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Impedance of Vertically Oscillating Circular Rigid Grillage Foundation on Half-Space

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Abstract **-** The dynamic impedance functions (force-displacement relationships) of a rigid circular grillage foundation resting on viscoelastic soil half-space is investigated in the context of linear finite element (FE) analysis. The rigid foundation is bonded to the half-space and is subjected to time-harmonic vertical excitations. The foundation is idealized as rigid massless single and multiple circular rings with varying spacing, and widths, and the results are compared against theoretically established rigid circular massless disk foundations. The half-space under consideration consists of a viscoelastic homogenous half-space with shear wave velocity of 200 m/s. The study is motivated by the need to develop an understanding of the behavior of skid-mounted vibratory machines supported directly on compacted grade, with varying contact areas with the supporting soil. Supporting equipment skids on compacted grade allows for easy relocation with minimal field effort and minimal cost, which is an emerging trend in multiple industries such as the oil & gas and utilities industries. The results revealed that circular grillage foundations behave similar to circular disk foundations with the same exterior diameter at low exciting frequencies; and behave similar to circular disk foundations with an equivalent diameter, derived from the net contact area, at high frequencies of excitation.

Keywords**:** Impedance, Grillage Foundation, Low Strain, Finite Element

1. Introduction

Most pump-houses in the oil & gas and utilities industries are designed as modular structures. These pump-houses are typically supported on piled foundations or mat foundations and are not designed to be relocatable. The emerging trend in these industries, however, is to use standardized modular pump-houses or skids supported on gravel pad foundations to allow relocation to other areas with minimal field work and costs. Because skids do not have solid cross-section (i.e., grillage foundation), their contact area with supporting soil are less than their gross area. Therefore, it is necessary to evaluate the behavior of skid-mounted vibratory machines supported directly on compacted grade with varying contact areas with the supporting soil.

Machine foundations are designed to withstand the static loads due to supported equipment weight and other live loads, and to perform satisfactorily when exposed to dynamic loads due to equipment operational dynamic loads. The response of the foundation to these dynamic loads depends on their frequencies (which is a function of machine speed) and the dynamic characteristics of the foundation system (i.e., stiffness and damping constants). Thus, thorough evaluation of foundation dynamic characteristics, excitation forces and associated frequencies are necessary for correct assessment of machine performance.

The dynamic behavior of vertically excited rigid circular or rectangular foundation on viscoelastic or elasto-viscoplastic soil half-space is well investigated. Bycroft [1] determined the impedances of a rigid circular plate attached to the free surface of a semi-infinite elastic space in vertical and horizontal translations as well as rotations about the horizontal and vertical axes. The variation of the plate impedance with frequency along the vertical and rocking degrees of freedom indicates that stiffness decreases as the frequency increases in a parabolic fashion, especially as Poisson's ratio of foundation soil increases, while radiation (geometric) damping increases proportional to frequency. Meanwhile, variation of stiffness along the horizontal axes with frequency is minimal. Resonance may occur in the machine-foundation system when the excitation frequency matches the natural frequency of the of the system along any of its degrees of freedom, and the response can be unacceptable if the available damping ratio is low.

Vrettos [2] investigated the vertical and rocking responses of rigid rectangular foundations resting on a linear-elastic, compressible, non-homogeneous half-space soil model. The soil non-homogeneity is described by a continuous yet bounded increase of shear modulus with depth. The mixed boundary value problem is solved by means of a semi-analytical method that involves subdividing the foundation/soil contact area into sub-regions and determining their influence functions by integrating the corresponding surface-to-surface Green's functions. Impedance functions are given for representative values of the soil shear modulus and Poisson's ratio and the foundation geometry over a wide range of frequencies. The method can be extended to treat foundations of arbitrary shape as well as of finite rigidity.

