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Reliability-Based Evaluation of Two-way Shear Design Reinforced Concrete Slabs with FRP Reinforcements

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Abstract – The two-way shear design of slabs is being revisited by researchers all over the world, especially, those with fiber reinforced polymers (FRP) reinforcements. Recently, researchers are investigating the development of mechanical models with physical meaning. The validation of these models was conducted based on the model variability, while ignoring the testing measurements and the basic variables variabilities. In this study, a reliability-based evaluation of the current design codes and guidelines for two-way shear of concrete slabs with FRP reinforcements. The reliability analysis method included the model variability and the variability of basic variables. For evaluating the probability of failure, Reliability indices (β) are calculated and compared to each other. Concluding remarks were outlined and discussed.

***Keywords*:** Reliability, Two-way shear, FRP, CSA, ACI

1. Introduction

Two-way shear strength of concrete slabs is a sudden failure [1, 2, 3, 4]. It is a complex behavior involving several mechanisms as shown in fig. 1, which includes but not limited to: (1) direct shear across the compression zone area ;(2) dowel action across the flexure reinforcements; and (3) aggregate interlock and friction across the concrete cracks. In addition, replacing conventional steel reinforcements with Fiber reinforced polymers (FRP) reinforcements is spreading worldwide due to its excellent properties. However, these reinforcements lack the ductility of the conventional steel reinforcements. Moreover, several design codes and guidelines are being developed for FRP-reinforced concrete slabs [5, 6, 4, 7, 8]. Thus, avoiding such disastrous failure, require sophisticated methods for assessing the reliability of these design codes [9, 10, 11, 12, 13].

Structural failures of well-designed structures are due to two types of variabilities, namely aleatory and epistemic variabilities. The aleatory variabilities, which are due to the variability of effective parameters affecting the two-way shear behavior. The epistemic variabilities, which are due to the lack of knowledge and understanding of the physical behavior. To assess a design model under both aleatory, and epistemic variabilities, precise evaluation of these design models is required. Mean Value First-Order Second Moment Method (MVFOSM) was selected [12, 14, 9, 15]. Reliability index (β) was used for evaluating the level of safety and thus the probability of failure. In this study, a reliability evaluation of the state-of-the-art design codes and design guidelines was conducted. Using the FOSM. Concluding remarks were outlined.

1 Selected Design Codes And Guidelines And Experimental Database

Selected models which are design codes, and design guidelines models developed for the case of FRP-reinforced concrete slabs under two-way shear. The CSA [5], where the shear strength is calculated such that:

$$V = b_{0.5d}\, d \begin{cases} 0.028\left(1+\frac{2}{\beta_c}\right)\left(E\rho f_c'\right)^{1/3} \\ 0.147\left(E\rho f_c'\right)^{1/3}\left(0.19+\alpha_s \frac{d}{b_{0.5d}}\right) \\ 0.056\left(E\rho f_c'\right)^{1/3} \end{cases} \tag{1}$$

Where $b_{0.5d} = 4(c+d)$, d is the effective depth, a, b are the concrete column dimensions, ρ is the flexure reinforcement ratio, E is the FRP young's modulus, f_c' is the concrete compressive strength, β_c is the ratio between the concrete column dimensions, α_s is factor for load eccentricity, taken 4 for interior columns. While the ACI [6] design guideline calculates the two-way shear strength such that:

$$V = 0.8\sqrt{f_c'}kdb_{0.5d},\ k = \sqrt{2\rho n + (\rho n)^2} - \rho n \quad (2)$$

Where $n = \frac{E}{E_c}$, and $E_c = 4750\sqrt{f_c'}$. And the JSCE [16] is such that:

$$V = \beta_d \beta_\rho \beta_r f_{Pcd} b_{0.5d} d,\ \beta_d = \left(\frac{1000}{d}\right)^{1/4} \leq 1.5 \quad (3)$$

Where $\beta_\rho = \left(100\rho\, {}^{\mathrm{E}}/_{E_s}\right)^{1/3} \leq 1.5$, $\beta_r = 1 + \frac{1}{\left(1+\frac{0.25b_{0.5d}}{d}\right)}$,and $f_{Pcd} = 0.2\sqrt{f_c'} \leq 1.2$. In addition, an extensive experimental database used in the calculation of the selected model variability which is detailed in other studies.

