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A Stone Bridge Assessed by Current Standards

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Abstract **-** A three span stone masonry bridge dating back to the middle of the 19tth century, still in the roadway network, is assessed to prevent its demolition and replacement by a reinforced concrete structure. Its industrial heritage value and ecological concerns were the principal reasons for the assessment. The owner demands that the bridge remains a full service part of the second class road net without limitations Its load capacity is therefore checked against available standards. A combination of a commercial software for linear 3D structural analysis analysis with a dedicated noncommercial 2D code for the dominant nonlinear phenomenon – tension cracking of the bed joints is employed. Interaction is accounted for of the three principal structural components of an arch bridge, the cracking stone barrel, the backfill of cohesionless soil and the reconstructed pavement slab of reinforced concrete. The bridge with the new pavement meets the national and European standards conditions. The owner decided to preserve the bridge and contribute to sustainable transportation.

Keywords**:** sandstone, bridge, arch, stress, displacement, finite element method

1. Introduction

The three span sandstone arch bridge was built in 1854 and remains an important part of the second class roadway network. It was considered for a general restoration with total demolition of the arches. In order to prevent an irrecoverable loss of the industrial heritage, an extensive load capacity assessment was conducted of the load capacity of the bridge with replacement of the pavement. Most load capacity assessments published determine load rating based on the ultimate load analysis. The load models and the reduction factors from the ultimate load to the load rating vary in different countries. The bridge owners prefer when the bridge passes the available standard's criteria since no traffic limitations are then necessary. This approach is used herein. The conditions of the *EN 1990: Basis of Structural Design* and *EN 1991-2: Actions on Structures Part 2 Traffic Loads on Bridges* have been checked to testify that the bridge with the valuable stone arches can remain a part of the roadway network without limitations. A number of clauses of the *EN 1996-1-1 Design of masonry structures - Part 1-1: General rules for reinforced and unreinforced masonry structures* could be applied although the standard is not designed for bridges.

'There is a multitude of assessment methods available for masonry arch bridges, yet there is no widely accepted framework for their application' quoted from [1] is a sad fact. The principal method selection argument used herein is simple: the assessment method can use only the data available in the usual bridge owner's and designer's reach and deliver results with reasonable cost. This admits relatively simple material models. On the other hand, there is no reason to avoid the finite element technology since it can model the components of the bridge and their interactions in a way that other methods and codes (modified MEXE, plasticity theorems like Ring [2], Castiligliano theorem like CTAP code [3] and others) cannot. At the same time it is widely available.

2. Analytical Models

2.1. Material models

The dominant material failure in the stone arch bridges is the tension cracking in bed joints that results in gradual development of virtual hinges. Cracking occurs often at service load levels and it must be represented in the material model. It is assumed that mortar in the bed joints has negligible tension strength and therefore no-tension material is assumed. Specific is the predestined orientation of the cracks which makes the no-tension models in general purpose codes almost unusable. Note that the desired function of the predestined cracks can effectively be achieved only in beam 1D continuum elements where the kinematic assumptions and cross-section normal stresses facilitate the specific no-tension model.

The other indispensable failure mode is the compression failure of the masonry. *EN 1996-1-1* provides support for determining the masonry compression strength from the compression tests of the core bored specimens of the stones and by nondestructive boring tests of the mortar (PZZ 01 device). The tests were performed for the Ponikla bridge masonry arches and the compression design strength was determined f_d =2 Mpa . The value is rather conservative since the sandstone ashlars of the barrel are in good condition.

The backfill and pavement are linear elastic. The attempts to assess the failure of the backfill with the Mohr-Coulomb and other material models are frequent in the academic community but almost never applicable in the design practice. There is hardly any way to obtain the parameters for the past failure behavior of such material models. The same applies to the models of potential contact failure between the fill and arch back. It is also emphasized that the bearing structure is the stone masonry barrel and local failures of the backfill do not affect the load capacity complex arch.

The elastic properties used in the FEM models are summarized in Table 1. They are deliberately chosen conservative for the backfill. A review of the backfill properties was conducted in the published literature [6] and these are the values at the low end.

