

# Structural Evaluation of Shiplap Hinge Joint Using Empirical and Strut-and-Tie Methods

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**Abstract** - Bridges designed before 1990 with shiplap hinge joints (SHJs) using classical approaches need to be evaluated to verify minimum reinforcing or anchorage and development length requirements to failure mechanisms that may occur as outlined in the *AASHTO LRFD Bridge Design Specifications* (2020). In addition, limited studies to date have focused on the consequences of these older bridge designs and their associated failure mechanisms when evaluating beam ledges with SHJs using classical approaches. In this study, the behaviour of SHJs in existing bridges is examined analytically using two methods, empirical and strut-and-tie, to demonstrate the potential application of each technique on assess existing structures. Most importantly, this study provides insight on how strut-and-tie methods can be applied to evaluate existing bridges with in-span hinge connections and how to adequately account for development lengths using the strut-and-tie method compared to the empirical method. Nonlinear finite element (FE) models are generated as a physics-based to represent the expected ultimate capacity and associated failure mechanisms of beam ledges. The results revealed that the estimated strength capacity of the SHJs using the strut-and-tie method was less than both empirical and FE methods, suggesting that the lower-bound solution may be the more critical evaluation method. Overall, the results illustrate the various governing failure mechanisms from the different methods when evaluating the section capacity, sufficient steel area, and development length, which influence the structural response of SHJs when loaded.

**Keywords:** shiplap hinge joints, strut-and-tie method, empirical method, finite element modelling.

## 1. Introduction

Shiplap hinge joints (SHJs) are pivotal elements of the overall structural system for in-span bridges, and their behaviour is one primary concern due to bringing irregularity to the span section. The De Concorde overpass structure in Laval, Québec is an example of a bridge designed with SHJs and suddenly collapsed in 2006 due to the thick cantilever slabs' lack of shear reinforcement, incorrect installation of the top bars, and the poor quality of the concrete [1]. This collapse raised concerns about evaluating other bridges designed with SHJs since these sections could be designed utilizing different analysis methods yet often result in different failure mechanisms. Consequently, a proper method should be used to evaluate the safety of existing bridges with similar detail, age, and period of construction. In this study, the behaviour of SHJ in existing bridges is examined using two methods, empirical and strut-and-tie, in order to demonstrate the potential application of each technique on assess existing structures to provide engineers an understanding how to evaluate existing complex concrete elements and why various failure mechanisms occur. Additional nonlinear FE analyses are carried out to validate the results from both methods.

## 2. Background

### 2.1. Empirical Method

The empirical method can be used to design an in-span hinge following the provision for beam ledges per *AASHTO LRFD Bridge Design Specifications* [2]. According to these provisions, four different failure mechanisms can occur: 1) shear failure and horizontal forces, 2) flexure failure, 3) tension failure mechanisms or failure in hanging the load up, 4) punching shear failure due to concentrated load, and 5) bearing failure. Each of these possible failure mechanisms needs to be inspected independently. More details about the empirical method can be found in Obayes and Head [3].

### 2.2. Strut-and-Tie Method

Strut-and-tie method provisions have been included in the *AASHTO LRFD Bridge Design Specifications* since 1994 [4]. It is a lower-bound theory of plasticity, and based on lower bound theorem. Accordingly, the results of the capacity

analysis obtained from the strut-and-tie method should be conservative as long as equilibrium and failure criteria are satisfied [5] [6]. Additionally, sufficient reinforcement anchorage should be provided.

The strut-and-tie method can provide a solution for elements that have irregular geometry with discontinuities. The discontinuity is either due to a concentrated load or abrupt change of the geometry, or both. A strut-and-tie model (STM) consists of compression struts that represent concrete stress, tension ties that represent one or more layers of tension reinforcement, and nodes that are link struts and ties together. The complex flow of the stress through structural components can be simplified into a truss model which idealizes how concrete elements disturbed by a load or geometric discontinuity may be represented. The load patterns can be expressed as struts for compression regions and ties for tension regions when load is transferred to the support through truss elements. However, this method has been a source of confusion for design practitioners [5] because there might be more than one solution for a particular problem.