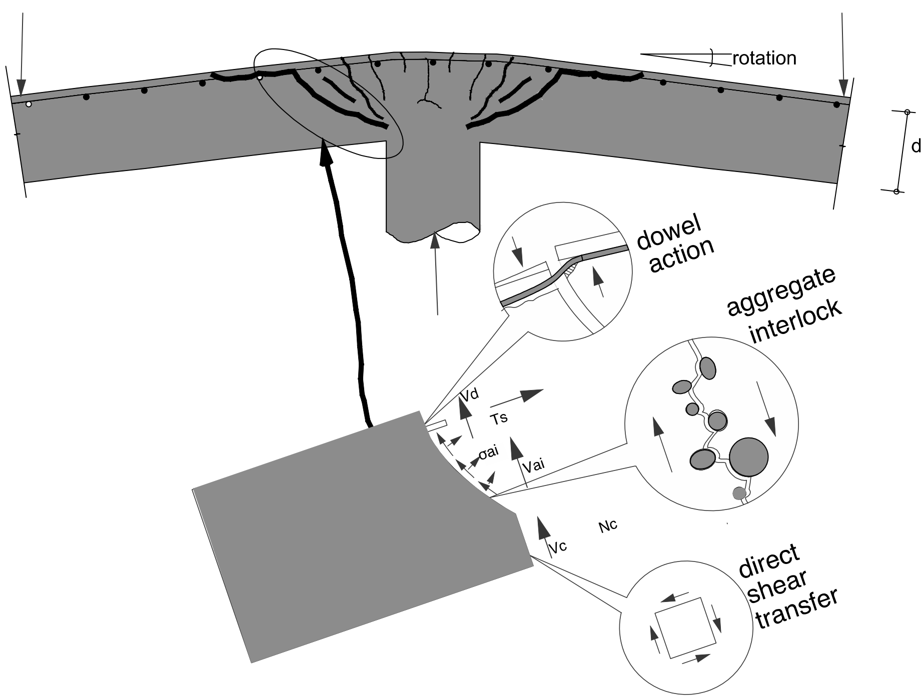

Fig. 1: Mechanisms for the two-way shear

2 Applying Mvfosm Reliability Techniques

The safety of design codes can be granted via reliability measures, where the reliability index β is the parameter representing the safety level and the probability of failure. The core of the structure's reliability is the verification of the equation $Resistance(R_d) \geq Actions(E_d)$, which is critical when $Resistance(R_d) - Actions(E_d) = 0$. The strength is calculated using the selected design codes. However, those design codes consider only limited number of the effective variables affecting the phenomena, thus, design codes have variabilities. In addition, those considered the variability of effective parameters.

2.1 Variabilities of the selected models

Selected models are used to calculate the strength and compared with the experimentally observed ones, which showed variation due to the model assumptions as well as the measurements of the testing. To evaluate this model variabilities, the actual strength is calculated such that:

$$R_d = \theta R_{dmodel} \tag{4}$$

Where R_{dmodel} is the strength calculated using the selected design methods and θ is a parameter that represent the model variability. The θ was calculated using a large experimental database where θ is calculated as the ratio between the measured strength and that calculated using the selected model [17]. Table (1) shows the statistical measures for the state of the art collected and used to calculate the model variability. The statistical values obtained from normal distribution need to be corrected because the strength is believed to be log-normal distribution and the test results variability [18]. The average (μ_θ) of θ is calculated such that:

$$\mu_\theta = e^{\mu+0.5\sigma^2} \tag{5}$$

Where μ is the average based on normal distribution and σ^2 is the standard deviation based on normal distribution. While coefficient of variation (CoV_θ) is such that:

$$CoV_\theta = \sqrt{{CoV_{LN,conv}}^2 + {CoV_{tests}}^2} = \sqrt{e^{\sigma^2} - 1 - 0.05^2} \tag{6}$$

Where $CoV_{LN,conv}$ is the coefficient of variation based on lognormal distribution and CoV_{tests} is the coefficient of variation for test results taken as 0.05.

2.2 Variabilities of the basic variables

Table (1) shown the statical measured based on the JCSS Probabilistic Model Code [19] for the various basic variables considered by the selected models.

2.3 Procedure

The Mean-Value First-Order Second Moment (MVFOSM) Method involves the linear approximation of a given limit state function and its derivatives to a first-order Taylor series, at the mean values of random input variables: $\mathbf{x}^* = (\mathbf{x}_1^*, \mathbf{x}_2^*, \ldots, \mathbf{x}_n^*)^{\mathbf{T}}$ The limit state function is:

$$\mathbf{g}(\mathbf{x}) = \mathbf{g}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \tag{7}$$

