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Pedestrian Induced Vibration Control of Bridges Based on MTMD

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Abstract **-** With the development of economy and aesthetic, the footbridges are no longer mere conduits for pedestrians across channels of traffic or water. The modern footbridge is often a focal point in the urban landscapes with its design inspired by the culture and character of the location. The introduction of new, high-performance construction materials, combined with advances in engineering design have led to ever lighter, slenderer and more elegant creations in our towns and cities. Nonetheless, the evolving demands being placed on bridge designers and the resulting daring designs being produced are not without drawbacks. This particular study delves into the discussion of pedestrian bridges and their chronological significance in initial phases. Then, the study takes into consideration about the varying forces applied on the footbridges from multi directions.

Keywords: Footbridge, Pedestrian forces, Mitigation devices, MTMD system

1. Introduction

As footbridges have become slenderer and lighter than ever before, new challenges have emerged, which are not properly addressed in the major codes of practice contemporarily in use. Currently, the prominent challenges are vibrations, both horizontal and vertical, which are induced by pedestrians walking across the bridge.

The reduction in mass to stiffness ratio of modern bridges implies that they are “livelier” than earlier structures. However, the issue of pedestrian excitation is not limited to these new structural forms. There have been several cases of bridges of various types listed in research, which have vibrated significantly under excitation from pedestrians, contributing to a feeling of discomfort experienced by the users. In the design of pedestrian bridges is that the bridges should conform to the universal design rules. No matter they are constructed on natural formations or roadway, the use of pedestrian bridges by everybody should be provided: The handicapped, old persons, children, pregnant, etc. Therefore, the vertical as well as horizontal circulation should conform to the universal design rules.

Our experience of different types of loading (for example from winds or earthquake) is constantly enhanced. Design guidance is constantly revised and risk is reduced. In cases of exceptional landmark structures, experimental studies on sophisticated models may be carried out using laboratory tools, i.e., wind tunnels, shaking tables, and centrifuges, further reducing the possibility for surprise.

2. Characteristics of Human Walking

Although human locomotion is a very complex process [1] describe walking or “gait” as a cyclic activity for which certain discrete events have been defined as significant. The following characteristics can be used to describe human walking: pacing frequency, stride length and step width.

The tests conducted by Deet et. Al [2] investigated the dynamic forces generated by mobile crowds. The forces were measured using a large instrumented force platform consisting of six independent honeycomb plates. A linear regression model was employed to derive the footstep force-time history and tests involved individuals as well as groups of two and four people walking across the platform at uncontrolled pacing frequencies and also at specified frequencies of 1.5 Hz, 1.75 Hz, 2.0 Hz, and 2.5 Hz.

Bachmann and Ammann [3] quoted their values from Matsumoto [4], which gave an average value for normal pacing frequency of between 1.5 Hz and 2.5 Hz. They also reported a slightly different value of 2.2 Hz from work done by Kramer[5] accordingly.

In order to measure the forces on a horizontal floor, a walkway was created containing a force plate, which gave a 4 meters’ lead-in and a 1-metre ‘lead-out’. Furthermore, from these tests, Kerr and Bishop[6] reported a relationship between the height of the test subject, h, and the stride length, ls, for footfall rates of 1.6 to 2.2 Hz. This relationship is defined as:

$$l_s = (0.24s)hf_s \tag{1}$$

Danion et al [7] studied the variability of stride parameters and the effect of stride length on variability. They conducted tests with 8 different test subjects at a range of pacing frequencies and found that, while human walking is in general a very consistent rhythmic motion, changes in stride length and stride frequency did have an influence on the spatial and temporal variability of gait. The Fig.1 graphically illustrates the definition of stride length.

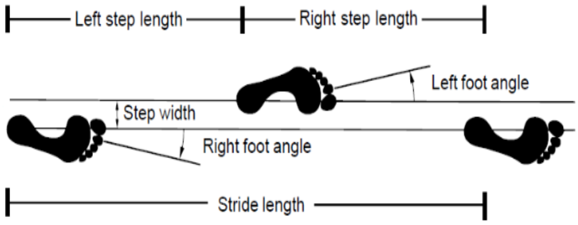

Fig. 1: Step Width & Stride Length[12]

Stride length and step width and reported this relationship between them based on the minimum metabolic cost involved in the action as:

$$w_s = 0.12 l_s \tag{2}$$

Several studies have been performed in order to quantify pedestrian walking forces. This is because until the opening of the Millennium Bridge, almost all documented problems with pedestrian-induced vibrations were associated with vertical forces and vibrations [8].