### 2.2.1. Anchorage of Ties

To allow ties to utilize their full capacity, the reinforcing bars must be correctly anchored, where the anchorage check and the development length for the tension ties are a crucial feature of nodal zones. According to AASHTO LRFD Bridge Design Specifications [2] and ACI PRC-445.2 [7], the anchorage length of a tie can be measured from the point when the resultant tension tie force enters the extended nodal zone. The tension force should be transferred to the node regions of the truss. Therefore, the anchorage of ties must be checked to achieve the resistance assumed by the STM and yield will occur.

A limit to the tie capacity should be applied if an insufficient development length is provided [8]. Based on previous research, there are two approaches to reduce the tie capacity to account for unsatisfactory anchorage of ties: 1) a simplified approach and, 2) rigorous approach. A simplified approach consists of multiplying the tension ties by a reduction factor  $\lambda$ . This is a simple approach but does not accurately satisfy the requirements of a lower bound method [9]. Another approach, deemed as a rigorous approach, consists of having the forces in the bars to not exceed  $F_{tie}$  specified in Eqs. (1) [9].

$$F_{tie} = \frac{\alpha\beta}{\gamma_{mb}} \sqrt{f_c} \pi d_b l_a \quad (1)$$

where  $l_a$  is the ratio of the actual provided anchorage length of the bar to the full anchorage length,  $\alpha$  is the residual bond strength factor,  $\beta$  is the bar type coefficient, and  $\gamma_{mb}$  is a partial safety factor equal [8].

## 3. Methodology

The SHJ for a midwestern United States bridge has been analyzed based on the empirical method and the strut-and-tie method implemented in AASHTO LRFD Bridge Design Specifications [2]. The design principles and criteria presented in the background section for the two methods, empirical and strut-and-tie, serve as a basis for the existing SHJs reinforced concrete assessment.

### 3.1. Bridge Description and Design Details

The bridge consists of six cast-in-place girders with the main span consisting of reinforced concrete box girders. The bridge was built in 1972 as two separate structures for the northbound and southbound directions with 4 lanes and was opened to traffic in 1973. The main span type of the bridge is a curved box girder reinforcement concrete. The total number of spans is 11 with a main span length of 29.8 m and total length of 283.7 m. The deck material is cast-in-place (CIP) concrete, and the wearing surface consists of latex modified concrete, which was installed in 2001 with a depth of 49 mm. The reinforcing layout of the SHJ is introduced in Figure 1, which is also used for the nonlinear FE analysis which will be described later. The joint supports six girders from the adjacent structure and, in turn, is supported by two circular columns with 1.22 m diameters at a distance of 3.0 m from the centreline of the bearing bed that has dimensions 305 x 229 x 76 mm<sup>3</sup>. The seat depth is 0.8 m and slab depth is 1.75 m. Figure 2 illustrates the dimensions of ledges of the joint with the position of the bearing bed. The bridge was designed according to AASHTO specifications[10].

