

# Detuning Effect of Multiple Tuned Mass Damper System in Vibration Control of Rail Bridge Girder

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**Abstract** – This study investigates the multiple tuning requirements of rail bridge girder and the effects of detuning in situations raised due to the presence or absence of the trainloads. At first, a generalized stepwise formulation is established for designing an optimum multiple-tuned-mass-damper (MTMD) system in the modal space of a continuous bridge girder for controlling a selected set of modes. To verify the performance of the MTMD system, a numerical study is conducted with an existing steel rail bridge girder considering with and without trainload conditions. For each of the loading condition, the primary bridge girder is considered i) for an attached MTMD system optimally tuned to the conforming load condition, ii) for an attached MTMD system optimally tuned to the other load condition, and iii) for an uncontrolled condition without any attached TMDs. Time history analysis of the system assembly is then performed under the seismic excitation along with the excitation caused by dynamic train movements. The response of the primary bridge girder for a particular trainload condition indicates that the MTMD system works satisfactorily when it is tuned based on the conforming load condition. However, the attached MTMD conforming to a different load condition was found to be inadequate in controlling the dynamic excitation of the primary bridge girder system.

**Keywords:** Multiple-tuned-mass-damper, detuning-effect, modal-control, optimal-control, transfer-function

## 1. Introduction

Bridges are subjected to severe vibration caused by heavy moving vehicles or environmental load, and are often required to transfer the vibration energy of the primary structural system to an auxiliary system like tuned mass damper. In the case of a railway bridge girder, application of tuned mass damper (TMD) in controlling the bridge vibration is well accounted. It is important to note that the full potential of a TMD can be achieved when its parameters are optimally tuned to the primary structure. As a development of the traditional form of optimum TMD, the concept of MTMD was introduced [1], and it was found to work efficiently in absorbing the vibration energy of the bridge structure. Later on, the MTMD system was successfully implemented for the rail bridge vibration control [2]. However, despite the widespread development, the performance of TMD is highly susceptible to the accuracy of tuning ([3]; [4]; [5]). Under miscellaneous circumstances, the tuning frequency of the TMD changes because of the variation in the primary structure's stiffness, inertia, or both. Consequently, the optimum tuning criteria of the designed TMD system changes, resulting in the reduction in the efficiency of the vibration control system. Such detuning condition of the TMD system often generates an undesired amplification in the dynamic response of the structure. This effect is relevant in the case of pedestrian bridges, grandstands, floors, sports stadia, liquid-retaining structures [5]. In the case of railway bridges, a significant modification in the dynamic properties of the system is likely to occur during the train movement as compared to the bare structure. This changes the optimum tuning requirements of the designed TMD system. This study investigates the effect of detuning considering an optimally designed MTMD system for an existing rail bridge girder. In the first part of the study, an optimum MTMD system is designed in the independent modal space of a continuous primary bridge girder for controlling a selected set of modes. Next, an existing steel rail bridge girder equipped with the designed MTMD system is modelled in OpenSees [6] framework under earthquake excitations with strong vertical components for the with and without trainload conditions. Finally, the performances of the MTMD systems as designed optimally for different trainload situations are compared to study the detuning effects in detail.

## 2. Design of Optimum MTMD System

In this section, a stepwise formulation is carried out to design an optimum MTMD system attached to a primary bridge girder for minimizing its dynamic vibration. The optimum MTMD design is conducted in the independent modal space of

the bridge girder for controlling the predominating modes contributing to the system vibration. In this formulation 'p' sets of MTMD systems are considered along the span of the bridge girder with each set comprised of 'q' number of TMDs uniformly distributed across the bridge girder. A schematic diagram representing the primary bridge girder with the attached MTMD system and a single TMD unit are shown in Fig 1.

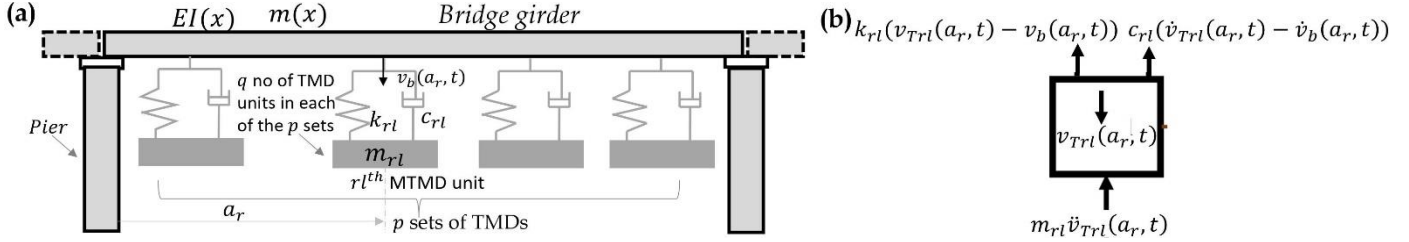


Fig. 1: Schematic diagram representing (a) the primary bridge girder with the attached MTMD system and (b) a single TMD unit