The typical pacing frequency for walking is around 2 steps per second, which gives a vertical forcing frequency of 2 Hz.

3. Vertical Forces Induced by Humans

Human walking exerts reaction forces on the walking surface in three directions. There are vertical as well as horizontal forces, which can be divided into lateral (or medio-lateral or transverse) and longitudinal (or sagittal) forces.

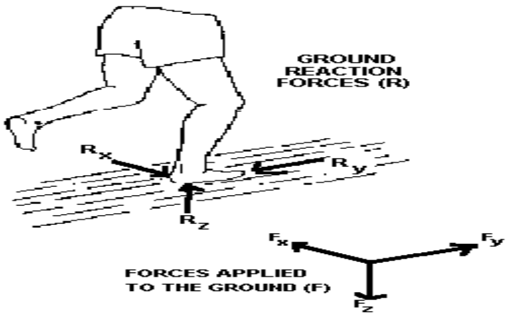

Fig. 2: Ground reaction force (forces from human walking)

The vertical force exerted from a single footstep can be quite easily measured in the laboratory by employing an effectively rigid force plate or some similar mechanism.

Several measurements have been conducted to quantify vertical loads induced by pedestrians on structures. Figure 3 shows a typical time-history for successive left and right footsteps. The human walking forces of one person are considered periodic and are represented by the following Fourier series:

$$F(t) = \sum_{i=1}^{n} G \sin(2\pi i f_p t - w_i) \tag{3}$$

where:

G is the person's weight (N); $\alpha_i$the Fourier's coefficient of the ith harmonic, i.e., dynamic load factor (DLF); $f_p$the activity rate (Hz); $\varphi_i$the phase shift of the ith harmonic; i the order number of the harmonic; nthe total number of contributing harmonics.

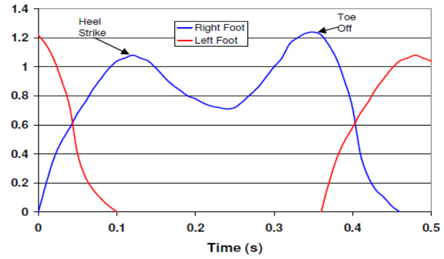

Fig. 3: Force-Time Function for Vertical Loading[1]

Figure 4 shows the total vertical force applied by both feet while walking.

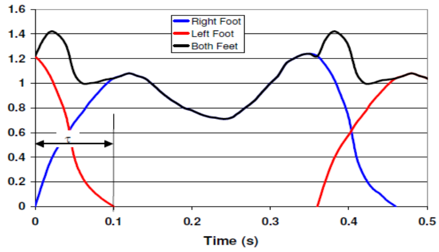

Fig. 4: Total Vertical Force from Successive Footsteps[1]

The temporal summation of successive left and right foot force-time histories yields the complete moving force-time history of a person walking. In order to do this, it is assumed that the time-history of both left and right feet is the same. Based on this assumption, one can overlap the footfall traces by an appropriate period of time, τ.

4. Horizontal Forces Induced by Humans

The earliest reported incidents of excessive lateral vibrations induced by crowds are dated back to the late 1950's, one involving a road/railway bridge in China (the Wuhan Yangtze Bridge) in 1957 and another one involving a pedestrian suspension bridge in Kiev following its opening in 1958[9]. However, the phenomenon of vibration caused by horizontal excitation has only relatively recently received attention, and there is less material published on work done in this area, and there is currently no provision for these effects in codes of practice.

A generic lateral force-time function for consecutive footsteps is shown in Figure 5. The dotted green line in Figure 6 represents a sinusoidal varying estimation of this force.

