

Finite Element Analysis of Damage-Induced Frequency Reduction in Reinforced Concrete Beams Using the Concrete Damaged Plasticity Model

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Abstract - This paper presents a finite element investigation of the static and dynamic responses of reinforced concrete beams subjected to progressive damage, utilising the Concrete Damaged Plasticity (CDP) model in ABAQUS. The study models a simply supported beam under four-point bending, capturing nonlinear behaviour, stiffness degradation, and cracking through advanced constitutive modelling. The simulation incorporates both tensile and compressive damage mechanisms, validated against experimental load-deflection data. Natural frequencies are extracted at various damage stages to analyse the correlation between structural degradation and dynamic properties. Parametric studies examine the influence of concrete compressive strength, reinforcement ratio, and beam geometry on the sensitivity of natural frequency to damage progression. Results demonstrate that lower concrete strength and reinforcement ratios accelerate frequency reduction, while beam geometry significantly affects the onset of dynamic response changes. The findings confirm that shifts in natural frequency serve as effective indicators of damage, supporting vibration-based structural health monitoring approaches. This work enhances the understanding of damage-sensitive dynamic parameters and provides a robust computational framework for assessing and monitoring the integrity of reinforced concrete structures.

Keywords: Reinforced concrete beams, Concrete damaged plasticity (CDP), Finite element analysis, Static and dynamic response, Structural health monitoring, Natural frequency, Damage progression, ABAQUS.

1 Introduction

Reinforced concrete is a composite material with nonlinear behaviour marked by distinct tension and compression responses, stiffness degradation, and cracking, making numerical modelling challenging. The Concrete Damaged Plasticity (CDP) model is widely used for its ability to capture both tensile cracking and compressive crushing under different loading conditions [1]. The foundation of modern concrete plasticity models was laid by a study in 1989, which proposed a constitutive model capturing stiffness degradation and matching experimental data [1]. Later, other researchers refined it by introducing a damage-based model with separate tensile and compressive damage variables and a multi-hardening yield function for cyclic loading [2].

Combining plasticity and damage mechanics offers a more complete representation of concrete behaviour. As Omidi and Lotfi noted [3], damage mechanics capture strain softening and stiffness loss, while plasticity accounts for residual strains. The Concrete Damaged Plasticity (CDP) model in software like ABAQUS uses this combined approach to simulate tensile cracking and compressive crushing, making it well-suited for analysing reinforced concrete under different loading schemes [4]. For example, a study confirmed CDP's superiority over the Concrete Smeared Cracking model in predicting reinforced concrete behaviour [5], especially under cyclic loads. Recent enhancements, like those accounting for low-cycle fatigue [6], improve the capture of triaxial compression responses and damage accumulation under repeated loading.

Dynamic properties like natural frequencies are critical for structural health monitoring, as damage-induced stiffness reductions strongly correlate with frequency shifts. Experimental studies, such as those on scaled reinforced concrete beams, revealed reductions in frequency under progressive flexural cracking [7]. Similarly, corrosion damage shows measurable frequency declines [8]. These findings reinforce the value of vibration-based methods, especially when combined with computational models, for detecting and evaluating structural damage, including shifts in natural frequencies that indicate damage location and severity in concrete structures [9, 10].

This paper presents a finite element study using the CDP model in ABAQUS to analyse the static and dynamic responses of reinforced concrete beams under progressive damage. It explores how parameters like concrete compressive strength,

reinforcement ratio, and beam geometry affect the relationship between damage progression and natural frequency reduction. By combining advanced constitutive modelling with dynamic analysis, this work enhances the understanding of damage-sensitive indicators for assessing and monitoring reinforced concrete structures.

