

Sustainability cement block selection based on interval-valued hesitant fuzzy group analysis for construction industry problems

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Abstract – Nowadays, sustainable material selection is an important issue for construction industry because of considering the environmental and social competencies according to quality and cost targets. Hence, multi criteria group decision making (MCGDM) is a known powerful tool to select the best potential alternatives based on group assessment of decision makers (DMs) in complex real-world cases. In traditional MCGDM methods, the relative significance of each criterion and the performance evaluating of potential alternatives are considered precisely. However, when the complexity of the real-world systems related to humans is increased, the future information of them cannot be precise / known completely. In this respect, decision making problems are one of the science fields that the information is often vague / uncertain. Moreover, If DMs cannot assign their opinions by expressing the linguistic terms regarding to the classical fuzzy sets, the Interval-Valued Hesitant Fuzzy Sets (IVHFSs) theory is a useful tool to help the DMs in these hesitant conditions and can present a more practical and accurate modeling. In this study, an Interval-Valued Hesitant Fuzzy Preference Selection Index (IVHF-PSI) method is presented to solve the sustainable cement block selection problems in construction industry. Finally, the process of the proposed IVHF-PSI method is performed by considering a real case study to represent the applicability and verification of the proposed approach.

Keywords: Green material selection problem, Preference selection index (PSI), Interval-valued hesitant fuzzy set (IVHFS), Sustainability, Group decision analysis.

1. Introduction

The multiple criteria decision making (MCDM) has provided an effective framework for the comparison regarding to the evaluations of multiple conflicted criteria [1]. In classical assessment of the MCDM problems the information is based on crisp values, but in fuzzy MCDM (FMCDM) we often determine the values by using the fuzzy terms [2]. However, in real-life the objects usually have been considered as fuzzy and uncertain, because the preferences decision makers' (DMs') judgments are imprecise / vague [3]. Thus, the criteria of decision making problems in some conditions are suitable to be defined by fuzzy terms [4], such as intuitionistic fuzzy sets (IFSs) [5, 6], interval values [7, 8], linguistic terms [9-11], and hesitant fuzzy sets (HFSs) [12, 13]. Meanwhile, the MCDM methodologies based on incomplete information have popular approaches that can solve the sustainable selection problems.

In this regard, Mousavi et al. [14] presented a hierarchical multi-criteria group decision making approach for ranking and assessments the sustainable new product ideas a under fuzzy environment. Vahdani and Zandieh [10] rating the candidate alternatives as linguistic variables that transformed to triangular fuzzy numbers for solved their fuzzy MCDM. Mousavi et al. [15] solve the decision making problem that considered the linguistic variables for establish the decision matrix regarding to interval-valued fuzzy numbers. In this approach, a group of experts / DMs are considered to cope with complexity of the management and engineering fields to discriminate the relevant aspects of sustainable construction decision-making problems. Furthermore, the multi criteria group decision making (MCGDM) methods consider the preferences DMs' judgments and also rating the quantitative and/or qualitative of criteria as well as weights of criteria. In the recent decade, some researcher focused on MCGDM problems to represent reliable such as [16-19].

Moreover, Xu [18] presented a distance based method under interval-valued intuitionistic fuzzy matrices for group decision analysis. In addition, introduce some relations and operations in interval-valued intuitionistic fuzzy numbers and defined interval-valued intuitionistic fuzzy equivalence and similarity matrices according to their characteristic extended. Yue [16] extended the TOPSIS method for calculating the DMs weight by considering the interval fuzzy number under group decision analysis. Yu and Lai [17] presented a distance-based method to solve the emergency problems in group decision-making analysis. Chen [19] presented a signed-distance based methodology for determine the relative significance of each criterion. In the proposed approach, the interval type-2 trapezoidal fuzzy number is considered for rating of alternatives and the relative significance of each criterion. In addition, in some hesitancy situations, the DMs for decrease the uncertainty risk and margin of errors assigned their preferences opinions by some membership degrees for an object under a set. Hence, hesitant fuzzy set (HFS) is a very powerful tool to cope with this situation that it has been first introduced by Torra and Narukawa [20] and Torra [21]. Torra and Narukawa [20] and Torra [21] have studied about the relationship between the IFS and HFS, and indicated that the IFS is assess by envelope of the HFS. To address the issue, HFS is very efficient producer. Therefore, each criterion can be expressed as the HFS and defined in terms of the preferences DMs' judgments. In addition, the properties of HFSs are more practical for modelling of vagueness to express the membership degree of an object. HFS has been much successfully implementation and attention in decision making problems [22-24].