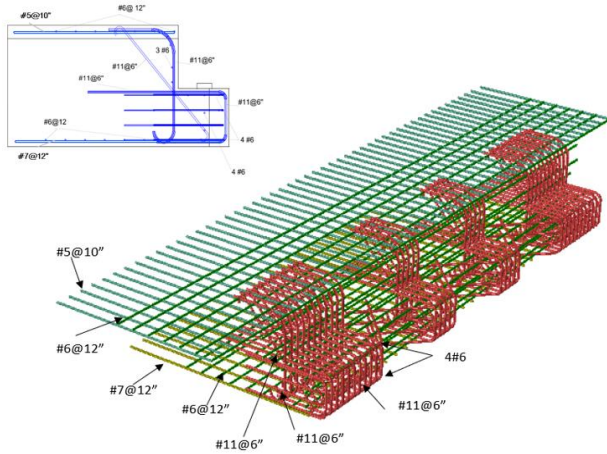


Fig. 1: Existing reinforcing layout of the SHJ

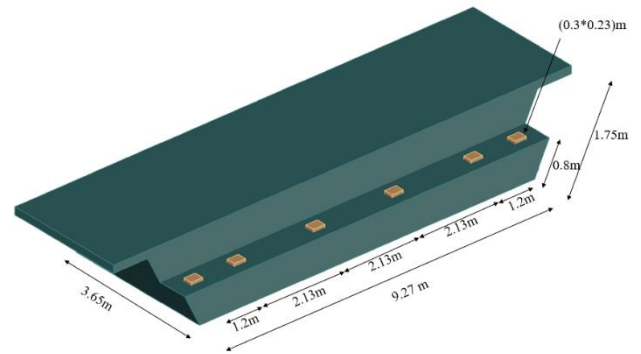


Fig. 2: The dimensions of ledges of the shi lap joint

### 3.1.1. Loading Details

Load reactions from adjacent frame are calculated on a critical neoprene pad at the joint using QConBridge. The dead load, including the weight of the girder, slab, asphalt, barrier, and steel bridge rail, was calculated as longitudinal distributed loads for each girder, and the highest load was picked since there is different length for each girder. Shear and moment distribution factors were calculated based on AASHTO LRFD Bridge Design Specifications [2]. The multiple presence factor and centrifugal forces due to the curvature of the bridge were also added. The dynamic load allowance (IM) for the truck load was 33% using the HL-93 notional loading. The unit weight of the reinforced concrete is assumed to be 22.7 kN/m<sup>3</sup> and the concrete strength is 27.5 MPa, which is less than 34.4 MPa [2]. The Strength 1 load combination governed, giving a value of 907 kN at the bearing pad.

### 3.2. Use of strut-and tie methods for assessment of existing SHJ

The strut-and-tie method was carried out to determine the loadings and capacity on a typical 1.22 m wide strip of the cantilever, corresponding to the width that governs the failure mechanisms using the empirical method, which approximately corresponds to the width of one box girder. The boundaries of the D-regions of the cantilever slab of the joint are determined. The factored concentrated applied load that has been used for the empirical method is also used for the strut-and-tie method.

A combination model, that is Model 1 and Model 2 (Figure 3), was developed to accurately represent the reinforcement layout of the bridge joint studied here. Model 1 is to represent the orthogonal reinforcement, while Model 2 is to represent the diagonal reinforcement bar that transfers part of the applied load to the full-depth section[8] [11]. However, both Model 1 and Model 2 are common STMs for SHJs and could be used independently. For example, Model 1 is a possible STM in case of reinforcement layout is diagonal such as in the Europe code SHJ design, and Model 2 is a possible STM in case of orthogonal reinforcement such as in the US SHJ design. Since the reinforcement layout of the joint is a combination of diagonal and orthogonal reinforcement, the applied concentrated load on the bearing bed from the adjacent structure's girder has been divided into two applied loads. These represent the portions of the girder load carried by two two-dimensional STMs, Model 1 and Model 2 [12].

In Figure 3, the dashed lines represent compressive struts, and the solid lines represent the tension tie. When the strut-and-tie method is used for the assessment, the layout and the orientation of the reinforcement govern the locations of the ties within the STM. The anchorage and the development length conditions of the reinforcement govern the location of the nodes. The ties were aligned with the layout of the reinforcement, and the width is twice the distance between the extreme tension fiber and the reinforcement bars' centroid. The intersection of the vertical bar close to the edge with the centroid of the strut

is determined to be the location of the Node C. Node E is determined as the intersection between Node F and Node A. Other ties and struts are connected to complete the stress flow with angles larger than 25 degrees, as required by AASHTO LRFD strut-and-tie method provisions. In Figure 4, the validity of the STMs for the joint was checked using stress trajectories obtained from 3D nonlinear FE analysis using ABAQUS software, which will be described later. Finally, the forces of the completed model are calculated using truss static analysis to compare with maximum capacities of the struts and ties. The forces acting on each of the boundaries of the D-region for the two models are equilibrated by the load path defined by the models. The struts, ties, and nodes have been labelled for references, and shown in Figure 3.

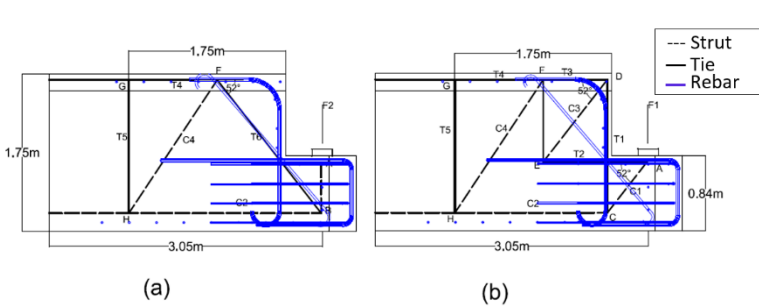


Fig. 4: STMs: (a) Model 1 (b) Model 2; with reinforcement details.