2 Methodology

2.1 Concrete damaged plasticity

A finite element simulation for the reinforced concrete beam was built using ABAQUS software. This simulation represents concrete nonlinearity through a model called concrete damaged plasticity (CDP). The model combines damage mechanics and plasticity theory by accounting for plastic strains and the reduction in stiffness of the concrete section due to damage. This makes the model suitable for various applications, such as monotonic, cyclic, and dynamic loading. Both tensile and compressive behaviour of concrete are incorporated into the model; for tensile behaviour, the tension stiffening effects and energy resulting from fracture are included to simulate post-failure stress-strain relationships. Under uniaxial tension, concrete follows a linear elastic behaviour until reaching a value of stress σ_{t0} , as shown in Fig. 1a, which corresponds to the formation of concrete microcracks. Compressive behaviour is built using stress-strain based calibrated data from lab experiments, with inelastic strain parameters and compressive strength integrated into the analysis. Under uniaxial compression, concrete exhibits linear elastic behaviour until reaching the value of initial yield stress σ_{c0} . After that, concrete enters the plastic behaviour phase, which is characterised by strain hardening before reaching concrete ultimate stress σ_{cu} , after which strain softening starts, as shown in Fig. 1.

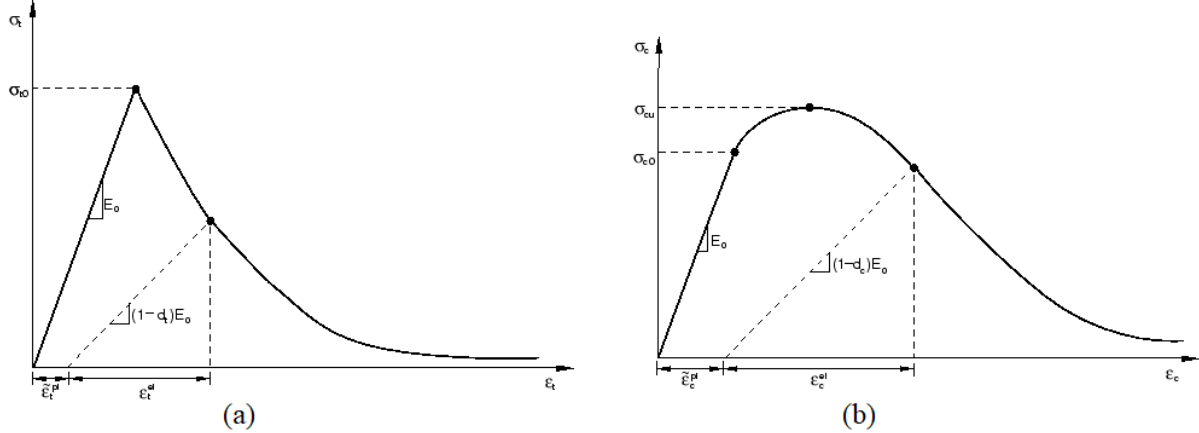


Fig. 1: Concrete response to uniaxial loading (a) tensile (b) compressive [4].

The CDP model accounts for stiffness degradation during unloading and provides accurate predictions of strain evolution under loading conditions. Degradation in stiffness is governed by damage variables d_t and d_c . The stress-strain relationship in Figure 1 can be described by the following equations:

$$\sigma_c = (1 - d_c)E_o(\tilde{\epsilon}_c - \tilde{\epsilon}_c^{pl}) \quad (1)$$

$$\sigma_t = (1 - d_t)E_o(\tilde{\epsilon}_t - \tilde{\epsilon}_t^{pl}) \quad (2)$$

Where σ_c , σ_t are compressive and tensile stresses, d_c , d_t are compressive and tensile damage variables, E_o is young's modulus of elasticity $\tilde{\epsilon}_c$, $\tilde{\epsilon}_t$ are compressive and tensile strains, $\tilde{\epsilon}_c^{pl}$, $\tilde{\epsilon}_t^{pl}$ are compressive and tensile plastic strains.