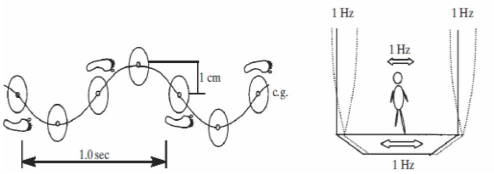

Fig. 5: Vibration mechanism

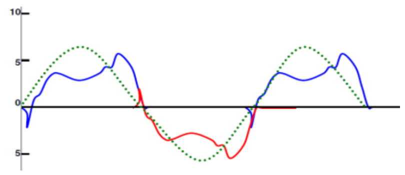

Fig. 6: Lateral force from consecutive footsteps[1]

However, lateral walking forces are often modeled as a truncated Fourier series with a fundamental frequency, f_ω, based on the simplified assumption that each foot step can be replicated from a single "characteristic" footstep

$$F(t) = \sum_{j=1}^{n} G_j \sin(2\pi j f_w t - \varphi_j) \tag{4}$$

where:

G is the person's weight (N) (usually G = 700 N); f_w is the pacing frequency (Hz); j the order number of the harmonic; n the total number of contributing harmonics.

In most cases, the load amplitude is defined through the bodyweight normalized dynamic load factor $DLF_j = G_j/W$. Despite this tremendous research effort, several questions relating to lateral pedestrian induced vibrations, which still remain unanswered.

A measurement of continuous walking has also been made. The measured time histories were near periodic with an average period equal to the average step frequency. General shapes for continuous forces in both vertical and horizontal directions have been constructed assuming a perfect periodicity of the force, see Figure 7[10].

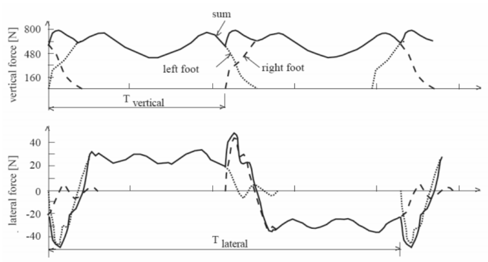

Fig. 7: Periodic walking time histories in vertical and horizontal directions[10]

5. Tuned Mass Dampers (TMD)

A Tuned Mass Damper (TMD), also called a "harmonic absorber", is a device mounted to a specific location in a structure, so as to reduce the amplitude of vibration to an acceptable level whenever a strong lateral force hit. There are two basic types of TMD; the Horizontal TMD, which is normally found in slender buildings, communication towers, spires and the like. The other type is the Vertical TMD, which is usually applied in long span horizontal structures such as bridges, floors and walkways. Both types have similar functions, though there might be slight differences in terms of mechanism.

5.1 How does a TMD work

Tuned Mass Damper (TMD) is among the oldest types of vibration absorbers. It was invented by Frahm in 1909 (Den Hartog (1956) and Chopra (2012)), which is described in detail in the literature[11] and [12] respectively. A TMD essentially has a mass, spring and a damping device, which dispels the energy caused by the motion of the mass itself. A simple undamped structure with attached TMD behaves like a simple two degree of freedom system without damping. The mechanism can be illustrated by the schematic diagrams below:

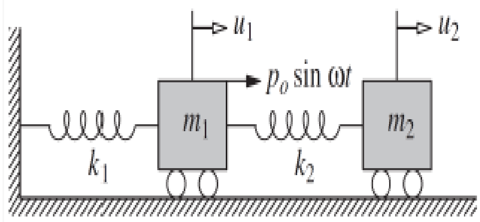

Fig. 8: A two DOF system

The equation of motion of such a system Figure 8 exited by a harmonic force of $\mathrm{P(t)} = P_0 \sin(\omega t)$ which act on the first mass (i.e., the main structure), can be written as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & -k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} p_0 \\ 0 \end{bmatrix} sin(\omega t) \tag{5}$$

The forced vibration of this system in the steady state condition can be assumed as:

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} u_{1_0} \\ u_{2_0} \end{bmatrix} sin(\omega t) \tag{6}$$

