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The Natural Flow of Stress in Shell Structures

Mitchell Gohnert
School of Civil and Environmental Engineering,
University of the Witwatersrand
P.O. Wits, Johannesburg, South Africa
mitchell.gohnert@wits.ac.za

Abstract **-** Structural optimization has been the subject of intense research for many years. However, the engineering profession seems to be oblivious to the important role of natural structures. Natural structures are those shapes created by nature and possess geometrical properties that are resistant to environmental conditions. Engineers of antiquity have recognized the importance of natural shapes and have incorporated these designs in many historical political and religious structures. Many of these structures are catenary or funicular in form, and are characterized by a pure compressive stress distribution—an essential requirement for masonry structures. Understanding the natural flow of stress provides insight in how historic structures are able to stand robust and stable.

Keywords**:** Catenary, shells, natural stress flow, natural structures, compression structures

1. Introduction

The evolution of structures has progressed from trial-and-error to structures with a scientific, experimental and theoretical basis. Over time, the engineering profession has developed an understanding of structures, expressed in mathematical or physical forms. Our technical abilities have expanded exponentially, with the development of new structural materials and shapes. Is it possible that our rapid path of development has blinded our eyes from reason and logic? Sometimes we fail to see the obvious, even when it stares us in the face. Fig. 1 is a photograph taken after the 2010 earthquake in Haiti. In the background, the Haiti National Palace lays in ruin. In the foreground, scores of the homeless inhabit the street. As engineers, it is natural to focus on the collapsed structure and form a hypothesis as to the cause of failure. However, we fail to see the obvious—the manmade structure has failed, but the natural structures have survived—the palm trees are still standing, without any outward signs of distress. Perhaps we will never work and live in palm trees, but the shape and flexibility of this natural structure is perfectly resistant to earthquakes and significant meteorological occurrences— thus, a structure based on nature is a more appropriate form for the Haitian environment, an gives us insight in how structures should be designed.

Natural structures have, perhaps, evolved over millions of years into optimum shapes that are resistant to local environmental conditions. Would it not be prudent to learn from these structures and emulate them in our designs [1]? Will an understanding of the natural flow of stress explain why historic structures have survived the test of time, or in danger of collapse?

Fig. 1: Manmade and biological forms in an earthquake zone (Wikimedia Commons file, Logan Abassi)

Nature possesses and persistently reproduces several universal features—all natural structures have curved surfaces and solid forms are not solid, but composed of cellular or tubular shell like substructures. As an example, Fig. 2 is a cross-section of a dinosaur femur bone and Fig. 3 is the cross-section of a Black Walnut leaf. In these examples (fauna and flora), the structure and sub-structure of the natural forms are curved shaped and composed of shells. The curvature of the shapes is directly related to the flow of stress, dictated by the imposition of environmental conditions. The curvature of the femur bone constitutes resistance to the action of walking, running, jumping, etc. Leaves, which are miniature solar panels engaged in the act of photosynthesis, are designed to resist wind flutter, rain, snow and the invasion of insects. Each shape takes on a suitable form, to optimize the resistance to environmental forces and to ensure the species ultimate survival.

Fig. 2: Cross-section of a dinosaur femur bone, illustrating the cellular form of the solid (photo by Ken Kolb)

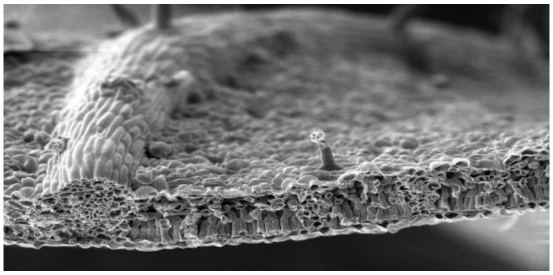


Fig. 3: Electron microscope photograph of a Black Walnut leaf illustrating the cellular and curved substructures through the thickness (Photo by Dartmouth College)

2. Examples from nature—the egg

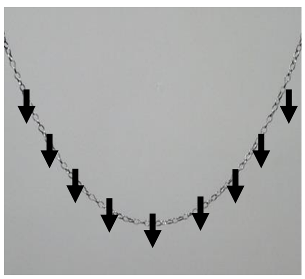
The chicken egg, similar to other eggs, has symmetry in one direction. The shape is referred to as an ovoid, which resembles a sphere at one end and an elongated sphere (or catenary) at the other, joined by an equator. The catenary end is in the direction of travel in the birth cannel (as the egg is laid), and is designed for impact. The catenary shape, dramatically improves the survival of the species and the prevention of damage to the eggshell. Furthermore, the circular shape is highly resistant to internal pressure, imposed during the "hatching" or incubation period.