Furthermore, Zhang [25] introduced some power aggregation relations and extended them in hesitant fuzzy setting, also used the novel aggregation relations to extend the techniques for green MAGDM problems. Chen et al. [26] extended an approach under interval-value hesitant preferences operations to solve the group decision problems based on sustainability competencies. Hence, Xia et al. [27] focused on the aggregation operators regarding to hesitant fuzzy information and then defined some aggregation relations and also studied about the relationship of them. Rodriguez, et al. [28] introduced a novel linguistic group decision method according to hesitant fuzzy linguistic term set for comforts the flexible linguistic illustration. Zhang & Wei [29] presented VIKOR and TOPSIS methods in hesitant fuzzy setting to solve the MCDM problem, and then the results compared by an illustrative example. Meanwhile, Liao & Xu [30] presented a hesitant fuzzy VIKOR method by utilizing the hesitant normalized Manhattan distance measure in procedure of the proposed method. Xu & Zhang [31] extended an approach according to TOPSIS and maximizing deviation methods by considering the incomplete criteria's weights to solve MCDM problem. Zhang, et al. [32] extended a series of aggregation operators in hesitant fuzzy and interval-valued hesitant fuzzy setting to solve the MAGDM problems.

In addition, Soares et al. [33] assessed the CO₂-cured mortars for heavy metals leachability by combining sewage sludge ash as filler. In this case, reactive magnesia, electric arc furnace slag, and Portland cement were used to produce three various CO₂-cured mortars that were cured though pressurized accelerated carbonation curing during a day. Soares et al. [34] evaluated the feasibility of combining seven various wastes sources as magnesia/filler replacement in Carbonated Reactive Magnesia Cement-based mortars.

In this paper, a group decision making model is designed under IVHFS to solve the sustainable cement block selection problem for construction industry by proposing a powerful MCGDM method under imprecisely. In classical fuzzy sets theory, it is usually difficult for the experts / DMs to express their opinions as an exactly number in [0,1]. Thus, it is more appropriate and functional to indicate this membership degree under an interval-valued. Often, DMs want allot some interval values of membership degrees for an object that IVHFS is a suitable tool to address with this issue for margin of errors. This paper, propose an IVHF-PSI method to solve complex group decision making problems, in which the ratings of the candidate alternatives are defined by linguistic terms and then are converted into IVHFS.

The rest of this paper defined as follows; in section 2, the relations / operations of IVHFS are illustrated. In section 3, the interval-valued hesitant fuzzy preference selection index (IVHF-PSI) technique is presented. In section 4, a real case study is considered to represent the feasibility of the presented method. Finally, some concluding remarks are provided in section 5.

2. Proposed IVHF-PSI method

In this section, the proposed IVHF-PSI method is presented based on following steps:

Step 1. Construct a group of DMs. These DMs assessment m possible alternatives A_i ($i=1,2,\dots,m$) among n criteria C_j ($j=1,2,\dots,n$) which are identified as benefit or cost types for evaluate the problem.

Step 2. Establish an interval-valued hesitant fuzzy decision matrix respecting to DM's judgment.

Step 3. Construct the normalized interval-valued hesitant fuzzy decision matrix for each DM.

$$M_k = \begin{bmatrix} \mu_{A_1}^k(x_1) & \mu_{A_1}^k(x_2) & \cdots & \mu_{A_1}^k(x_n) \\ \mu_{A_2}^k(x_1) & \mu_{A_2}^k(x_2) & \cdots & \mu_{A_2}^k(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{A_m}^k(x_1) & \mu_{A_m}^k(x_2) & \cdots & \mu_{A_m}^k(x_n) \end{bmatrix} \quad \forall k \quad (1)$$

Step 4. Specify the interval-valued hesitant fuzzy preference value ($IVHFPV_j^k$) by considering the Euclidean distance measure.