The mean value of the basic variable is: $\boldsymbol{\mu} = (\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \ldots, \boldsymbol{\mu}_n)^{\mathbf{T}}$ and the expected value of $\mathbf{m_z} = \mathbf{E}(\mathbf{Z})$ is given by:

$$\mathbf{m_z} = \mathbf{g}(\boldsymbol{\mu}) \tag{8}$$

The variance $\boldsymbol{\sigma_z}^2 = \mathbf{Var}(\mathbf{Z})$ is given by:

$$\boldsymbol{\sigma_z}^2 = \sum_{i=1}^{n} \left(\left. \frac{\partial g}{\partial x_i} \right|_{\mu_i} \right)^2 \boldsymbol{\sigma_{x_i}}^2 \tag{9}$$

The reliability index $\boldsymbol{\beta}$ is determined by:

$$\boldsymbol{\beta} = \frac{\mathbf{m_z}}{\sigma_z} \tag{10}$$

Table (1) Variability of basic variables based on JCSS PROBABILISTIC MODEL CODE.

Variable	Distribution	Nominal Value	Mean value	Standard deviation	Coefficient of Variation
Concrete compressive strength (f'_c, MPa)	Log-normal	25	38.8*	4.67*	-
		35	47.2*	4.26*	-
		45	53.6*	3.76*	-

Effective depth ($\boldsymbol{d}$, mm)		Normal	d_n	$d_n + 10$	10	-
Column dimension ($\boldsymbol{c}$, mm)		Normal	c_n	$1.003c_n$	$4+0.006c_n$	-
Flexure reinforcement ratio ($\boldsymbol{\rho}$, %)		Normal	ρ_n	ρ_n	-	0.02
Young's modulus ($\boldsymbol{E}$, MPa)		Normal	E_n	E_n	-	0.15
Model variability ($\boldsymbol{\theta}$, -)	JSCE	Log-normal	-	2.87*	-	0.35*
	CSA		-	1.20*	-	0.38*
	ACI		-	2.20*	-	0.38*

* Corrected values to include the log-normal distribution.

3 Reliability Evaluation Of The Selected Model

To assess the reliability of the selected design codes, the reliability index was calculated twice. (1) a case study level of only single design vector of a flat slabs was selected such that: $\boldsymbol{E}$ = 80,000 MPa, $\boldsymbol{d}$ = 150 mm, ($\boldsymbol{f'_c}$) = 30 MPa, ($\boldsymbol{\rho}$) = 1%, and ($\boldsymbol{C}$)= 100 mm. (2) a parametric study for the full range of values for each parameter. There was no significant difference in the reliability index value for all selected models. Although the CSA design code is the most reliable, while the ACI is the least, however, all design models had a reliability index lower than the target value of 3.95 [12]. Thus, further investigation is needed in the reliability of current design codes. Moreover, a parametric study was conducted, where the reliability index is calculated for a one variable and the values of the single vector for other variables. Figs 2-5 show the variation of the reliability index with respect to the variables d, $\boldsymbol{f'_c}$, $\boldsymbol{\rho}$, and $\boldsymbol{E}$. For all selected models, the reliability index variation with the change in the effective depth, young's modulus, and the flexure reinforcement ratio is quite similar. The reliability index was constant with the increase in the concrete compressive strength. For the effect of size, the reliability index increases with the increase in the effective depth; however, the rate of variation decreases with the increase in depth. This pattern was recognized in previous studies for two-way shear using other design codes [12]. For the effect of flexure reinforcement ratio, the reliability increases with the increase in the reinforcement ratio; however, the rate of that increase in the reliability decreases with the increase in the reinforcement ratio. For the effect of compressive concrete strength, the reliability slightly increases with the increase in the compressive concrete strength; however, the rate of that reliability increase is constant with the increase in the compressive concrete strength. For the effect of young's modulus, the reliability slightly increases with the increase in the young's modulus; however, the rate of that reliability increase is constant with the increase in the young's modulus. It can be concluded that selected model is less reliable at slabs thickness less than 200 mm and for reinforcement ratio under 1%. On the other hand, the reliability index is constant for the concrete compressive strength and young's modulus.