The radius to thickness ratio of the chicken egg is about 100 [2]. The structural performance of the chicken egg is exceptional, despite the material of the shell being composed of a relatively weak material (calcium carbonate crystals), and is semipermeable. Yet, the strength of an egg is, without question, the result of its shape. If one takes an egg and squeezes it in hand, membrane stresses are produced in the shell, and it is almost impossible to break it. To crack an egg, a deliberate sharp blow is required. Despite having a large radius to thickness ratio, an egg is able to resist loads that far exceed typical scaled up manmade structures. However, it is not the egg shell material that resists the loads, but its strength is derived from its shape. Thus, the key to structural strength lies primarily in its shape. Most importantly, the flow of stress matches perfectly the geometrical shape of the egg.

3. Pure compression shapes

Engineers of the past commonly used hanging chains as a tool to design arches and domes. The physical properties of a hanging chain are ideally suited to determine the shape of optimal structures—chains are only capable of resisting a tension force, and are free of bending moments and shears. This is ideal, since structural members that are subjected to bending are highly uneconomical. As depicted in Fig. 4, a hanging chain is in pure tension. If the links are locked and the chain is turned upright, the force is reversed and in pure compression. A pure compression structure is advantageous—the stresses are lower (since bending is eliminated) and most construction materials are able to resist compressions far better than tensions, especially masonry. Historic structures that are in pure compression have a greater propensity for survival.

In Fig. 4, two types of loading are applied to a chain—a continuous load and a point load. If a chain is only subjected to a uniform distributed load (i.e., self-weight), the profile of the chain is the catenary. However, if a point load is applied, the shape of the chain is triangular. In both of these cases (uniform and point loads), the chain is in pure tension. If the links are locked and the shape is flipped upright, the stresses will be in pure compression. Thus, with these two examples, the basic optimal shapes for structures are defined. In addition, we can add a third optimal shape—the column, where loads flow directly into the support (see Fig. 5).



a

b

Fig. 4: Natural structural shapes for a continuous load (a.) and a point loads (b.)

The shape of a structure should be directly related to the loading, to optimize the design. A point load will require a triangular supporting structure and a uniform distributed load will require a catenary shaped structure. A combination of the two loads will require a pointed arch or dome. St. Paul's Cathedral in London is an example of the combination of the two types of loads. The dome is constructed of masonry, and must resist its self-weight. In addition, the dome must resist the lantern, which is placed on top of the dome. The combination of both types of loads necessitates a conical, or pointed shaped dome as the structural support. In Fig. 7, the conical internal dome is the main supporting structure. The outer and inner circular domes are decorative and non-structural.

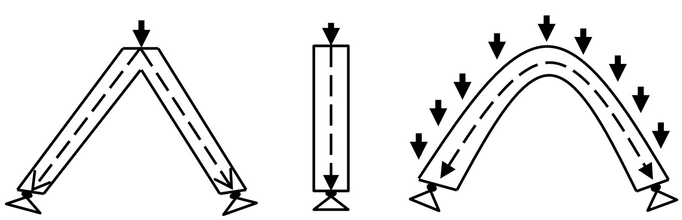


Fig. 5: Basic optimal shapes in pure compression.

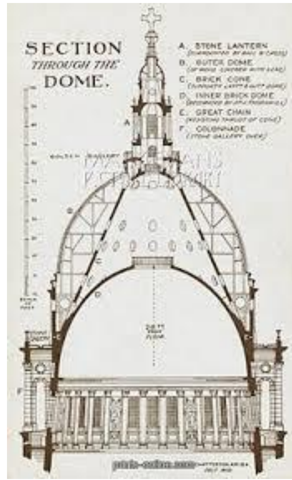


Fig. 6: Christopher Wen's structural drawings of St. Paul's Cathedral in London.

4. Structural efficiency of resisting forces

An important concept in understanding the strength of curved structures is to understand how loads are efficiently carried by structures. To illustrate this concept, a simple example is given in Fig. 7.