$$IVHFPV_j^k = \sum_{i=1}^m \left(\sqrt{\frac{1}{2l} \sum_{j=1}^l |R_{ij}^{lk\sigma(j)} - \bar{R}_j^{k\sigma(j)}|^2 + |R_{ij}^{uk\sigma(j)} - \bar{R}_j^{k\sigma(j)}|^2} \right) \quad \forall j, k \quad (2)$$

where \bar{R}_j^k is the mean of normalized value of criteria j that calculated as follow:

$$\bar{R}_j^k = \frac{1}{2m} \sum_{i=1}^m (R_{ij}^{lk} + R_{ij}^{uk}) \quad \forall j, k \quad (3)$$

Step 5. Determine the interval-valued hesitant fuzzy overall preference value (ψ_j^k) for each DM by regarding to deviation (ϕ_j^k) in interval-valued hesitant fuzzy preference value as follow:

$$\phi_j^k = 1 - IVHFPV_j^k \quad \forall j, k \quad (4)$$

$$\psi_j^k = \frac{\phi_j^k}{\sum_{j=1}^n \phi_j^k} \quad \forall j, k \quad (5)$$

The total interval-valued hesitant fuzzy overall preference value should be one, $\sum_{j=1}^n \psi_j^k = 1 \quad \forall k$.

Step 6. Determine the interval-valued hesitant fuzzy preference selection index ($[IVHFI_i^{lk}, IVHFI_i^{uk}]$) for each DM as follow:

$$[IVHFI_i^{lk}, IVHFI_i^{uk}] = \left[1 - \prod_{j=1}^n (1 - R_{ij}^{lk} \psi_j^k), 1 - \prod_{j=1}^n (1 - R_{ij}^{uk} \psi_j^k) \right] \quad (6)$$

Step 7. Determine the final interval-valued hesitant fuzzy preference selection index ($IVHFI_i$).

$$HIVFA(\tilde{h}_1, \tilde{h}_2, \dots, \tilde{h}_k) = \left(\bigoplus_{k=1}^K \left(\frac{1}{k} \tilde{h}_k \right) \right) = \cup_{\tilde{\gamma}_1 \in \tilde{h}_1, \tilde{\gamma}_2 \in \tilde{h}_2, \dots, \tilde{\gamma}_k \in \tilde{h}_k} \left\{ \left[1 - \prod_{k=1}^K (1 - \gamma_k^L)^{\frac{1}{K}}, 1 - \prod_{k=1}^K (1 - \gamma_k^U)^{\frac{1}{K}} \right] \right\} \quad (7)$$

Step 8. Rank the possible alternatives by choosing the maximum value of interval-valued hesitant fuzzy preference selection index by ordering equation as follows:

Let two IVHFSs as \tilde{M} and \tilde{N} on X . Two types of ordering of IVHFEs are suggested as component-wise ordering and the total ordering defined respectively.

$$\tilde{M} \leq \tilde{N} \text{ if } h_{\tilde{M}}^{\sigma(j)L}(x_i) \leq h_{\tilde{N}}^{\sigma(j)L}(x_i), h_{\tilde{M}}^{\sigma(j)U}(x_i) \leq h_{\tilde{N}}^{\sigma(j)U}(x_i) \quad \forall i=1,2,\dots,m; j=1,2,\dots,n \quad (8)$$

$$\tilde{M} \prec \tilde{N} \text{ if } Score(\tilde{M}) \leq Score(\tilde{N}) \quad (9)$$

$$Score(\tilde{M}) = \frac{1}{n} \sum_{i=1}^n \left(\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left[\frac{h_{\tilde{M}}^{\sigma(j)L}(x_i) + h_{\tilde{M}}^{\sigma(j)U}(x_i)}{2} \right] \right) \quad (10)$$

3. Case study

In this section, a real case study is presented to demonstrate the novel PSI method under IVHFS. The provided problem is defined for considering the environmental competencies according the costs and quality targets. In this respect, three possible alternatives ($A_i, i=1,2,3$) versus 10 conflicted criteria ($C_i, j=1,2,\dots,10$) are evaluated by a group of three DMs ($DM_k, k=1,2,3$). The candidate cement blocks are defined as A_1 : MgO+Cement+Sand+Filler(waste)+Water, A_2 : CaO+Cement+Sand+Filler(waste)+Water, A_3 : Cement+Sand+Water. The evaluation process to gather the information is implemented based on Delphi method. Then, the hesitant linguistic variables are defined by IVHFE that represented in Table 1. Also, the decision matrix is constructed by DM's opinions according to hesitant linguistic variables and then the matrix transform to normalized interval-valued hesitant fuzzy decision matrix by considering the IVHFEs. The results are represented in Table 2.