Finite element (FE) modelling is used to determine stiffness and damping of embedded foundations [3-5]. The derivation of dynamic FE formulations and the introduction of viscous and transmitting boundaries made it possible to simulate response of infinite media to dynamic loading reasonably, and to analyze the response of surface and embedded foundations. Therefore, the dynamic response of foundations with complex geometry can therefore be readily evaluated employing dynamic finite element formulations, and it can be used to validate and calibrate analytical solutions. Realistic simulations of machine foundation problems were successfully achieved considering plane strain and axisymmetric simplifications. Borja [5] investigated the dynamic response of vertically excited rigid foundations on an elastic/viscoelastic half-space in employing FE analysis. Time-domain analyses can be used to investigate linear and nonlinear responses of vertically oscillating circular and square foundations to harmonic loads using two- and three-dimensional FE models.

Numerous studies investigated the dynamic behaviour of vibratory machines supported on rigid circular or rectangular foundations. However, to the Authors knowledge, the dynamic behaviour of vertically excited grillage foundation resting on viscoelastic soil half-space has not been investigated. Therefore, this paper investigates the dynamic impedance functions (force-displacement relationships) of a rigid circular grillage foundation resting on viscoelastic soil half-space employing dynamic FE analysis. In this study, the grillage is simulated by considering single and multiple circular rings with varying spacing and widths resting on soil half space.

2. Finite Element Model

The vertical impedance of a grillage shallow foundation was investigated by comparing the vertical impedance of a rigid circular ring foundation against the well-established impedance values of a circular rigid foundation. Finite element models (FEM) were developed employing the general finite element program ABAQUS [6] to estimate the dynamic response and corresponding vertical impedance of a massless rigid circular grillage foundation resting on an elastic half-space. The finite element models were two-dimensional exploiting the axisymmetric nature of the problem.

The soil half-space was numerically approximated as a finite domain with defined boundaries. The domain boundaries were placed far enough from the source of oscillations in order to reduce any wave reflections at the boundaries, which may affect the accuracy of the results. This was achieved by varying the domain size incrementally until the results were no longer affected by the proximity of the boundaries. This approach tends to yield reliable results [5]. Alternatively, transmitting boundaries with energy dissipation mechanism (viscous damping) or the infinite element boundaries [6] could be used to further reduce the boundary effects. A combined approach considering the former approach and the infinite element boundary approach was implemented in the current study. The mesh was refined sufficiently by reducing the elements size to ensure proper wave propagation throughout the elements. The maximum element size is a function of the wavelength, which in turn is a function of the soil stiffness and the frequency of excitation. The maximum element size was 1/5 to 1/8 of the shortest wavelength of the dynamic problem [7].

The developed 2D axisymmetric finite element model is shown in **Error! Reference source not found.**. The domain was discretized into finite linear quadrilateral elements with reduced integration, (CAX4R), and infinite linear quadrilateral elements (CINAX4) as shown in **Error! Reference source not found.**. The foundation was modelled as massless rigid circular disk tied to the ground surface. The soil was modelled as linear elastic material, which was considered appropriate for the level of load and associated soil shear strain produced by typical vibrating equipment such as pumps. The soil properties considered in this study are summarized in Table 1. A wide range of excitation frequencies is considered up to 65 Hz to capture the most common operating frequencies of pumps. The element size was limited to 100 mm or less within an area extending to a distance equal to the foundation diameter (D) around the foundation and did not exceed 300 mm to a zone extending 10 D from the center of the foundation. The maximum element size everywhere else in the domain was less than 850 mm.

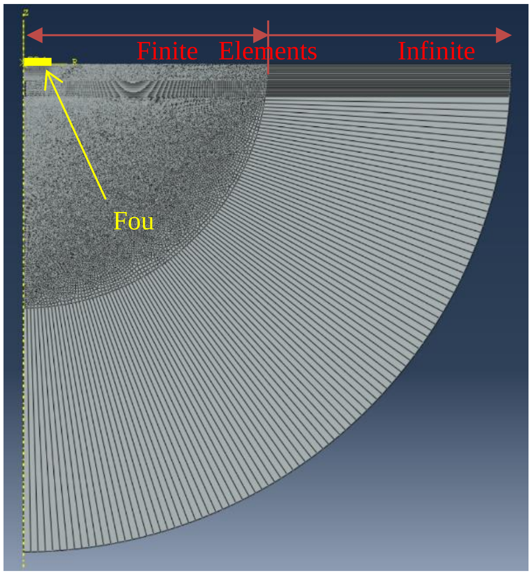

Fig. 1 Numerical Domain

Table 1 Dynamic soil properties considered in the analysis

Density (kg/m^3)	Shear Modulus, G_{max} (MPa)	Poisson's ratio	Damping (%)	Shear Wave Velocity, Vs (m/s)
2000	80	0.3	3	200