The equation of motion of the overall system assembly can be written as follows:

$$\frac{\partial^2}{\partial x^2} \left[ EI(x) \left( \frac{\partial^2 v_b(x,t)}{\partial x^2} + a_1 \frac{\partial^3 v_b(x,t)}{\partial x^2 \partial t} \right) \right] + m(x) \frac{\partial^2 v_b(x,t)}{\partial t^2} + c(x) \frac{\partial v_b(x,t)}{\partial t} = p(x,t) + \left[ \sum_{r=1}^p \delta(x - a_r) \sum_{l=1}^q k_{rl} (v_{Trl}(a_r, t) - v_b(x, t)) \right] + \left[ \sum_{r=1}^p \delta(x - a_r) \sum_{l=1}^q c_{rl} \left( \frac{\partial v_{Trl}(a_r, t)}{\partial t} - \frac{\partial v_b(x, t)}{\partial t} \right) \right] \quad (1)$$

$$m_{rl} \ddot{v}_{Trl}(a_r, t) + k_{rl} (v_{Trl}(a_r, t) - v_b(a_r, t)) + c_{rl} [\dot{v}_{Trl}(a_r, t) - \dot{v}_b(a_r, t)] = 0 \quad (2)$$

where,  $m(x)$ ,  $c(x)$  and  $EI(x)$  represent the mass per unit length, damping force per unit length and the flexural rigidity of the continuous system, respectively at a distance  $x$  along the span;  $a_1$  is a constant corresponding to stiffness proportional damping;  $p(x, t)$  denotes the external excitation; the delta function represents the control force exerted by the MTMD system at distinct locations; the distance of the  $r^{\text{th}}$  set of MTMDs along the bridge span is denoted as  $a_r$ ; the TMD unit corresponding to the  $r^{\text{th}}$  set and  $l^{\text{th}}$  number across the bridge girder is denoted by the subscript  $rl$ ; the stiffness and the damping of the  $rl^{\text{th}}$  TMD unit are indicated as  $k_{rl}$  and  $c_{rl}$ , respectively, and  $v_b(a_r, t)$  and  $v_{Trl}(a_r, t)$  are the vertical displacements of the bridge girder and the  $rl^{\text{th}}$  TMD unit, respectively, at a distance  $a_r$  at time  $t$ . The vertical displacements are measured in reference to the equilibrium position of pier base as a datum. The system assembly thus obtained in Eqs 1-2 are taken to the independent modal space of the primary bridge girder by following the modal superposition as  $v_b(x, t) = \sum_{i=1}^{\infty} \phi_i(x) Y_i(t)$  where  $\phi_i(x)$  and  $Y_i(t)$  are the mode shapes and the modal coordinate for the  $i^{\text{th}}$  mode, respectively. The equation of motion for the  $i^{\text{th}}$  mode can be represented as

$$M_i \ddot{Y}_i(t) + 2M_i \omega_i \zeta_i \dot{Y}_i(t) + \omega_i^2 M_i Y_i(t) = F_i(t) + F_{Ti}(t) \quad (3)$$