Table 1. Hesitant linguistic variable for rating the possible alternatives

Hesitant linguistic variable	Hesitant interval-valued fuzzy element
Extremely good (EG)	[1.00, 1.00]
Very very good (VVG)	[0.90, 0.90]
Very good (VG)	[0.80, 0.90]
Good (G)	[0.70, 0.80]
Moderately good (MG)	[0.60, 0.70]
Fair (F)	[0.50, 0.60]
Moderately poor (MP)	[0.40, 0.50]
Poor (P)	[0.25, 0.40]
Very poor (VP)	[0.10, 0.25]
Very very poor (VVP)	[0.10, 0.10]

Table 2. Decision matrix that established by hesitant linguistic terms

Criteria	Candidate cement blocks	Decision makers		
		DM_1	DM_2	DM_3
Price (C_1)	A_1	G	MG	G
	A_2	MG	F	F
	A_3	F	MG	F
Recyclable (C_2)	A_1	MG	MG	MG
	A_2	F	F	MP
	A_3	MP	F	F
Durability (C_3)	A_1	G	MG	G
	A_2	G	F	F
	A_3	F	MG	F
Eco-friendly (C_4)	A_1	G	G	MG
	A_2	MG	F	MG

Criteria	Candidate cement blocks	Decision makers		
		DM_1	DM_2	DM_3
Job creation (C_5)	A_3	F	F	F
	A_1	MG	MG	F
	A_2	MP	MP	P
	A_3	P	P	P
Air pollution (C_6)	A_1	MP	MP	P
	A_2	P	MP	MP
	A_3	P	P	P
Implementation risks (C_7)	A_1	MG	F	MG
	A_2	F	MG	MG
	A_3	MP	F	F
Efficiency (C_8)	A_1	VG	VG	G
	A_2	G	G	F
	A_3	F	F	G
Water consumption (C_9)	A_1	F	MP	MP
	A_2	MP	P	MP
	A_3	F	F	P
Technology requirements (C_{10})	A_1	MP	MP	P
	A_2	MP	P	P
	A_3	VP	MP	VP

The mean of normalized values for each criterion regarding to each DM are computed by using the Eq. (1), then, the interval-valued hesitant fuzzy preference values are determined by utilizing the Eq. (2) (Step 4). Give the results in Tables 3 and 4, respectively. As indicated in Table 5, the interval-valued hesitant fuzzy overall preference values are specified by considering the Eqs. (4) and (5) (Step 5). In this regard, we used the Eq. (6) to determine the interval-valued hesitant fuzzy preference selection index (Step 7). The results represented in Table 6.

Table 3. The mean normalized value of each criterion regarding to each DM (\bar{R}_j^k)

$C_j \backslash k$	$k=1 (\bar{R}_j^1)$	$k=2 (\bar{R}_j^2)$	$k=3 (\bar{R}_j^3)$
C_1	[0.300, 0.350]	[0.283, 0.333]	[0.283, 0.333]
C_2	[0.250, 0.300]	[0.266, 0.316]	[0.250, 0.300]
C_3	[0.316, 0.366]	[0.283, 0.333]	[0.283, 0.333]
C_4	[0.300, 0.350]	[0.283, 0.333]	[0.283, 0.333]
C_5	[0.208, 0.266]	[0.208, 0.266]	[0.166, 0.233]
C_6	[0.150, 0.216]	[0.175, 0.233]	[0.150, 0.216]
C_7	[0.250, 0.300]	[0.266, 0.316]	[0.283, 0.333]
C_8	[0.333, 0.383]	[0.333, 0.383]	[0.316, 0.366]
C_9	[0.233, 0.283]	[0.191, 0.250]	[0.175, 0.233]
C_{10}	[0.150, 0.208]	[0.175, 0.233]	[0.100, 0.175]

Table 4. The interval-valued hesitant fuzzy preference value ($IVHFPV_j^k$)

$C_j \backslash k$	$k=1 (IVHFPV_j^1)$	$k=2 (IVHFPV_j^2)$	$k=3 (IVHFPV_j^3)$
C_1	0.691607	0.656277	0.656398
C_2	0.586010	0.621041	0.586010
C_3	0.726923	0.656277	0.656398
C_4	0.691607	0.656398	0.656277
C_5	0.512880	0.512880	0.434107
C_6	0.397762	0.440289	0.397762

