutions of a polymer in water, the geometry of the annulus is fixed at AR=1 and K=2.

Furthermore, in order to validate the numerical code used for the present study, the steady-state solutions obtained as time-asymptotic solutions for an untilted square cavity with differentially heated sidewalls and adiabatic top and bottom walls, have been compared with the benchmark results by De Vahl Davis (1983). In particular, the average Nusselt numbers obtained at Rayleigh numbers in the range between 10^4 and 10^5, and the maximum horizontal and vertical velocity components on the vertical and horizontal midplanes of the enclosure, have been found to be within 1%-3% of the benchmark data.

In the Figs. 2 and 3 we present the variation of the mean Nusselt number as a function of the flow index for two values of Reynolds number (Re=100 and Re=200) at Gr=2000.

In the mixed convection (Re=100 or Re=200, Gr=2000), the heat transfer is more important on the case where the inner cylinder is rotated to compare with the case where the external cylinder is rotated.

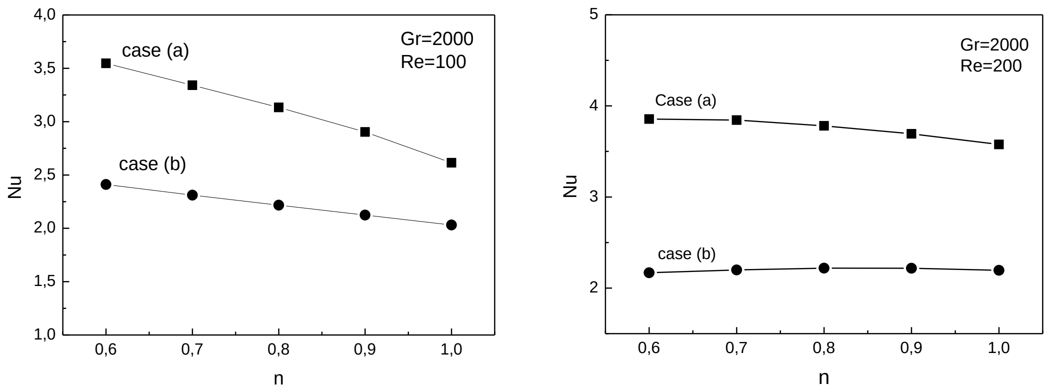


Fig. 2. Average Nusselt number as a function of the flow index at Re=100 and Re=200 for two cases.
(a) inner cylinder is rotated
(b) external cylinder is rotated

The fig. 4 shows the streamlines for the mixed convection flows. In such case of regime, the increase of the flow index leads to the apparence of a second cell flow occupying the high internal zone of the annular space in the case (a). But this second cell flow occupies the high external zone of the annular space in the case (b). The intensity of this cell flow grows with the flow index for the two cases.

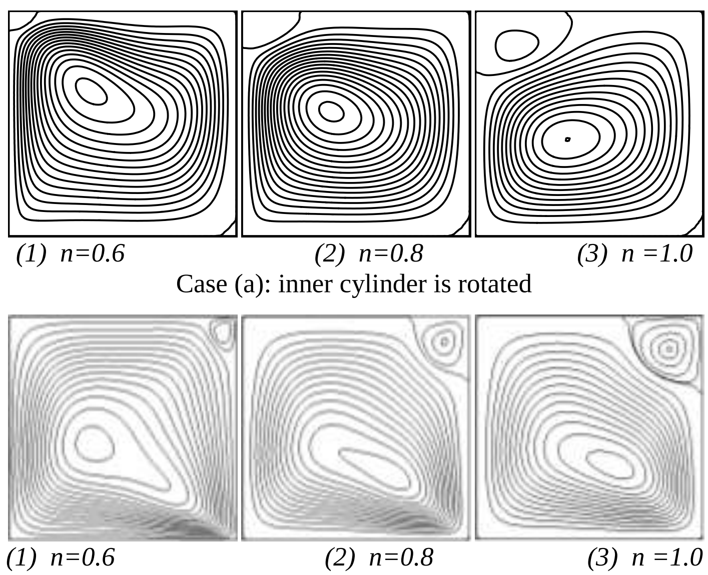
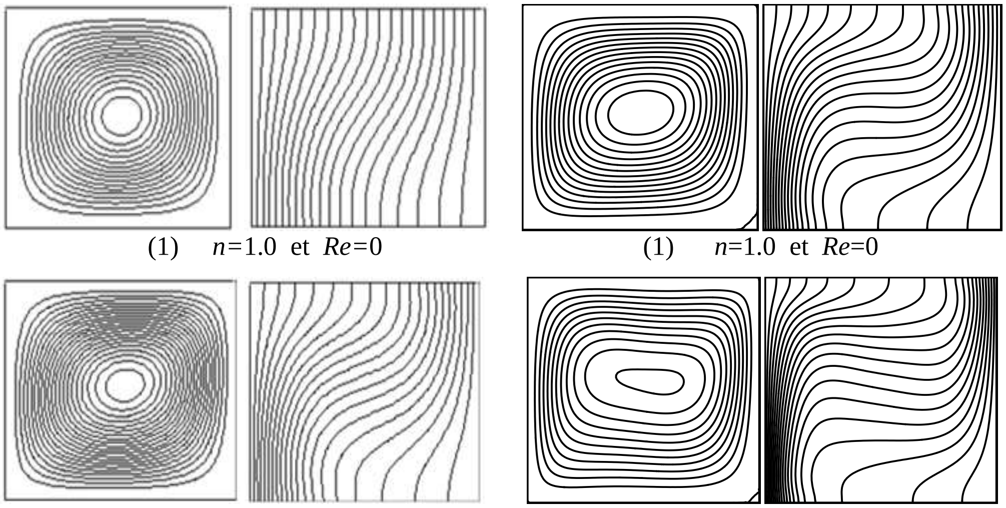


Fig. 4. Streamlines at Gr=2000

Fig. 5 displays the streamlines and isotherms for natural convection flows. We can see on Fig. 7(a) and (b), that the recirculating flow is more intense and the thermal gradients are more important on the inner cylinder for a fluid with a flow index lower than unity.



(1) n=1.0 et Re=0 (1) n=1.0 et Re=0

(2) n=0.8 et Re=0 (2) n=0.8 et Re=0

Case (a): Ra=200 Case (b): Ra=2000

Fig. 5. Streamlines and isotherms.

5. Conclusion

A numerical study of a fluid flow and a heat transfer is presented for non-Newtonian fluids confined in a differentially heated annular cylindrical space with rotating inner cylinder or with rotating external cylinder. The shifted Carreau constitutive was adopted to model the rhological fluid caracteridtics. Two flow regimes are considered according to the speed of rotation of the inner cylinder: mixed, and natural convections. The results show that the non-Newtonian effects are important on the structure of the flow and on the heat transfer. The effects of inner and outer cylinder rotation on heat transfer are examined. A parametric study of the effects of the flow index is presented to describe the flow behaviour.

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