3. Analysis And Discussion

The analysis was divided into three steps: first the numerical FEA results of a massless rigid circular foundation resting on elastic half-space were compared against theoretical solutions. The theoretical solutions were obtained with the aid of the program DYNA6 [8]. This step validated the adequacy of the numerical model including adequate domain size, element type, mesh density, and boundary conditions. DYNA6 [8] program calculates the impedance of a rigid circular solid foundation resting on visco-elastic half-space soil medium employing the model proposed by Veletsos et al. [9 - 11].

The second step was to perform a FEA parametric study by varying the geometry of the foundation as well as the shear modulus of the half-space soil. The third and final step involved comparing the results of the different foundation geometries against the classical solution of rigid circular foundation resting on half-space.

In the validation step, the considered foundation radius was 2 m. The domain size (R) was varied to 40 m, 80 m, and 120 m in three separate analyses. The results were compared to the theoretical values obtained by DYNA6 and are shown in Fig. 2.

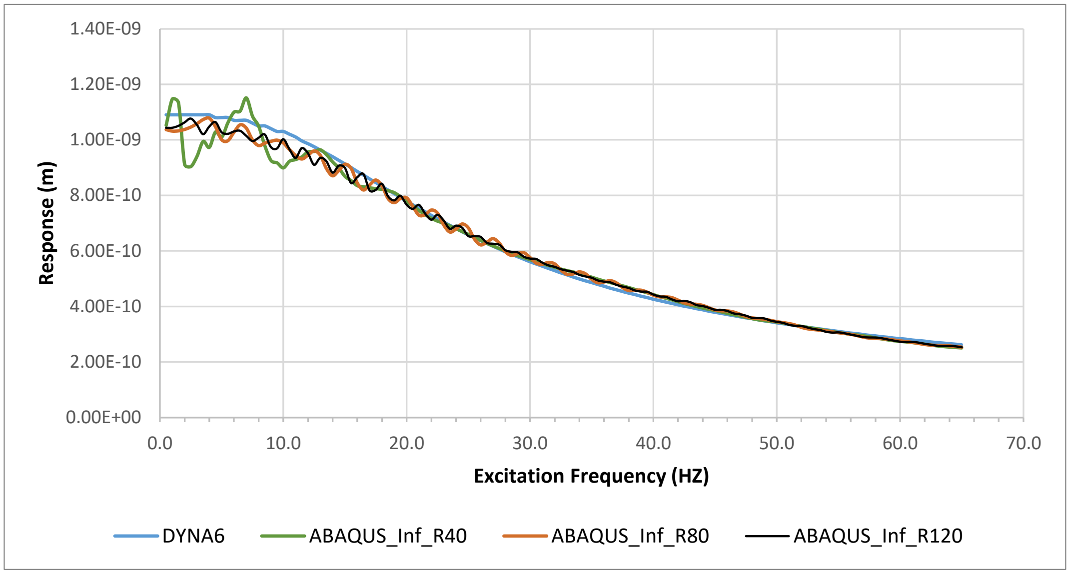

Fig. 2 The response of a rigid massless circular foundation modeled in ABAQUS considering different domain sizes, and theoretical values obtained by DYNA6

As shown in Fig. 2, the FEA results are almost identical to the theoretical results, regardless of the domain size at frequencies greater than 20 Hz, and the response varied depending on the domain size at low frequencies up to 20 Hz. This observation is reasonable given that the wavelength is largest at low frequencies allowing the waves to reach the finite boundaries while the wavelengths are shortest at high frequencies which results in significant attenuation of the waves before they reach the finite boundaries. It is also observed that the response contains large oscillations for the smallest domain size and less oscillations at the largest domain size. The oscillations are a result of wave reflections into the domain. Furthermore, it is observed that the response for domain sizes R80 and R120 are almost identical.