Substituting Eqs. (6) in equation Eqs. (5), the result would be an algebraic equation:

$$\begin{bmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & -k_2 - m_2\omega^2 \end{bmatrix} \begin{bmatrix} u_{1_0} \\ u_{2_0} \end{bmatrix} = \begin{bmatrix} p_0 \\ 0 \end{bmatrix} \tag{7}$$

or

$$[k - \omega^2 m] \begin{bmatrix} u_{1_0} \\ u_{2_0} \end{bmatrix} = \begin{bmatrix} p_0 \\ 0 \end{bmatrix} \tag{8}$$

that can be solved for u_{1_0} and u_{2_0} as:

$$\begin{bmatrix} u_{1_0} \\ u_{2_0} \end{bmatrix} = [k - \omega^2 m]^{-1} \begin{bmatrix} p_0 \\ 0 \end{bmatrix} \tag{9}$$

that gives:

$$\begin{cases} u_{1o} = \frac{p_o(k_2 - m_2\omega^2)}{m_1 m_2(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} \\ u_{2o} = \frac{p_o k_2}{m_1 m_2(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} \end{cases} \tag{10}$$

By defining the following dimensionless symbols:

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} \text{an d} \quad \omega_2 = \sqrt{\frac{k_2}{m_2}} \text{ and } \mu = \frac{m_2}{m_1} \tag{11}$$

in which ω_1, ω_2 and μ are: the natural frequency of the first mass, the natural frequency of the second mass and the mass ratio of the two masses respectively. By substituting these symbols in the Eqs. (10), we can obtain:

$$\begin{cases} u_{1o} = \frac{p_o}{k_1} \frac{1-(\omega/\omega_2)^2}{[\,1+\mu(\omega_2/\omega_1)^2 - (\omega/\omega_1)^2][1-\mu(\omega/\omega_2)^2] - \mu(\omega_2/\omega_1)^2} \\ u_{2o} = \frac{p_o}{k_1} \frac{1}{[\,1+\mu(\omega_2/\omega_1)^2 - (\omega/\omega_1)^2][1-\mu(\omega/\omega_2)^2] - \mu(\omega_2/\omega_1)^2} \end{cases} \tag{12}$$

According to the first part of this equation, u_{1o} is zero, when $1 - (\omega/\omega_2)^2$ is zero. In other words, when the natural frequency of the second mass (i.e., the vibration absorber) is equal to the frequency of the force, the vibration of the first mass (i.e., the main structure) is zero. Thus, by tuning the vibration absorber the frequency of the external force, vibration of the main structure can be prevented. Figure 10 shows the response amplitude of the first mass m_1, when ω/ω_2(i.e., the second mass tuned to the natural frequency of the main mass), and the mass ratio is u_{2o}= 0:2and $u_{1sto} = p_o = k_1$.

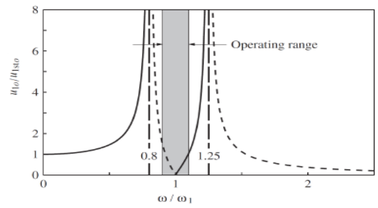

Fig. 9: Response amplitude of the first mass versus the exiting frequency whenω/ω_1andu_{1o}/u_{1sto} = 0:2 [12]

Furthermore,

$$u_{2o} = -\frac{p_o}{k_2} \tag{13}$$

that means, while the motion of the first mass is zero, the vibration of the second mass is p_o=$k_2$sin(ωt), which produce the force of p_o sin(ωt) in second mass spring that is in fact equal and opposite to the external force. This behavior is the main concept behind the employment of the TMDs, in which by attaching a tuned relatively small mass to the main structure, the undesired vibration due to an external force can be prevented.

Eqs.(13) also imply that since the force acting on the absorber is:

$$\mathrm{k_2 u_{2o}} = \omega^2 \mathrm{m_2 u_{2o}} = -\mathrm{p_o} \tag{14}$$

The mass and stiffness of the TMD depend on the permissible motion of the TMD, since the smaller mass led to larger amplitude of the TMD. Moreover, while the larger mass of the TMD is difficult to handle, the smaller mass decreases the operating range of the TMD.