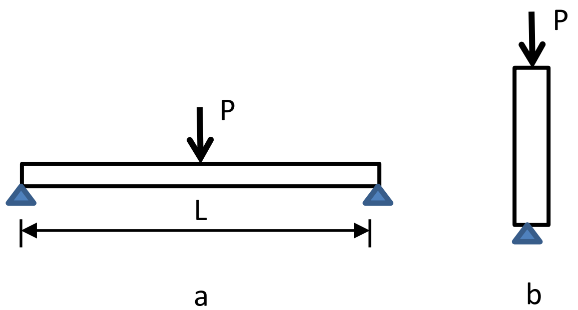


Fig. 7: Load carrying efficiency of bending and axial compression

The maximum bending stress (σ_b) in Figure 10a is expressed by Eqn. (1).

$$\sigma_b = \frac{3PL}{2bh^2} \tag{1}$$

Where L is the length, h is the depth of the beam and b is the width.

If the same load is resisted by an axial member, the maximum compressive stress (σ_c) (Fig. 10b.) in the column is given by Eqn. (2).

$$\sigma_c = \frac{P}{bh} \tag{2}$$

If the bending stress (σ_b) is divided by the column stress (σ_c), the ratio of the stresses (r) is given by Eqn. (3).

$$r = \frac{3L}{2h} \tag{3}$$

The value (L/h) is the span-to-depth ratio, which usually falls within prescribed values [3], usually dictated by codes of practice. Table 1 is a list of stress ratios for a variety of simple load and beam support conditions. The stress ratio represents the increase in stress compared to an axially loaded member. As indicated in the first row of Table 1, the stress in a simply supported beam subjected to a point load is 24 times higher than the stress in an axially loaded member. Similarly, the stress increase in other beam load/support configurations range from 12 to 18 times the stress in a column.

Resisting forces by bending is highly uneconomical compared to axial resistance. Avoiding bending, however, is not always feasible with our current practice of designing box-shaped linear structures. Table 1 demonstrates that greater efficiency is achieved by directing the flow of force along the axis of the member. The axis, however, does not necessarily have to be straight. A stress will flow along an axis of a curved member, or along the curvature of a shell if the shape is catenary. The catenary is a natural form, where stresses will flow in pure compression or tension. Unlimited variations of the catenary are also possible by varying the loads and corresponding shape, referred to as funicular forms.

Table 1: Stress ratios of various loading and support configurations

Load condition	*L/h*	Stress ratio (*r*)
Simply supported and single point load at mid-span	16	24
Simply supported and uniform distributed load	16	12
Fixed supports and uniform distributed load	28	14
Fixed supports and single point load at mid-span	28	21

5. Forgotten lessons of our forefathers

The discovery and importance of natural structures seems to have occurred around the 17th century from the work of Robert Hooke [4], the same who developed the theory of elasticity (i.e., Hooke's law). Hooke, a celebrated scientist, discovered the significance of a hanging chain and its relationship to structural forms. Hooke recognised that a hanging chain is in complete tension, without bending and shears. The shape has since been referred to as a catenary. If the chain is locked and inverted, the stress in the chain is reversed and in pure compression. The importance of this discovery is that loads may be carried in pure compression, eliminating shears and bending.

Strangely, Hooke's discovery was written in the margins of his scientific works—seemingly, a fleeting thought. Translated from Latin, "...as hangs the flexible line, so but inverted will stand the rigid arch."

One of the first notable applications of Hooke's arch theory was applied to St. Peter's cathedral in Rome. The dome exhibited serious cracking and the safety and stability of the structure was questioned. In 1743, Pope Benedict XIV appointed Giovani Poleni to assess the structure [5]. Poleni hung 32 weights on a chain to represent the self-weight of the dome walls. The resulting profile of the chain represented the natural thrust line, which was inverted and superimposed on a scaled cross-section of the dome walls (see original drawings Fig. 8). Since the chain profile fit within the walls of the dome, the structure was deemed stable and safe for occupation. According to Poleni, if the thrust line fell outside of the walls, tension cracking would be expected and the structure would be unstable. He concluded that the source of cracking was the result of inferior materials used in construction, rather than a mismatch of the thrust line. Nearly three centuries later, St. Peters cathedral remains stable as predicted, substantiating Poleni's analysis. More recent solutions, however, indicated that the cracking was due to hoop tensions, rather than the meridian flow of stress.