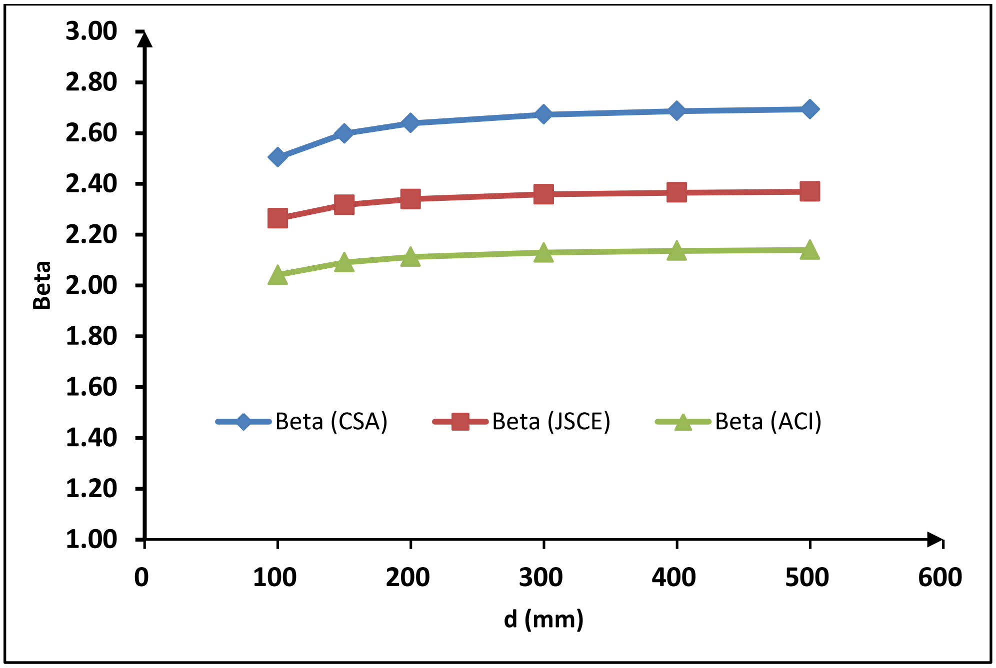

Fig. 2: Reliability indices in dependence of effective depth (d).

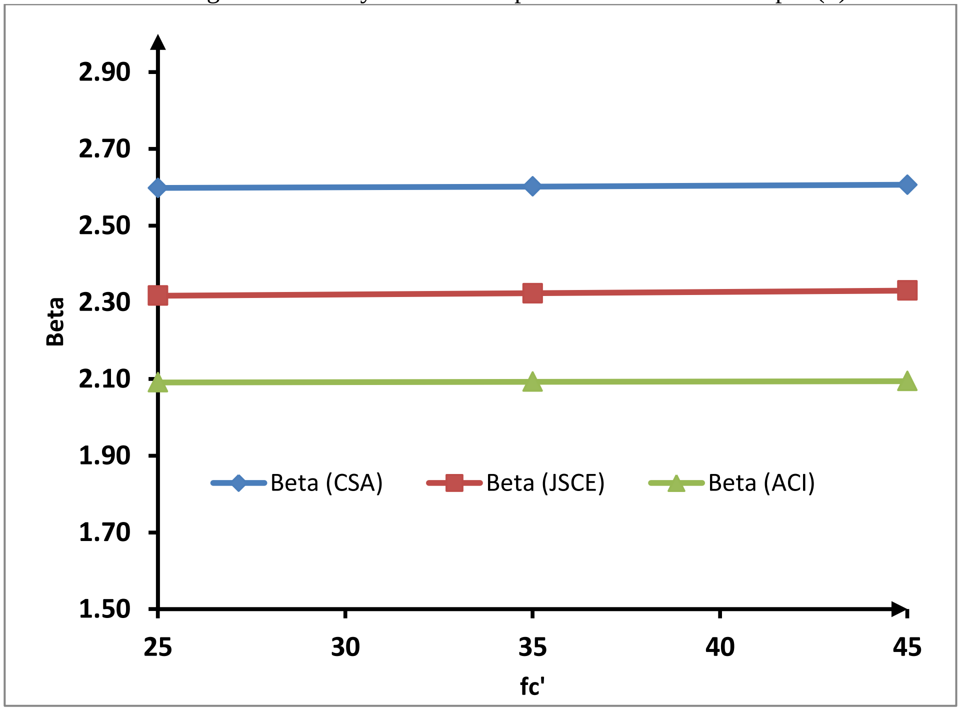

Fig. 3: Reliability indices in dependence of fc’