TMD can be highly effective in suppressing the resonance response due to the external force, when it is tuned to the resonance frequency. Den Hartog[13] (1956) suggested a method to determine the optimal parameters of a damped TMD that attached to an undamped SDOF system, to minimize the steady state response of the main mass subjected to a harmonic load as following:

$$f = \frac{1}{1+\mu} \tag{15}$$

$$\zeta = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \tag{16}$$

Where f and ζ are frequency ratio and damping ratio of the TMD, defined as:

$$f = \frac{\omega_2}{\omega_1} \qquad and \qquad \zeta = \frac{c}{2\sqrt{k_2 m_2}} \tag{17}$$

where, ω_2, k_2, m_2 and c, are the natural frequency, stiffness, mass and the damping of the TMD respectively and $\omega_1$is the natural frequency of the main structure. In the structures with damping, however, the optimal TMD parameters should be obtained by using numerical methods[14].TMDs have been applied in tall buildings to mitigate the vibration induced by wind and moderate earthquakes. Gutierrez Soto and Adeli (2013) presented several real cases of application of TMDs in existing high-rise buildings and towers around the world as described in their relevant study in the research[15]. Lin et al.[16] investigated the seismic application of the TMDs and suggested the optimal design parameters of the TMDs in their study accordingly.

To summarize, the main idea of the system is to match the frequency of the TMD to the inherent frequency of the structure itself, that is to say, if the structure's frequency is 0.2 Hz for example, the TMD should be designed to a frequency that is exactly the same or close to this value, hence "tuned". If this is achieved, then the TMD will be effective and the vibration of the structure will be controlled, likewise, acceleration will subside more efficiently.

Multiple tuned mass dampers (MTMDs) are more effective and more robust than single tuned mass dampers (STMDs). Previous studies have shown the excellent effectiveness of the MTMDs in suppressing the footbridge vibrations induced by pedestrian induced forces such as walking, running, and jumping excitations with different frequencies.

6. MTMD System

An MTMD system, which consists of several TMDs with different frequencies, has a wide application in the vibration control of footbridges.

6.1 Schematic Diagram and Dynamic Analysis

For a simply supported footbridge, the first vertical mode is usually dominated. Therefore, footbridge can be simplified as a single-degree-of-freedom (SDOF) system. When it is coupled with an MTMD system, the schematic diagram is shown in Figure 12, under a vertical harmonic excitation.

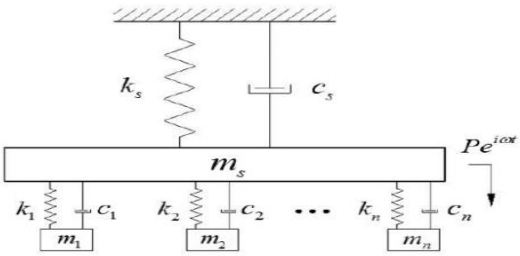

Fig. 10: The schematic diagram of a single-degree-of-freedom (SDOF) primary structure with a multiple tuned mass dampers (MTMD) system[17]

$$\begin{bmatrix} m_s & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & 0 & m_n \end{bmatrix} \begin{bmatrix} \ddot{x_s} \\ \ddot{x_1} \\ ... \\ \ddot{x_n} \end{bmatrix} + \begin{bmatrix} c_s + \sum_{j=1}^{n} c_j & -c_j & 0 & -c_n \\ -c_1 & c_1 & 0 & 0 \\ 0 & 0 & ... & 0 \\ -c_n & 0 & 0 & c_n \end{bmatrix} \begin{bmatrix} \dot{x}_s \\ \dot{x}_1 \\ ... \\ \dot{x}_n \end{bmatrix} + \begin{bmatrix} k_s + \sum_{j=1}^{n} k_j & -k_j & 0 & -k_n \\ -k_1 & k_1 & 0 & 0 \\ 0 & 0 & ... & 0 \\ -k_n & 0 & 0 & k_n \end{bmatrix} \begin{bmatrix} x_s \\ x_1 \\ ... \\ x_n \end{bmatrix} = \begin{bmatrix} Pe^{i\omega t} \\ 0 \\ ... \\ 0 \end{bmatrix} \tag{27}$$