Table 1 Elastic properties considered in the case study

	E(GPa)	**G (GPa)**	**ν**	**m (kN/m^3)**
Arch	10	5	0.25	20
Backfill	0.1	0.043	0.15	18
Pavement	31	12.917	0.2	25
Spandrel	0.1	0.04	0.25	22
Piers/Abutments	10	4	0.25	18

2.2. Finite element bridge models

Three models have been used. The checks required by the standards are performed in a non-linear 2D (plane strain condition) model of a single span featuring the no-tension material specified above. The simplest Timoshenko beam elements are used for the arch so that the normal stresses in the cross-section planes are used in the above failure conditions. The backfill is meshed by standard CST triangles and the pavement by linear Timoshenko beam elements. Clamped springings are assumed. The results are slightly obscured by the 2D idealization which neglects all transverse effects and by the restrained horizontal displacements of the springings. A non-commercial in-house code was developed for the purpose and is available under the GNU license [7].

The other two models are 2D and 3D linear models of one span with the same boundary conditions as in the non-linear model and of the whole bridge including piers and abutments. They are primarily used to assess the differences between the 2D and 3D results and develop correction factors for the simple 2D solutions. The factors are then used to correct the results of the non-linear model. A front view of the bridge and its 3D FEM model are shown in Fig 1. The spans are all approximately 11.4 m, the full width of the bridge is 7.6 m. The single span 3D model is an extraction of the full bridge separated by adjacent pier's symmetry planes. On the planes, the symmetry boundary conditions are applied (zero normal and free in-plane displacements). The 2D single span model is the normal projection of the 3D model. The elements are basically isoparametric bricks/quadrilaterals enriched by rotational degrees of freedom at all nodes. Commercial package RFEM of Dlubal Software is used.