The impedance function (i.e., stiffness and damping) of the foundation was also calculated and is compared against the theoretical values. As presented Fig. 2, similar to the observations made to the response, as the domain size increases, the FEA values approach the theoretical values. Therefore, it can be concluded that the FEA yields reliable results and that any increase in domain size beyond 120 m will have marginal improvement to the results. Therefore, domain size R120 is considered appropriate for this study.

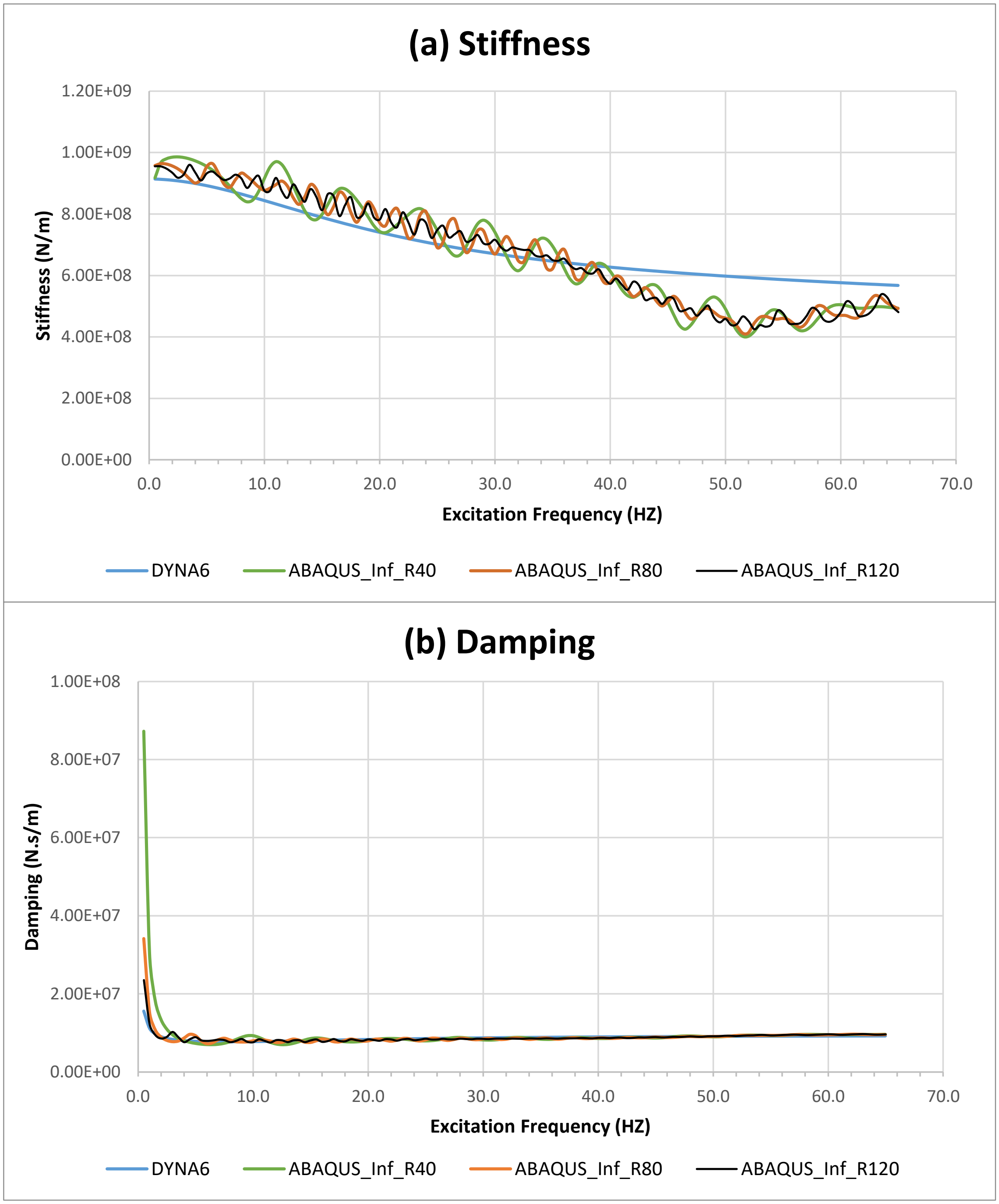

Fig. 3 Calculated impedance functions of a rigid massless circular foundation considering different domain sizes compared with theoretical values obtained by DYNA6: a) stiffness and b) damping