Plasticity theory in concrete describes the permanent deformations that occur after exceeding the elastic limits. It is characterised by several parameters: the dilation angle (ψ), which controls volumetric expansion; the flow potential eccentricity (ϵ), which describes the shape of the plastic surface and affects the flow potential; the ratio of biaxial to uniaxial compressive yield stress (σ_{b0}/σ_{c0}); the ratio of the second stress invariant on the tensile to compressive meridian (Kc); and the viscosity parameter (μ). The values for these parameters that are used in the model are summarised in Table 1.

Table 1: CDP model parameters.

Parameter	ψ	ε	σ_{b0}/σ_{c0}	K_c	μ
Value	35°	0.01	1.16	2/3	0.002

2.2 Model description

A simply supported reinforced concrete beam with a rectangular cross-section was modelled in ABAQUS, based on the experimental setup from previous [11]. The beam geometry, including the cross-section and steel reinforcement details, is shown in Fig. 2 and Table 2. The beam was subjected to four-point bending to simulate different damage levels under static loading. The analysis was performed in two steps: a static general step for load application, followed by a frequency extraction step to determine the natural frequencies corresponding to each damage level.

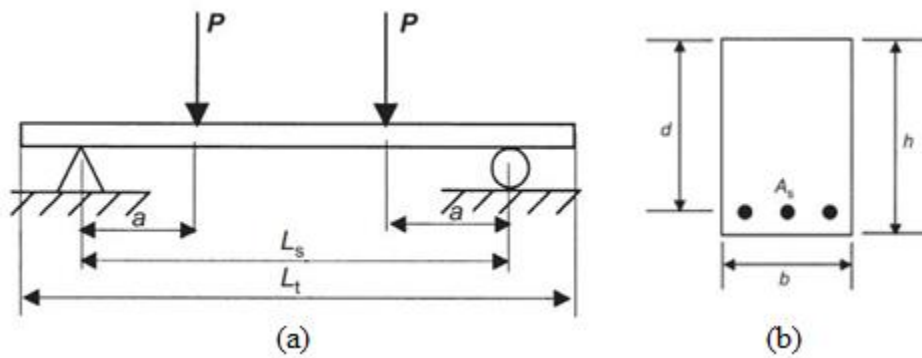


Fig. 2: Simply supported reinforced concrete (a) four-point bending test configuration (b) beam cross section [11].

Table 2: Geometry of the reinforced concrete beam.

Dimension	L_t	L_s	a	b	h	d
Value (mm)	3353	3048	1143	127	254	210

The material properties used in the finite element model are summarised in Table 3. Concrete was modelled using its mechanical characteristics, including density, Young's modulus, Poisson's ratio, compressive strength, and modulus of rupture. Steel reinforcement was modelled as an elastic-perfectly plastic material, with its relevant properties also listed in Table 3. The steel stress-strain behaviour was idealised as elastic-perfectly plastic, as illustrated in Fig. 3.

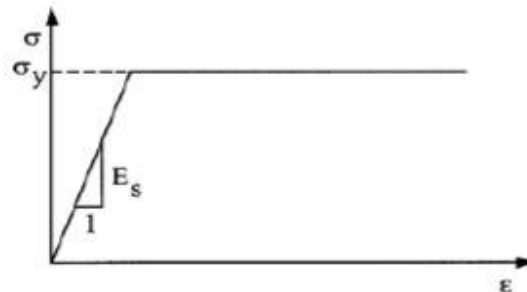


Fig. 3: Steel reinforcing bars elastic perfectly plastic model [12].

Table 3: Material Properties of Concrete and Steel Reinforcement.