where,  $M_i = \int_0^L \phi_i(x)^2 m(x) dx$ , indicates the  $i^{\text{th}}$  modal mass;  $F_i(t) = \int_0^L \phi_i(x) p(x, t) dx$ , indicates the  $i^{\text{th}}$  modal excitation;  $F_{Ti}(t) = \int_0^L \phi_i(x) \left[ \sum_{r=1}^p \delta(x - a_r) \sum_{l=1}^q k_{rl} (v_{Trl}(a_r, t) - v_b(x, t)) \right] + \left[ \sum_{r=1}^p \delta(x - a_r) \sum_{l=1}^q c_{rl} \left( \frac{\partial v_{Trl}(a_r, t)}{\partial t} - \frac{\partial v_b(x, t)}{\partial t} \right) \right] dx$  indicate the  $i^{\text{th}}$  modal control force from the MTMD system,  $\omega_i$  and  $\zeta_i$  indicate the  $i^{\text{th}}$  modal frequency and damping ratio, respectively. The system transfer function in the modal domain can be expressed as  $\bar{Y}(s) = \mathbf{G}(s) \bar{F}(s)$ , where  $\bar{Y}(s)$  and  $\bar{F}(s)$  are the Laplace transformation of modal displacement  $\mathbf{Y}(t)$  and modal excitation vectors  $\mathbf{F}(t)$ , respectively and  $\mathbf{G}(s)$  is the transfer function matrix. The  $ij^{\text{th}}$  diagonal element of  $\mathbf{G}(s)^{-1}$  can be written as  $M_i s^2 + 2M_i s \omega_i \zeta_i + \omega_i^2 M_i - \sum_{r=1}^p \sum_{l=1}^q \phi_i^2(a_r) k_{rl} \left( \frac{(k_{rl} + s c_{rl})}{m_{rl} s^2 + k_{rl} + s c_{rl}} - 1 \right) - \sum_{r=1}^p \sum_{l=1}^q \phi_i^2(a_r) s c_{rl} \left( \frac{(k_{rl} + s c_{rl})}{m_{rl} s^2 + k_{rl} + s c_{rl}} - 1 \right)$  and the non-diagonal element corresponding to the  $i^{\text{th}}$  row and  $j^{\text{th}}$  column can be written as  $-\sum_{r=1}^p \sum_{l=1}^q \phi_i(a_r) \phi_j(a_r) k_{rl} \left( \frac{(k_{rl} + s c_{rl})}{m_{rl} s^2 + k_{rl} + s c_{rl}} - 1 \right) - \sum_{r=1}^p \sum_{l=1}^q \phi_i(a_r) \phi_j(a_r) s c_{rl} \left( \frac{(k_{rl} + s c_{rl})}{m_{rl} s^2 + k_{rl} + s c_{rl}} - 1 \right)$ . The optimum MTMD system parameters for the  $rl^{\text{th}}$  TMD unit, such as the optimum tuning ratios, i.e.,  $\gamma_{rl} = \sqrt{k_{rl}/m_{rl}}/\omega_1$  and the optimum damping ratios, i.e.,  $\eta_{rl} = c_{rl}/(2\sqrt{k_{rl}m_{rl}})$  can be identified

by minimizing the infinity norm (i.e.,  $\|G\|_\infty = \sup_\omega \sigma_{max}(\mathbf{G}(s))$ ) of the system transfer function. Here,  $\sigma_{max}(\mathbf{G}(s)) = \lambda_{max}(\mathbf{G}(s) \mathbf{G}(s)^T)$  denotes the maximum Eigen value of the matrix  $(\mathbf{G}(s) \mathbf{G}(s)^T)$  and  $\sup_\omega$  denotes the supremum or least upper bound over all real valued frequencies  $\omega$ . In case of a known excitation statistics, the system transfer function can be further modified based on the frequency regimes of excitation for obtaining the optimum MTMD parameters.

### 3. Numerical Investigation

#### 3.1. Bridge modelling

In this part, an existing thirty-five-year-old steel plate girder rail bridge classified as bridge number 656 [7] is considered for designing the optimum MTMD system for resisting the earthquake-induced vibration. The length of each plate girder is 24.4 m. In the bridge system, the connections are primarily welded connections with riveted bracings and intermediate stiffeners. A detailed description of the bridge with all the sectional dimensions is provided in [7]. The three-dimensional prototype of the bridge superstructure is modelled in OpenSees [6] for the cases of with and without trainloads. For without trainload case, the total primary mass is estimated to be 41000 kg, mainly coming from the self-weight of the bare deck of the bridge. In the case of with trainload, a 16.25-ton axle load enhances the total mass of the structure to 106000 kg. The main bridge girder is modelled using the ‘beamWithHinges’ element, and the bracings are modelled as the ‘truss’ elements in OpenSees. The damping ratio of the primary structure is taken as 2% under service conditions and modelled as Rayleigh damping. Apart from the earthquake excitation, the vibration due to the moving trainload with an average speed of 60 km/hour [8] is also accounted in this study. The dynamic load due to train movement is modelled as time-series load considering the train speed and time delay for each node on the girder. Eigenvalue analysis for the bridge girder considering both the trainload cases shows a significant deviation (i.e., 36%) between the corresponding modes of the systems. In order to design the optimum MTMD parameters, only the fundamental vertical modes of the bridge girder are considered on the basis of a preliminary time-history analysis with an impulse and white noise excitation. Six TMDs on the symmetric edges across the bridge girder at distances L/4, L/2, and 3L/4 from the bridge pier are considered for obtaining the maximum control efficiency. The mass ratio of each unit of the MTMDs is considered to be 0.83% with respect to the without trainload case. Optimum tuning ratios for without and with trainload cases are obtained as 0.7 and 0.68, respectively, and the optimum damping ratios for without and with trainload cases are obtained as 0.05 and 0.02, respectively for each unit of MTMDs. MTMD units are modelled in OpenSees using ‘UniaxialMaterial Parallel’ elements by connecting uniaxial elastic and viscous material in parallel.