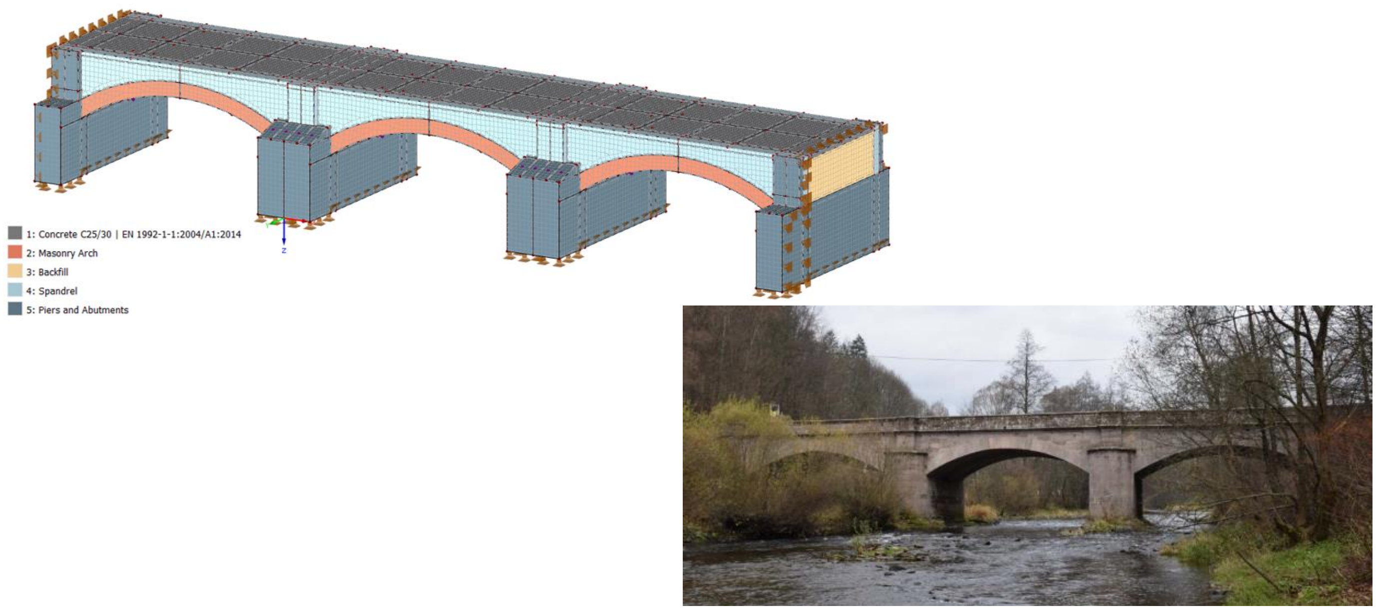


Fig. 1 Full 3D bridge model with two tandem axle forces and front view of the sandstone bridge at Ponikla

2.3. Load model

The load model LM1 of the *EN 1991-1-1* is used. It is known to be critical for bridges with these space parameters. The same position of the two axle tandems is considered in all calculations, as shown in Fig.1. The position above the quarter span is generally acknowledged to be the most adverse for the arch. The uniform continuous live loads of LM1 are not shown in Fig. 1 but applied on the loaded half of the span in the calculations. It is worth mentioning that LM4, 1800kN of the *EN* standard gives approximately the same total load on the half span but is better distributed. The dead load is included via mass densities of the bridge components materials. The load factors of the ultimate limit state are used according to the *EN* and *National Application Documen*t.The load in the 2D model is the average over the bridge width of the 3D loads.

3. 2D/3D comparative analysis

In order to compare results accurately between the 2D and 3D models, rigorous consistency was the primary focus when preparing models and considering parameters. Throughout the model analysis, the same or equivalent settings and values were considered throughout. Additionally, in order to be able to accurately determine ratios and differences in result values the location of the result analysis needed to be the same. In this way, the location where results could be extracted and compared are displayed in Fig.2. Points along the intrados and extrados of the arch were considered at the springings of the arch, the crown and at the quarter span, according to possible locations of plastic hinge developments. The location of the studied sections on the center span in the span direction *x* remains the same across the 2D and 3D models.