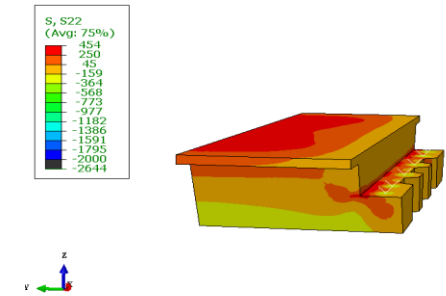


Fig. 3: Stress trajectories (in psi unit) in y-direction obtained from 3D Nonlinear finite element analysis using ABAQUS.

### 3.3. Nonlinear FE Modelling Of SHJ

To investigate the performance of the ledge beam, a nonlinear FE using a commercial software ABAQUS has been used. The node type used to model the concrete is an eight-node solid 3D continuum element with incompatible modes and linear bricks (C3D8I); this element has three translational degrees of freedom at each corner node. The reinforcement bars are modelled using two, node linear, three-dimensional truss elements (T3D2). The rebars elements are embedded in the concrete using an ABAQUS constraint function called “Embedded Region”, which allows for a full bond between concrete and reinforcement. The embedment feature in ABAQUS allows both the embedded elements nodes (rebar) and host elements nodes (concrete) to have the same translational degrees of freedom. Because of the way the longitudinal column reinforcement is ended within the slab (i.e., straight bar anchorage), the columns are expected to behave as fixed supports, and the back face of the model is restraining from the movement in longitudinal direction to simulate the continuous of the span. The load is applied as displacement control on a reference point constraint to act as a rigid body with pin nodes connections to simulate the bearing bed. The material behaviour of the concrete follows the concrete damage plasticity model (CDP), which is suitable for both nonlinear compressive and tensile behaviours [13]. The compressive behaviour of the concrete is modelled using the modified Hognestad stress-strain formulation [14]. The material behaviour of the reinforcing steel is modelled as an elastic-perfectly plastic material hardening; considering the reinforcement in size and distribution in the SHJ. The average aspect ratio for the FE model was 1.25.

First, girder loads were applied on the bearing bed (i.e. girder load that was used for the empirical and strut-and-tie analyses) to compare the stress trajectories with the two proposed STMs. Figure 4 presents the stress trajectories in y-direction due to concentrated force of 907 kN at each bearing bed; the stress trajectories show good agreement with the proposed STMs. Then, displacement load control was used to calculate the collapse load and find the capacity of the section, where the displacements increased with time at the locations of the bearing load until failure occurred.

## 4. Results

### 4.1. Empirical Method Calculation

The calculations of the different failure mechanisms of the empirical method were carried out to determine the loadings and capacity using the section geometry, 27.5 MPa concrete strength, and 413 MPa for the reinforcement. The lowest

estimated strength of the aforementioned failure mechanism yields the total capacity of the ledge. Table 1 summarizes the capacities of each failure mechanism that could occur using the empirical method. It is worth noting that the failure mechanism based on the lowest of the estimated capacities for SHJ was punching shear.

Table 1: Capacity of joint 8SB using empirical method (kN) [3].

Bearing	Shear Fraction	Nominal Interface Shear	Punching Shear	Hanger Reinforcement
2286	4826	4492	<b>1993<sup>s</sup></b>	2998

<sup>s</sup> Note: Controlling failure mechanism

## 4.2. Strut-and-Tie Method Calculation

The upper and lower chords resist the boundary bending moment with equal but opposite forces computed to satisfy the equilibrium. Once the forces in all components of STMs have been calculated, the individual members' permissible stresses must be validated and should not exceed the developed stresses due to applied load [8].