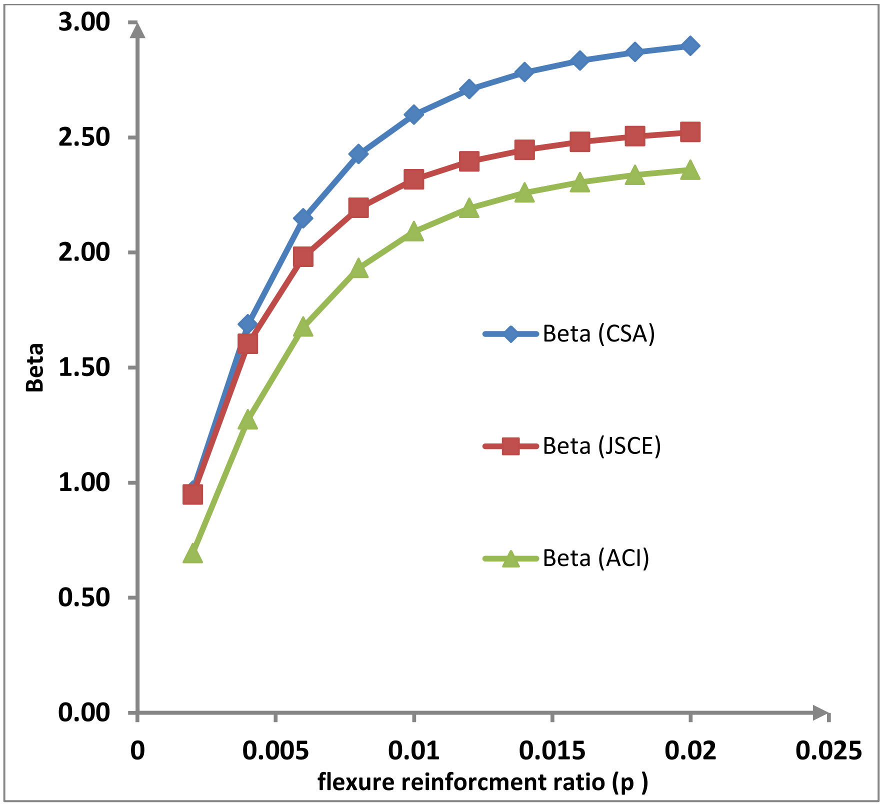

Fig 4: Reliability indices in dependence of flexural reinforcement ratio (p)

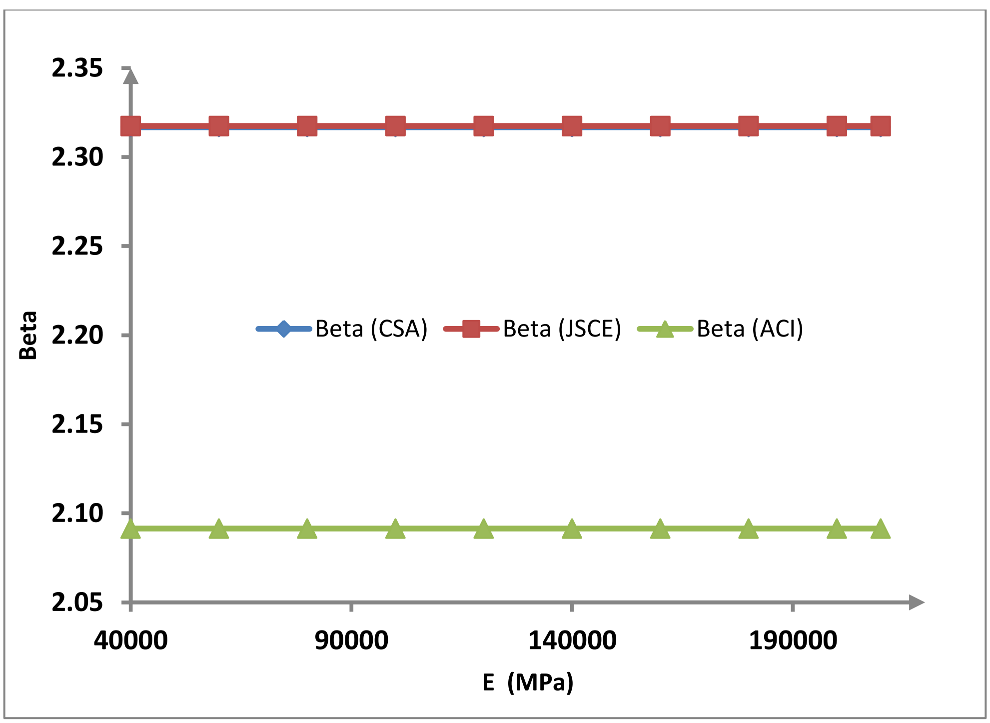

Fig. 5: Reliability indices in dependence of Young's Modulus (E)

4. Conclusion

A reliability-based evaluation was conducted, the selected models was found to need further investigation. The CSA being the most reliable and the ACI the least reliable. For all selected models, the reliability index variation with the change in the effective depth, young’s modulus, and the flexure reinforcement ratio is quite similar. The reliability index was constant with the increase in the concrete compressive strength, the effective depth, the effect of flexure reinforcement ratio, and the young’s modulus. All selected models are less reliable at slabs thickness value less than 200 mm and for reinforcement ratio under 1%. On the other hand, the reliability index is constant for the concrete compressive strength and young’s modulus.

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