C_7	0.586010	0.621041	0.656277
C_8	0.762301	0.762301	0.726923
C_9	0.550668	0.476407	0.440289
C_{10}	0.416690	0.440289	0.307916

Table 5. The interval-valued hesitant fuzzy overall preference value (ψ_j^k)

$C_j \setminus k$	$k=1 (\psi_j^1)$	$k=2 (\psi_j^2)$	$k=3 (\psi_j^3)$
C_1	0.052893	0.057814	0.056708
C_2	0.071004	0.063740	0.068324
C_3	0.046836	0.057814	0.056708
C_4	0.052893	0.057793	0.056727
C_5	0.083546	0.081933	0.093394
C_6	0.103290	0.094143	0.099392
C_7	0.071004	0.063740	0.056727
C_8	0.040768	0.039981	0.045068
C_9	0.077065	0.088068	0.092374
C_{10}	0.100044	0.094143	0.114220

Table 6. The final interval-valued hesitant fuzzy preference selection index

$IVHFI_i^k$	$IVHFI_i^1$	$IVHFI_i^2$	$IVHFI_i^3$	$IVHFI_i (I_i)$
A_1	[0.45081, 0.50535]	[0.43371, 0.49090]	[0.41038, 0.47450]	[0.43187, 0.49040]
A_2	[0.38364, 0.44734]	[0.38208, 0.44990]	[0.36337, 0.43216]	[0.37643, 0.44319]
A_3	[0.32916, 0.40375]	[0.37518, 0.44298]	[0.33957, 0.41643]	[0.34827, 0.42129]

Finally, the final interval-valued hesitant fuzzy preference selection index is determined by Eq. (7), (Step 7), and then the potential alternatives are ranked by considering two type of ordering (Step 8). The results of proposed IVHF-PSI method compare with fuzzy compromise solution method that proposed by Vahdani et al. [35]. Both methods, selected the first cement block as a most suitable alternative among the candidate alternatives versus conflicted criteria. Gives the obtained results in Table 7. Although, the same results of comparison analysis show the feasibility and validity of the proposed method, the proposed approach of this study has some merits and advantages that could lead to reliable results. Meanwhile, IVHFS theory could help DMs to analysis complex real problems by covering the existed imprecise information. Moreover, the proposed methodology is tailored based on last aggregation concept that could prevent the data loss and reach to precise ranking results.

Table 7. Ordering the final IVHFEs by two types of ordering and comparison solution

	Component-wise ordering	Total ordering	Ranked by IVHF-PSI method	Ranked by Vahdani et al. [35] method
I_1	$I_1^l \geq I_2^l \geq I_3^l, I_1^u \geq I_2^u \geq I_3^u$	0.461142	1	1
I_2	$I_1^l \geq I_2^l, I_1^u \geq I_2^u$	0.409814	2	2
I_3	$I_2^l \geq I_3^l, I_2^u \geq I_3^u$	0.384784	3	3

4. Conclusions and future directions

This study has introduced a novel group decision making procedure-based on preference selection index under IVHFS environment to solve the imprecise / vague group decision problems. The presented IVHF-PSI method can help the DMs / experts to evaluate the possible alternatives versus the selected criteria in real world applications. The proposed approach could solve the complex group decision making problems without the criteria weights under uncertainty. In addition, the proposed IVHF-PSI method not only considers the linguistic terms to assessment the alternatives among the conflicted criteria, but also employed the IVHFSs that can prepare more flexibility than classical fuzzy sets. Furthermore, the proposed method has been introduced as a capable decision aid to assessment as well as suitable tool to cope with group decision problems under imprecisely for construction industry problems. Consequently, the obtained results of solving the sustainable cement block selection problem indicate that the first candidates (MgO+Cement+Sand+Filler (waste)+Water) can consider as a suitable alternative. Moreover, the aforementioned provided problem solved by a recent and relevant methodology from literature that same ranking results obtained. Although, same ranking results obtained, the proposed IVHF-PSI method has some merits and advantages such as preventing data loss, IVHFS theory, etc. that could lead to reliable and precise solutions.

For future research, the presented method could enhance by considering the relative importance of each DM as different. Thus, the DM's weights will be computed in interval-valued hesitant fuzzy setting and will be apply in proposed approach. In addition, defining the hierarchical

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