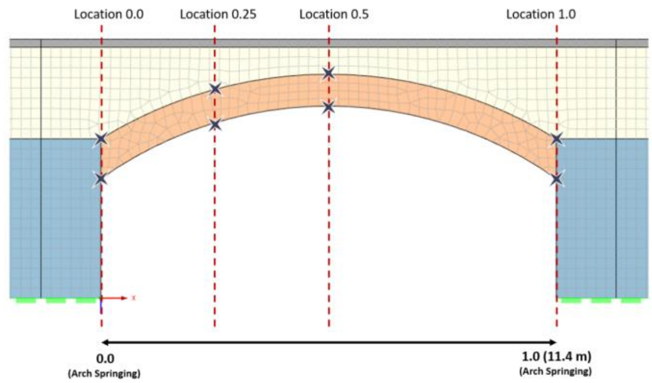
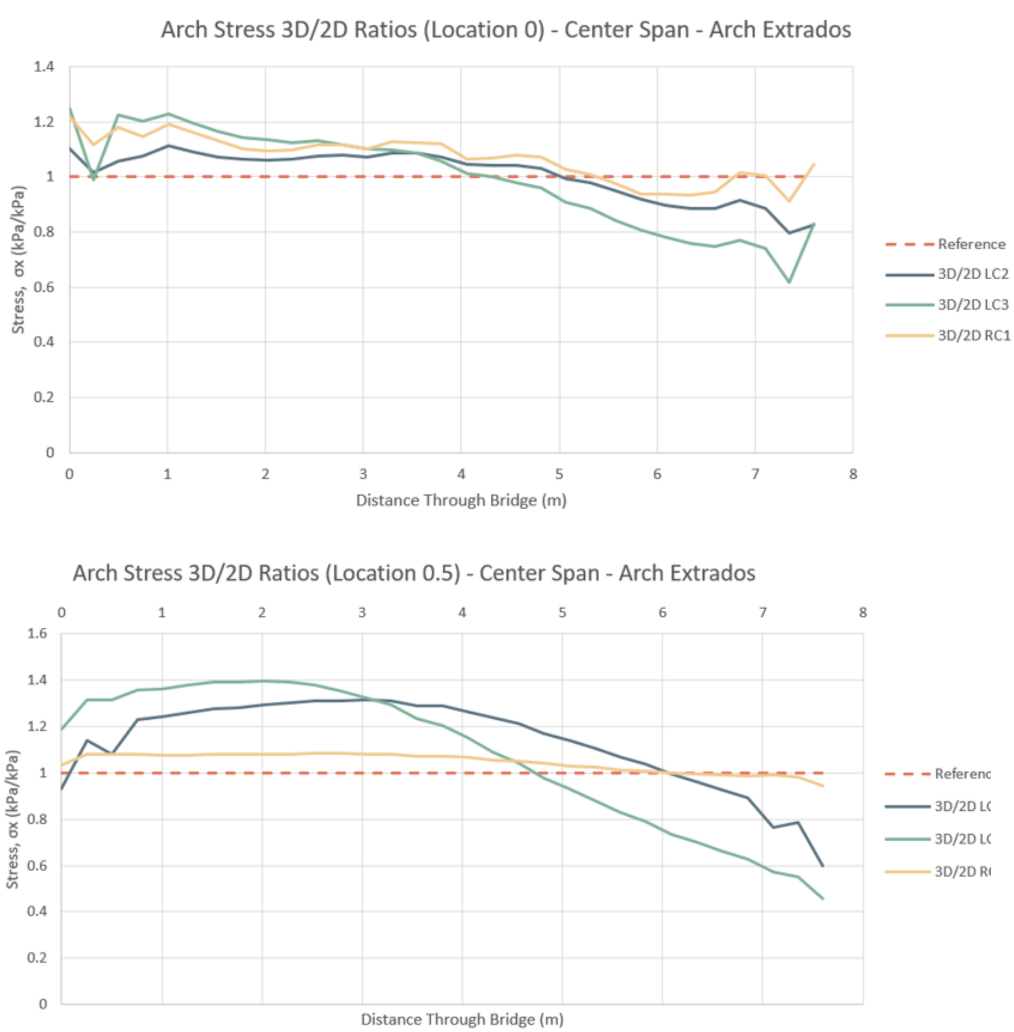
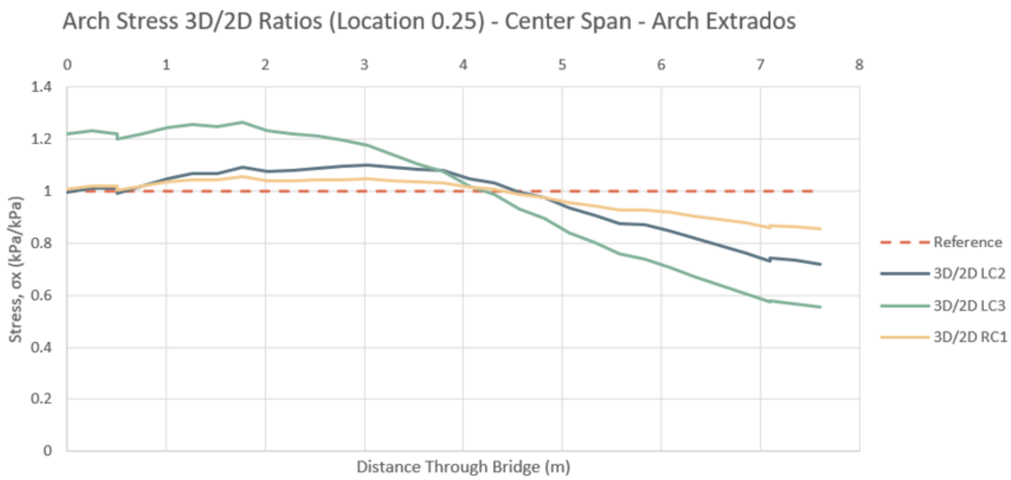


Fig. 2: Sampling points for stress comparisons in the span direction.