### 4.2.1. Anchorage of Tie and Development Length

Although it was determined that the area of the U-shaped hanger bars, and the diagonal bars was sufficient, the anchorage details of this reinforcement need to be checked. The anchorage length of the size, no.11 transverse bars needs to be checked at node A, E, and F in tension and node B and C in compression. Figure 5 shows the available development length for all nodes. Based on the chosen geometry of Node A, the available length is 508 mm. For Tie 2, the required amount of reinforcement surpasses the amount of reinforcement provided, and by applying the modification factors following the simple approach, the modified tension length is therefore calculated to be 210 mm. There is enough development length supplied at Node A. Available length of the reinforcement that exits the extended nodal zone of Node E is checked and it was found to be adequate. For Node F, two checks for the development length are required for the straight and the 180-degree hooks. The straight development length for the Node F was less than what is required, so the capacity of the tie should be modified by 0.506 reduction factor. For the standard hooks that are used is 180-degree bend, check will be following AASHTO LRFD Bridge Design Specifications [2]. Regarding the standard hooks in tension, the development will be determined as the basic development length of a standard hook in tension. In the compression, hooks are not considered effective in developing bars [2], subsequently, hooks in Node B and Node C, may not have effective developing lengths. The straight development length in compression in Node B and C were found to be adequate. Table 2 explains the determination of the controlling failure mechanism based on the elements and the nodes of the model; where the resistance of the three node faces both at the slab and ledge were checked. The strength of the two models' ties was also checked.

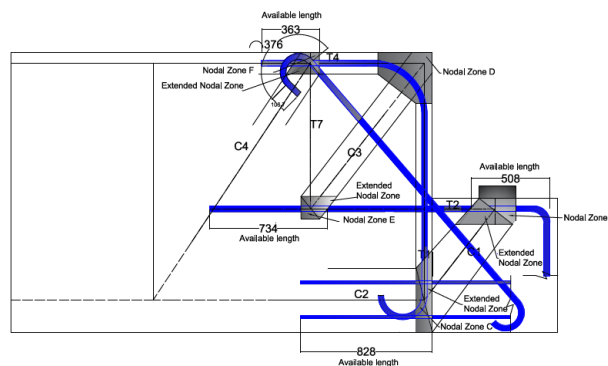


Fig. 5: Available development length for ties (dimensions in mm)

Table 2: Capacity of joint 8SB using AASHTO LRFD strut-and-tie method (kN)

<b>Bearing</b>	<b>Resistance of the node face at the slab</b>	<b>Resistance of the node face at the ledge</b>	<b>Strength of Ties</b>
4840	2148	2909	<b>1517</b> &

& Note: Controlling failure mechanism

### 4.3. Nonlinear FE Analyses Results

The failure load indicated from the FE results is 3114 kN, while the maximum capacity from the empirical and strut-and-tie methods are 1993 kN and 1517 kN, respectively at the ledge. The failure mechanism is assumed by CDP to be either tensile cracks or compressive crushing, which are the dominated failure for the concrete under low confining pressure [15]. Since the SHJ location for the bridge is under low confining pressure, it will behave in a brittle manner and, as expected, the cracking under tension and crushing under compression were the main failure mechanisms. The concrete reached its maximum strength capacity before the bars reached their yield. The cracks initiated at the ledge to slab interface which indicated that shear friction failure was governed and it was not governed by the yield stress of the reinforcement.

Since there is principal compressive stress due to the bearing bed loads and the principal tensile stress at the interface reaches the tensile strength of concrete, a crack is initiated parallel with the direction of the principal compressive stress, which was expected since the interface is considered to be the highest shear stress region. Diagonal failure or shear failure is the term for the sort of failure brought on by these cracks, which typically occurs in an extremely brittle and sudden manner [16]. In addition, SHJs were expected to experience diagonal tension failure which was the governing failure mechanism due to the limited number of stirrups. However, the SHJ has a sufficient longitudinal reinforcement ratio at the seat, which led to forming a compression zone. Therefore, shear cracks can start from previous flexural cracks with ease, but they cannot pass through the compression zone which is also the case in the FE model. Moreover, in the ABAQUS FE model, the section force due to applied load at shear friction plane was approximately 2891 kN when the force section at the punching plane reached approximately 2002 kN (controlling failure mechanism using empirical method). This indicated that the controlling failure is not punching shear and no yielding at the reinforcement, where the hanger reinforcement only reached 16% of its yield strength at the failure.

Table 3 gives a comparison between the failure mechanism using the empirical method, the strut-and-tie method, and FE analysis. Resistance of the node face controlled the capacity of the ledge at Node E for the strut-and-tie method for the slab, and the capacity of the ledge at Node A. The estimated failure load indicates that the result from the empirical was closer to the FE analysis. However, both results from the empirical and strut-and-tie methods lower than the result produced from the FE model.