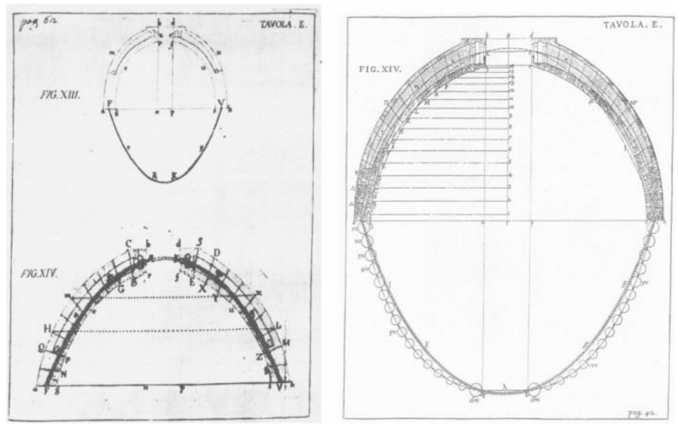


Fig. 8: Poleni’s study of St. Peters Cathedral, 1747

Application of the catenary shape to structural forms was furthered by Culmann and Stevin [6], who developed a graphical analysis method, referred to as the force polygon method (in later years, called the “tip-to-tail” method) [7]. The graphical analysis method was the dominant method of structural analysis until the early 1900s. For this reason (until recently), graphical methods were taught in most engineering programmes.

The catenary shell is, perhaps, the most time enduring shape known to humankind. The Pantheon in Rome is nearly two millenniums old, yet proudly remains intact and serviceable. Although the inner surface of the dome is a hemisphere, the secret of the Pantheon is its outer profile, which is catenary; thus, permitting the flow of stress in pure compression [8]. Although Hooke is credited with the discovery of the catenary, evidence suggests that knowledge of the catenary may have been known much earlier by the Romans, the master builders (considering the remedial work on the Pantheon—the stepped outer profile of the dome).

Historically, the catenary dome was only used in a few notable structures, advocated by antiquity’s most prominent engineer-architects, who recognized and understood the structural importance of these shapes. By far, the majority of shells were hemi-spheres. These shapes, however, commonly cracked due to hoop tensions, bending and shears at the base (stress characteristics of a hemi-sphere). For this reason, hemi-spherical domes were frequently cladded with sheets of copper, lead or tiles to make the shells water-tight, cover unsightly cracks and improve durability.

Unfortunately, the practice of implementing natural forms seem to have faded in the early 1900s—an incomprehensible digression in structural design. Utilizing natural forms should be a logical choice, since structural and material efficiency is of paramount importance.

6. Conclusions

Nature should be the engineer’s mentor. The shapes and forms we see in nature have evolved over time and are optimal load resistant structures. Many of these shapes are catenary, which are resistant to environmental conditions. Ironically, architects endeavour to replicate building forms which harmonize with the environment. The correctness of this approach is far more apparent when structural resistance and the economy of materials is considered.

Natural shapes have inspired and enlightened the engineering profession for centuries. Catenary concepts have been incorporated into numerous historical buildings; the majority of which are religious, with the intension of lasting for eternity (and many of these structures probably will!). Why natural shapes have become foreign to our profession is an absolute mystery. We have deviated from the obvious, have forgotten the lessons of the past and ignored the lessons taught by our forefathers.

References

[1] A. Bejan, S. Lorente, ”Constructional theory of generation of configuration in nature and engineering”, J *App Phy*, Vol. 100, 2006.

[2] J. Heyman, J, *The Stone Skeleton: Structural Engineering of Masonry Architecture*. Cambridge University Press, 1997.

[3] SANS 10100-1, “The structural use of concrete, Part 1: Design”, *The South African Bureau of Standards*, 2000.

[4] D. O’Dwyer, “Funicular analysis of masonry vaults”, *Comp & Struct*, Vol. 73, pp.187-197, 1999.

[5] G.M. Lopez, Poleni’s manuscripts about the Dome of Saint Peter’s, Available at: http://www.arct.cam.ac.uk/personal-page/james/ichs/Vol%202%201957-1980%20Lopez.pdf, pp. 957-1979, 2005.

[6] P. Block, M. DeJong, J. Ochsendorf, “As hangs the flexible line: equilibrium of masonry arches”, *Nexus Networking J*, Vol. 8, No. 2, pp. 9-19, 2006.

[7] D.A. Firmage, *Fundamental Theory of Structures*, 2nd Edn., Hutington, New York: Robert E. Krieger Pub. Co., 1980.

[8] R. Goshima, H. Ohmori, N. Hagiwara, N. Aoki, N “Investigation of the cross-section of the Pantheon dome through catenary mechanisms”, *IASSS*, Vol. 52, No. 1, pp. 19-23, 2011.