Stresses and deflections were sampled in the 3D models at the extrados and intrados nodes of the cross-section meshes. Normal stresses in cross-section planes (bed joints planes) should be compared. The software does not offer extraction and output of these stresses. The normal stresses in *x* direction are thus compared instead. The values may differ considerably but the 2D/3D ratios are believed approximately correct. Graphic representation of the comparison is shown in Fig.3 for three load cases.





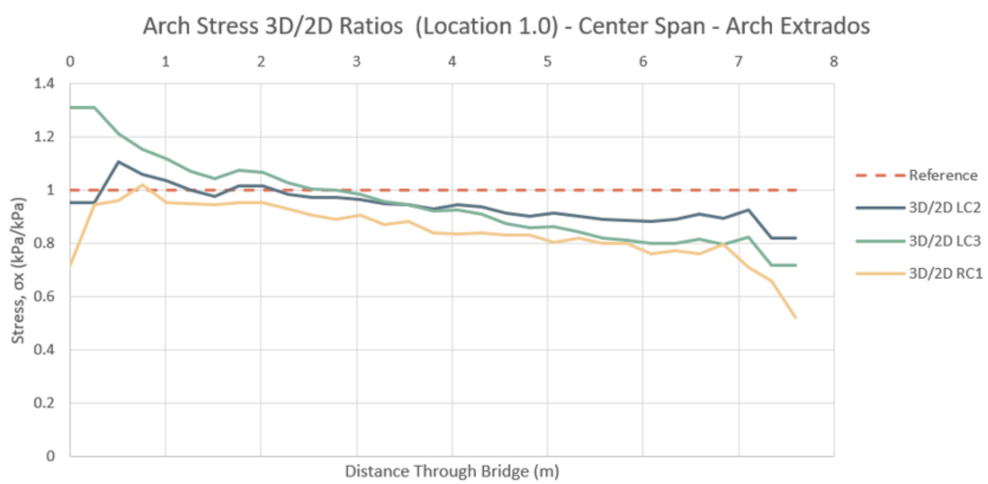


Fig. 3: Stresses variations across the barrel width extracted from the 3D model solutions compared to 2D reference (dashed straight lines).

Important for the final correction factors are just the RC1 curves which show the full design load stresses. The other curves are for simple load cases with characteristic load factors. They were used for result consistency checks. 2D reference values for the correction factors evaluation are indicated by the dashed horizontal lines.

4. 2D/3D Stress Correction Factors

The raw data extracted from the FE analyses exhibit some irregularities inherent to any FEM models. For instance curves at location 0 and 1 (arch springings) in Fig. 4 show erratic stress fluctuations at the barrel edges. They are the consequence of the discretization. At those locations an 'exact' continuum solution would give infinite stresses owing to sharp concave corners. FEM discretized model tends to approximate the 'exact' one and produces those fluctuations. The raw data were therefore further processed by averaging to obtain the final smoothed diagrams in Fig. 4.
The correction factors to be applied to the 2D non-linear solutions are summarized in Table 2.