The second step of the analysis was performed considering the domain and mesh size that were established in the validation step. In addition, the following foundations geometries shown in Fig. 4 were considered in the analysis: a circular foundation with 2 m radius, a ring foundation with an internal radius equal to 1.5 m and outer radius of 2 m; and a circular foundation with 1.323 m radius. The second and third foundations have the exact same contact area with the soil surface.

The three foundation sizes above were chosen to replicate the ratio of full area to contact area associated with a real-life application of a rectangular steel skid foundation, which has a full (i.e. footprint) area equal to 116.5 m2 and a contact area with the soil equal to 50.5 m2. The area ratio of such foundation is equal to approximately 0.43. The area ratio between foundation FDN-1 and FDN-2, or FDN-3 foundations is approximately 0.44 which is almost identical to the real-life application.

Given that the numerical model was validated against theoretical values, for simplicity the response and impedance of solid foundations FDN-1 and FDN-3 were obtained theoretically employing DYNA6. The response and impedance of ring foundation FDN-2 was obtained numerically via FE analysis. The results shown in Fig. 5 indicate that the response of a ring foundation (FDN-2) is identical to the response of a solid foundation with the same full area (FDN-1) at low frequencies, and that the response of a ring foundation (FDN-2) approaches that of an equivalent solid foundation with the same contact area (FDN-3) at high frequencies.

This observation seems reasonable given that at low frequencies (up to 40 Hz) the wavelength is large, which implies that the ring foundation, including the soil within the ring, will move together in phase as one foundation, resulting in an effective foundation with an area bounded by the exterior perimeter of the ring foundation. The shear wavelength at 40 Hz is approximately 200 m/s / 40 Hz = 5 m which is close to the outer diameter of FDN-2.

On the other hand, at high frequencies (starting at about 62 Hz), the wavelength decreases to a point where the ring foundation is moving out-of-phase with the soil within the ring, which can be approximated by a solid foundation with the same contact area as the ring foundation. The shear wavelength at 62 Hz is approximately 3 m which is equal to the inner diameter of FDN-2.