Property	Concrete	Steel Reinforcement
Density (kg/m ³)	2368	7850
Young's Modulus (MPa)	30,480	200,000
Poisson's Ratio	0.19	0.3
Compressive Strength (MPa)	41.37	—
Modulus of Rupture (MPa)	4.0	—
Yield Strength (MPa)	—	276
Stress-Strain Behavior	Nonlinear	Elastic-perfectly plastic

A three-dimensional finite element model was developed using ABAQUS to simulate the behaviour of a reinforced concrete beam under four-point bending. Concrete was modelled using C3D8R elements (8-node linear brick elements with reduced integration and hourglass control), while the steel reinforcement was represented by T3D2 elements (2-node linear 3D truss elements). The steel bars were embedded within the concrete using the embedded region constraint in ABAQUS, ensuring perfect bond behaviour between the two materials. The analysis consisted of two main steps. First, a static general step was used to apply the loading, simulating a four-point bending test, as shown in Fig. 4. The load was varied across different analyses to induce multiple levels of damage in the beam. This was followed by a frequency extraction step to determine the natural frequencies corresponding to each damage level.

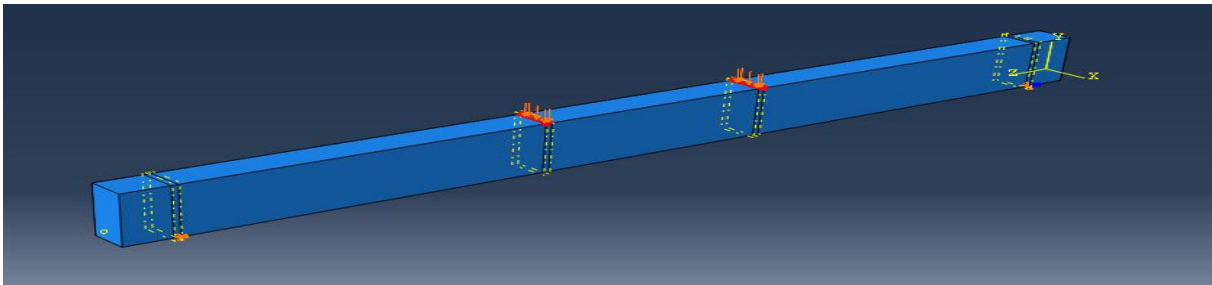


Fig. 4: Reinforced concrete beam finite element model showing four-point bending.

3 Results and discussion

3.1 Static load

A four-point bending configuration was adopted for testing the reinforced concrete beam, as illustrated in Fig. 2 and Fig. 4. In this setup, the applied load P was incrementally increased at two symmetric loading points to induce progressive levels of damage in the beam. At each loading level, the corresponding natural frequency was recorded to assess the dynamic response under increasing damage conditions.

To validate the finite element model, a comparison was made between the simulated deflection results from ABAQUS and the experimental data obtained from the second beam model. As shown in Fig. 5, the finite element results closely follow the experimental trend across the full range of loading. The model demonstrates strong agreement with the measured deflections, particularly in the linear elastic range (up to approximately 4 kN), and continues to align well even as the beam enters the nonlinear response phase.

It is important to note that the onset of visible cracking was observed at a load level of approximately 4 kN, as depicted in Fig. 6. This transition marks the beginning of nonlinear behaviour in the load-deflection curve, where stiffness degradation becomes apparent. The finite element model successfully captures this behaviour, indicating its capability to simulate both elastic and inelastic damage responses in reinforced concrete beams under flexural loading.