#### 3.2. Time history analysis and detuning

Earthquake ground motions having predominant vertical components are considered from the PEER Ground Motion database [9] for analysing the response time history for the bridge structure (Table 1). The data set consists of vertical and orientation independent GMRotD50 spectra with 5% damping [9].

Table 1: Response Summary.

| S.No. | Name of the ground motion | PGA (g) | Percentage reduction in acceleration responses |         |                |         |
|-------|---------------------------|---------|--|---------|----------------|---------|
|       |                           |         | Without trainload                              |         | With trainload |         |
|       |                           |         | Tuned  | Detuned | Tuned          | Detuned |
| 1     | Imperial Valley (1940)    | 0.178   | 55   | 10      | 64             | -33     |
| 2     | Kobe (1995)               | 0.34    | 50   | 13      | 52             | -25     |
| 3     | Northridge (1994)         | 0.32    | 35   | 12      | 50             | -20     |
| 4     | Loma Prieta (1989)        | 0.30    | 44   | 10      | 50             | -21     |
| 5     | Cape Mendocino (1992)     | 0.74    | 40   | 2       | 48             | -24     |

Time history analysis is carried out for both the trainload conditions. For each condition, the primary bridge girder is considered with an attached MTMD system i) optimally designed for the conforming load condition, denoted as the tuned case, ii) optimally designed for the other load condition, denoted as the detuned case, and iii) for an uncontrolled condition. The responses are obtained in terms of the acceleration of the bridge girder at its mid-span. The percentage reduction in the responses for an attached MTMD condition is estimated with respect to the uncontrolled condition as summarized in Table 1. The table clearly indicates that a significant percentage reduction in the responses is observed for the tuned condition of the attached MTMD system considering both the trainload cases. However, at a detuned condition of the attached MTMD

system, deterioration in the performance of the control system can be observed. Moreover, for with trainload condition, the attached detuned MTMD system was found to amplify the bridge girder responses as compared to the uncontrolled structure. The illustrative time history plots are shown for both the trainload conditions for the Imperial Valley earthquake in Fig 2.

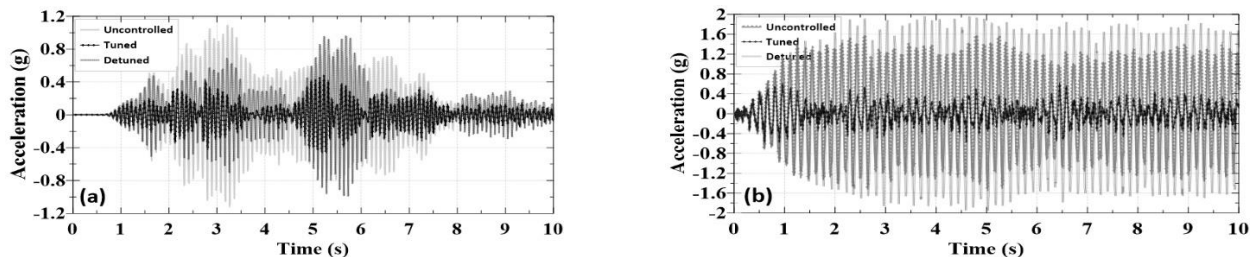


Fig 2: Acceleration responses for the (a) without trainload and (b) with trainload case for Imperial Valley earthquake.

#### 4. Conclusion

The present study mainly focuses on the multiple tuning requirements of rail bridge girders and the effects of detuning in situations raised due to the presence or absence of the trainloads. The optimum MTMDs as analytically designed in the modal domain of a continuous system are implemented by numerical modelling of an existing steel rail bridge under earthquake excitations at different trainload conditions. Time history analysis of the system assembly shows that the attached MTMD performs satisfactorily while tuned to the conforming trainload condition. However, the same MTMD becomes detuned at a different trainload condition, and it fails to provide the desired response reduction of the bridge girder system. Moreover, in certain cases, the responses are found to be amplified as compared to the uncontrolled system. The study thus demonstrates the effect of detuning for an MTMD system in the context of a rail bridge girder at varying trainload conditions and necessitates the multi-tuning requirement of the control system.

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