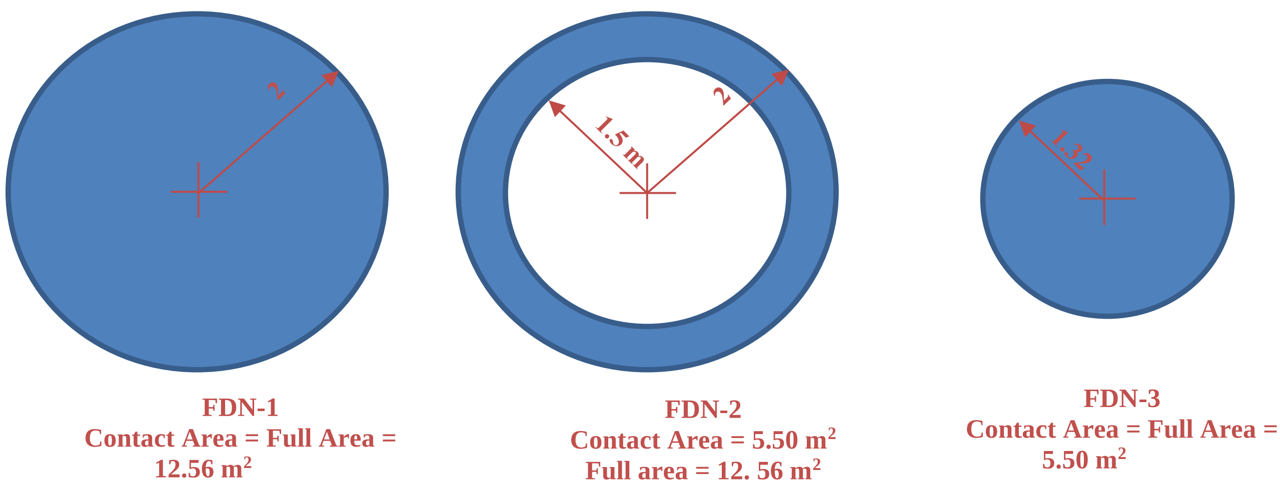

Fig. 4 Foundation geometries considered in the analysis

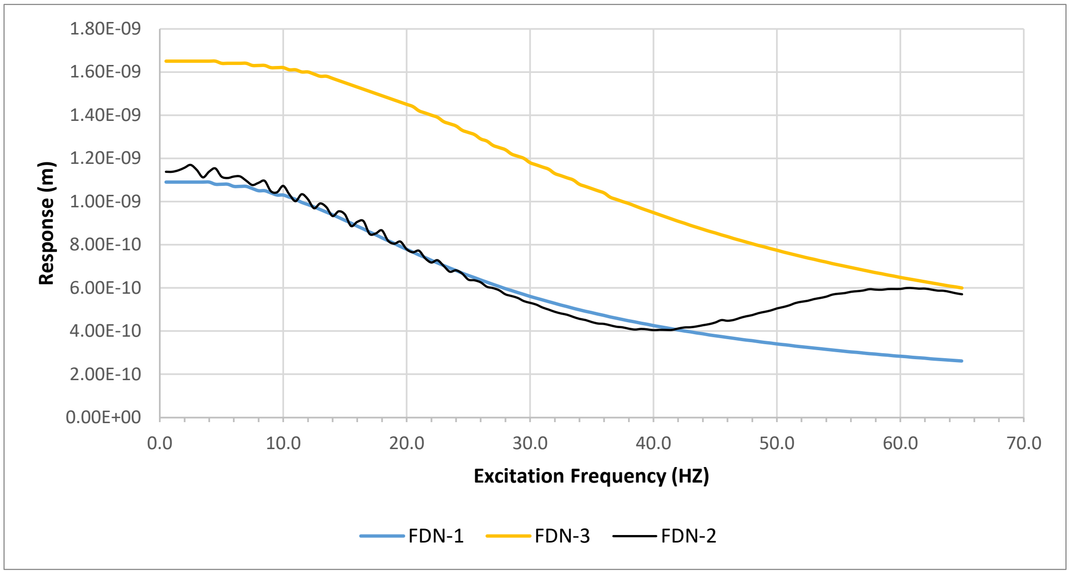

Fig. 5 Response of foundations FDN-1, FDN-2, and FDN-3

The foundation impedance calculated considering the different geometries are compared in Fig. 6. The damping values follow the same observations made to the response (i.e., at low frequency, the ring foundation damping is almost identical to a solid foundation with the same full area; and at high frequencies, the ring foundation damping is almost identical to a solid foundation with the same contact area). The stiffness variations, however, do not follow the same trends as the response and damping. This is attributed to the fact that the stiffness increased as the response of the soil within the ring became increasingly out-of-phase with the ring foundation response up to a peak value at which the ring foundation is 180 degrees out-of-phase with the soil within the ring. This behavior is similar to the behavior of foundation groups within close proximity.