Table 3: Comparative analysis using various methods

<b>Method</b>	<b>Capacity (kN)</b>	<b>Predicted failure mechanism</b>
Strut-and-tie method	1517	Strength of Ties
Empirical method	1993	Punching Shear
ABAQUS nonlinear analysis	3114	Shear Friction

## 4. Discussion

In this study, two design and evaluation methods were investigated analytically for evaluating the accuracy of using empirical and strut-and-tie methods. It was not a trivial task to compare the advantages and disadvantages of the two methods. Each method is characterized by its own pros and cons. All the equations of the different failure mechanisms per the empirical method are explicitly stated in AASHTO LRFD Bridge Design Specifications [2] to estimate the capacity of the seat of SHJ and are easier to use with a good level of confidence. In addition, experience related to predicting the stresses' flow through the section when using strut-and-tie method is not required. On the other hand, the strut-and-tie method requires different steps that must be completed before starting the analysis. One example would be knowing the stresses' trajectories, which requires engineering judgment and experience about the reinforcement concrete behaviour to develop the truss model. In addition, the strut-and-tie method is an iteration method that can produce different results for different models. However, iterations can help in the optimization of the design and utilize the maximum section capacity. The hanger reinforcement for the empirical method must be added to the shear reinforcement required on the slab supported by the reactions, which might overestimate the reinforcement required. The strut-and-tie method can directly estimate the reinforcement required for transferring the compression load to the tension chord at the top of the slab and the reinforcement to resist shear stresses in the slab. As a result, the strut-and-tie method might suggest itself to be more economical than the empirical method.

Regarding the development length, it is required to be calculated and checked for both methods, however, in both methods the critical sections are different. For the empirical method, the first critical section is located at the interface of the shear transfer on both sides. The other critical section identified from the empirical method is determined by the punching shear failure, where the critical section is located around the bearing bed at a distance equal to the distance from top of ledge to compression reinforcement. For the strut-and-tie method, the available development length can be determined based on the location of the nodal zone and the extended nodal zone. All ties are required to be properly anchored for all appropriate nodes of the truss model. Bearing capacity must be checked when using the strut-and-tie method and the empirical method.

## 5. Conclusion and Future Research Needs

Both the empirical design approach and the strut-and-tie were used to estimate the capacities of the SHJ. The analytical results were compared to the computational FE model using ABAQUS software to predict the failure mechanism. The following conclusions can be made based on the failure mechanisms and capacity estimations resulting from the comparison of the empirical method, strut-and-tie method, and FE analyses:

- **Failure mechanisms:**

- o The failure mechanisms were identified based on the strength capacity of the ledge, and a comparison of results and their corresponding controlling failure mechanisms were as follows: strength of hanger tie (strut-and-tie method), punching shear (empirical method), and shear friction (FE analysis).

- o Based on the FE analysis, strains indicating cracking of concrete were observed on the interface between the seat and the slab, which refers to the ledge failure mechanism, and called shear friction. Hence, shear friction failure is likely to be a governing ledge failure mechanism. For the bridge evaluated in this study, the prediction of the governing failure mechanism from the empirical method occurred at a lower strength capacity estimate 1993 kN than the results obtained from FE analysis 3114 kN. In addition, the section force due to applied load at the shear friction plane in the FE model was approximately 2891 kN, which was larger than the 1993 kN section force along the punching (shear) plane as predicted by the empirical method. Consequently, punching shear was not the governing failure mechanism revealed by the FE model.

- **Strength capacity:**

- o As expected, the strut-and-tie method provided the lowest estimate of the strength capacity of the ledge 1517 kN since it is a lower-bound design and based on lower bound theorem.

- o From the FE model, it was verified that the hanger reinforcement only reached 16% of its yield strength at failure and that the controlling failure was not due to yielding of the reinforcement.

- o Comparing the strength capacity of the ledge from physics-based ABAQUS FE model to the estimated capacity from the two methods revealed that the estimated strength capacity of the SHJs using the strut-and-tie method was less than both empirical and FE methods. This may suggest to practitioners and researchers that while requiring a bit more insight, there

may be benefits to using the strut-and-tie method to provide efficient estimates (from a lower-bound solution perspective) when evaluating the existing structural capacity of SHJs.

o Experimental testing can be conducted in the future to verify structural behaviour and failure mechanisms predicted by the two methods and FE analysis.

## Acknowledgements

The authors would like to express gratitude to Mr. Daniel Baxter for providing bridge details for the case study. Financial support for this study was provided by the University of Delaware.

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