In Eqs. (27), j is the numerical order of a TMD in the MTMD system. Under a harmonic excitation, the dynamic response in the steady state can be written as

$$\begin{cases} x_s = H_s e^{i\varpi t} \\ x_j = H_j e^{i\varpi t} \end{cases} \tag{28}$$

In Eqs. (28) , H_s and H_j are the response amplitude of the primary structure and a TMD with the numerical order j, respectively. To analyze the relationship between the primary structural response and the parameters of an MTMD system, H_s can be firstly calculated as:

$$H_s = \frac{P}{k_s - \varpi^2 m_s + i\varpi c_s - \varpi^2 \sum_{j=1}^{n} m_j \frac{k_j + i\varpi c_j}{k_j + i\varpi c_j + \varpi^2 m_j}} \tag{29}$$

For simplification, it is defined that primary structural circular frequency $\omega_s = \sqrt{\frac{k_s}{m_s}}$;circular frequency of the TMD with the numerical order j,$\omega_j = \sqrt{\frac{k_j}{m_j}}$;damping ratio of the primary structure$\zeta_s = \frac{c_s}{2m_s\omega_s}$;excitation frequency ratio$g = \frac{w}{\omega_s}$;TMD frequency ratio$f_j = \frac{\omega_j}{\omega_s}$;;single TMD mass ratio with the numerical order j$\mu_j = \frac{m_j}{m_s}$;MTMD mass ratio$\mu = \sum_{j=1}^{n} \mu_j$;central frequency ratio$f_T = \frac{\omega_T}{\omega_s}$ where ω_T is the central frequency of the TMDs in an MTMD; frequency bandwidth ratio $X = \frac{\omega_n - \omega_1}{\omega_T}$, where $\omega_n$is the circular frequency of the last TMD with the maximum frequency, and ω_1 is the circular frequency of the first TMD with the minimum frequency; TMD damping ratio$\zeta_s = \frac{c_s}{2m_j\omega_j}$,where $m_j$and $k_j, c_j$are the mass, stiffness and damping coefficient of the TMD with the numerical order j, and it is assumed that all TMDs have the same damping ratio.

As for the serviceability problem, the primary structural acceleration response is usually used as the evaluation index. Then, the acceleration dynamic amplification factor can be calculated as:

$$DMF_{acc} = \frac{\mathrm{g}^2}{\sqrt{R_z^2 + I_z^2}} \tag{30}$$

In Eqs. (30) , R_z and I_z mean that

$$\begin{cases} R_z = 1 - \mathrm{g}^2 - \mathrm{g}^2 \sum_{j=1}^{n} \mu_j \frac{f_j^2\left(f_j^2 - \mathrm{g}^2\right) + (2\xi_j f_j \mathrm{g})^2}{(f_j^2 - \mathrm{g}^2)^2 + (2\xi_j f_j \mathrm{g})^2} \\ I_z = 2\xi_s \mathrm{g} + \mathrm{g}^4 \sum_{j=1}^{n} \mu_j \frac{(2\xi_j f_j \mathrm{g})^2}{(f_j^2 - \mathrm{g}^2)^2 + (2\xi_j f_j \mathrm{g})^2} \end{cases} \tag{31}$$

Therefore, in order to decrease the primary structural acceleration response, it is necessary to discuss and optimally design these parameters. Considering that the damping ratio of the slender footbridge is low and for simplification, in the following parametric discussion, $\boldsymbol{\xi_s}$ is ignored and set to be zero.

7. Conclusion

This particular study initiates by taking into consideration about the historical prospective of contemporary footbridges. After describing the introductory literature, the study focuses on the types of multifarious footbridges which are employed on global scale. In addition, the study focuses on the varying forces applied on the bridges from multi-dimensions. The study then concentrates on employment of multiple mitigation devices and given detail about the relevant force being applied to achieve such mitigation. Finally, the study presents certain modeling approaches which are critical for the computations of varying footbridges in contemporary employments.

Recommendations for Future Work

My future course of action will comprise, but not limited, to the following: I will take into consideration the other types of bridges, i.e., steel or concrete bridges which are contemporarily employed for the heavy traffic. Similarly, my approach of mitigation will focus on the aforementioned concrete and/or steel bridges.

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