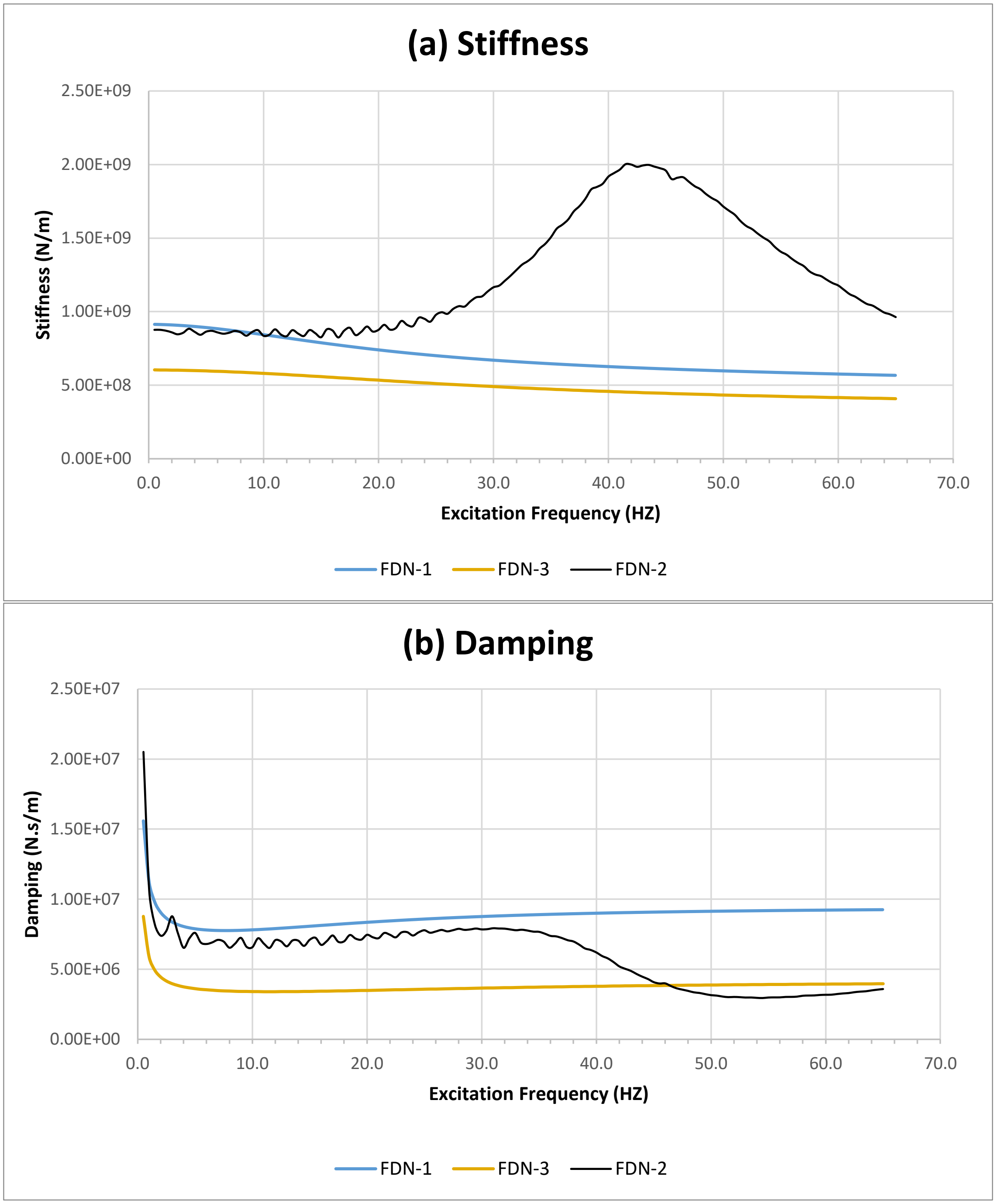

Fig. 6 The estimated damping and stiffness of foundations FDN-1, FDN-2, and FDN-3: a) stiffness and b) damping

4. Conclusions

The vertical impedance and response of a grillage foundation due to small strain steady-state harmonic excitation is dependent on the excitation frequency, the ratio of full area to contact area, the dimensions of the grillage foundation and the stiffness of the supporting soil medium.

It is found that if the shear wavelength is greater than the largest dimension of the soil area within the grillage, that the vertical performance of the grillage foundation may be approximated with the impedance of a foundation that has the same full area as the grillage foundation. However, if the shear wavelength is less than the smallest dimension of the soil area within the grillage, the grillage foundation vertical behaviour may be approximated by the impedance of a foundation that has the same contact area as the grillage foundation.

For shear wavelengths between the two scenarios above, the vertical response may be approximated by considering a foundation that has the same contact area as the grillage foundation. This approach should result in a conservatively high vibration response. However, if the foundation natural frequency is the main design consideration, a finite element analysis shall be conducted to capture the exact foundation geometry. Alternatively, the impedance of both foundation cases may be considered, provided that the excitation frequency is sufficiently greater than the natural frequency obtained from the impedance of the full area approach (i.e., FDN-1) or lower than the natural frequency obtained from the impedance of the contact area approach (i.e. FDN-3).

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