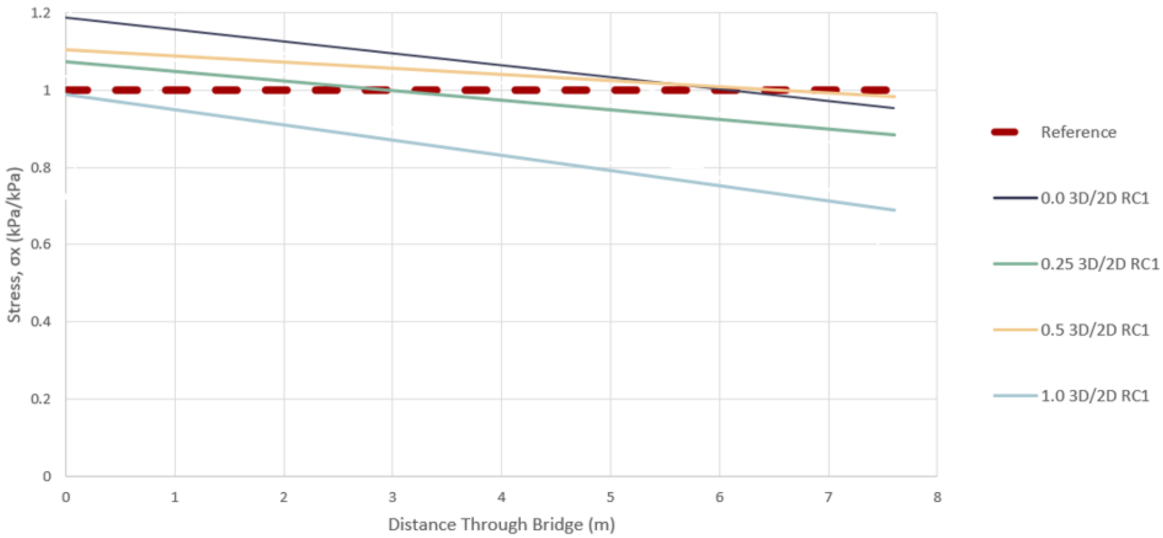


Fig.4 Final smoothed 3D/2D stress ratios at the arch extrados, full design load

Table 2 The 2D/3D correction factors

Variable		**Cross Section Location (fig.2)**			
		0	**0.25**	**0.5**	**1**
Deformation ux, Extrados	Min	0.55	0.85	0.92	0.59
	Max	0.90	1.00	1.02	0.92
Deformation ux, Intrados	Min	0.55	0.85	1.14	0.63
	Max	1.05	1.00	1.25	0.95
Stress σx, Extrados	Min	0.95	0.88	0.98	0.69
	Max	1.18	1.08	1.10	0.98
Stress σx, Intrados	Min	0.96	-	0.78	0.53
	Max	1.05	-	0.94	0.55

The correction factors were derived for a specific space parameters and material properties of the bridge at Ponikla. The author's survey [6] indicates that bridges of the industrialization period exhibit quite similar parameters. The arches all have approximately the same ratios rise/span and width/span, the backfill always is a cohesionless soil with considerable stone content. These similarities justify the application of the correction factors in other bridges assessments.

5. Interaction of the Spans

Besides corrections of the non-linear 2D model for the stress variation across the bridge width a correction is necessary for the interaction/compliance of the neighbor spans which is not accounted for in a single span model with restrained horizontal displacements at the left and right boundary planes. When the central span is loaded by the design live load the piers and the neighbor spans give in to the thrust so that the central arch widens. The widening distance must be assessed and possibly accounted for by corrections of the 2D nonlinear model. The simplest way to do that is to impose the widening distance detected in the whole bridge model as the boundary plane horizontal displacement. The wideming distance u_w=0.12 mm was computed as the difference of the left and right springing's horizontal displacements of the central span. The latter are the averages of the extrados and intrados displacements across the arch barrel width.. Somewhat surprisingly the horizontal displacements of the extrados and intrados at the same springing have opposite signs since the cross-sections rotations more than outweigh the translations of the pier heads. This occurs in spite of the springings embedded in the massive pier's heads. The model in Fig. 1 is used for the whole bridge analyses.

6. The 2D nonlinear solutions

6.1. Model features

The arch is modelled by the simplest straight Timoshenko finite elements so that the shear deformation is accounted for. In order to facilitate a compatible connection to the 2D continuum model of the fill, instead of the standard three node nodal degrees of freedom of theTimoshenko elements, two translations and one rotation of the end cross-section referred to the central beam axis, three translational DOFs are used. Two of them are assigned to the top point of the end cross-section. This enables seamless connection with the simple constant strain triangles or isoprametric quadrilateral elements of the fill region. The third DOF of the node is the component in the beam axis direction of the bottom end of the end cross-section. The element thus is a 'hybrid' of the 1D beam and 2D continuum elements. Experience confirmed that in non-linear problems the element accelerates the convergence of the iteration as compared to the classic beam elements with rotational DOFs.

The no-tension material model for the normal stress in the beam axis direction together with the assumption of the rigid cross-sections allow for an explicit closed form integration of the normal stress across the cross-section to the beam. No numeric integration is necessary. The material model is history independent (nonlinear elastic) so that no state variables need be stored. It is claimed that the element is the simplest possible to model the cracks development in the bed joints of the stone masonry arches. The full description of the element and the formulas for the nodal forces and stiffness matrix elements can be found in [4]. The concept was also generalized there to a 3D thick shell triangular facet element in the reference. That element is based on Mindlin shell kinematic assumptions and has five translational DOFs per node.

The solution for one span of a stone masonry bridge is coded in a self contained Octave/Matlab script. Data for individual problems are supplied by modifications of an input dedicated function. Templates are provided for some arch geometries, the backfill meshing is then automatic by an open source mesh generator [5]. A simple purely vector graphics output is provided.

The deformed mesh, the (algebraic) minimum stresses in each element of the arch and the depths of cracks in the bed joints are shown in the graphic output. The values of the stress are inserted at the face (top/bottom) where the minima occur wheras the depths of cracks are presented in a graphic form (Fig.6)

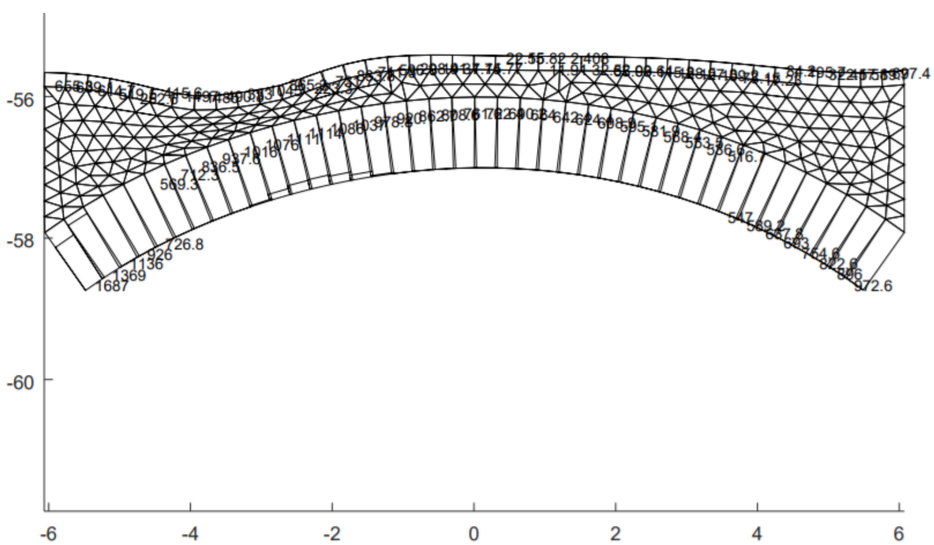


Fig. 6 The minimum stresses and crack depths in the Ponikla central span arch.

6.2. Results for the design load of the Ponikla bridge

The solution of the Ponikla bridge for the design loads is shown in Fig. 6 with axle tandems at a quarter span of the arch. The imposed displacement of the right springing and right boundary plane as specified in section 5 is already applied.

The displacements are 1000 times scaled up.

7. Conclusions

The assessment of the Ponikla bridge by available standards was conveyed to the bridge owner. Together with activities of other institutions and individuals in the cultural heritage protection it helped to avert the bridge arches demolition. A monument of the craftsmanship of the previous generations remains preserved for our descendants. This is the immediate and 'material' result of the research. Besides, a simple material model and a dedicated finite element were developed for the 2D (plane strain) models of the stone masonry arches. A single arch model consisting of the barrel, fill and roadway was coded in a Octave/Matlab script and is available for free at [7]. The combination of a linear 3D finite element solution of the whole bridge with the non-linear single span arch model is proposed and applied to account for the 3D phenomena and inetraction of the spans of multispan bridges.The extension of the non-linear single span arch model to 3D is in progress. The parallel linear 3D model will then not be necessary.

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