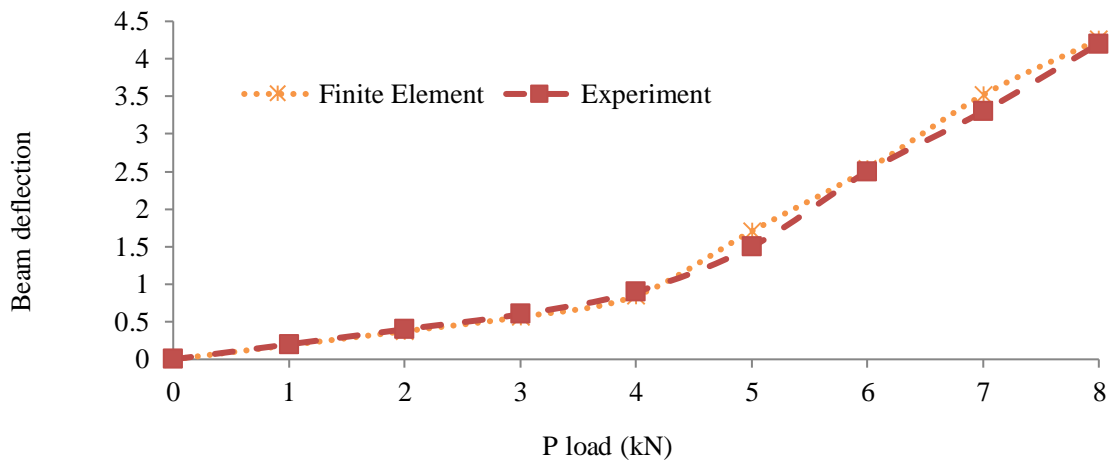


Fig. 5: Mid-span deflections versus load.

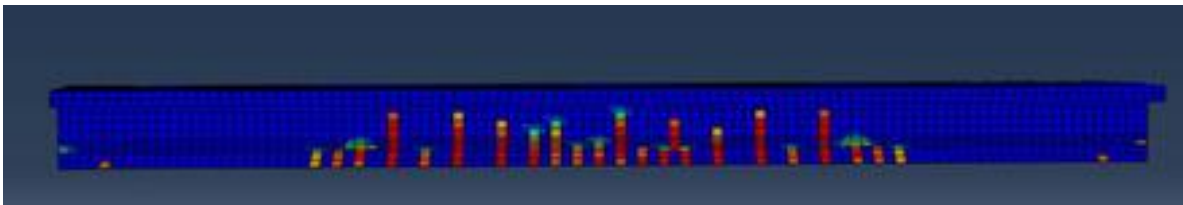


Fig. 6: Cracking damage in the reinforced concrete beam.

3.2 Free vibration

Building upon the previous analysis of the beam's response under four-point bending, the extracted natural frequencies at various loading levels were used to investigate the relationship between structural damage and dynamic behaviour. A parametric study was conducted to examine the sensitivity of the first natural frequency to both concrete compressive strength and tensile reinforcement ratio.

Fig. 7 illustrates the variation in natural frequency as a function of applied load for beams with different concrete compressive strengths. The original model, with a compressive strength of 41.37 MPa, serves as the reference case. As shown, beams with lower compressive strengths exhibit a more significant and earlier reduction in frequency. This trend indicates that concrete with reduced strength initiates cracking at lower load levels, leading to a faster degradation in stiffness and, consequently, a sharper decline in dynamic response. The onset of frequency reduction—associated with the appearance of cracks—is clearly load-dependent and occurs earlier for weaker concrete mixes. Fig. 8 presents a complementary analysis comparing the influence of tensile reinforcement ratio while keeping the concrete compressive strength constant. Results reveal that beams with lower steel reinforcement ratios experience a greater reduction in natural frequency, particularly after the onset of cracking. This behaviour is attributed to the reduced capacity of the lower-reinforced beams to restrain crack propagation and maintain stiffness. At higher load levels, all models exhibit a converging trend as damage becomes more dominant, and material strength differences have a diminishing effect on the frequency response. These findings reinforce the effectiveness of using natural frequency as a damage-sensitive parameter in reinforced concrete structures. The correlation between damage progression—observed in both the static response and cracking behaviour—and the dynamic response provides a valuable non-destructive indicator for structural health monitoring.

In addition to material properties, the effect of beam geometry on dynamic response was also investigated. Fig. 9 presents a comparison between two beams with different width-to-depth ratios (b/h), specifically $b/h = 0.5$ (reference case) and $b/h = 1.0$. The results indicate that the beam with the lower b/h ratio (i.e., lower width, fixed depth) exhibits an earlier onset of

frequency reduction compared to the squarer section. This behaviour can be attributed to the fact that the cracking moment is reached at a lower load level in the $b/h = 0.5$ configuration, leading to earlier stiffness degradation and a more rapid decline in natural frequency.

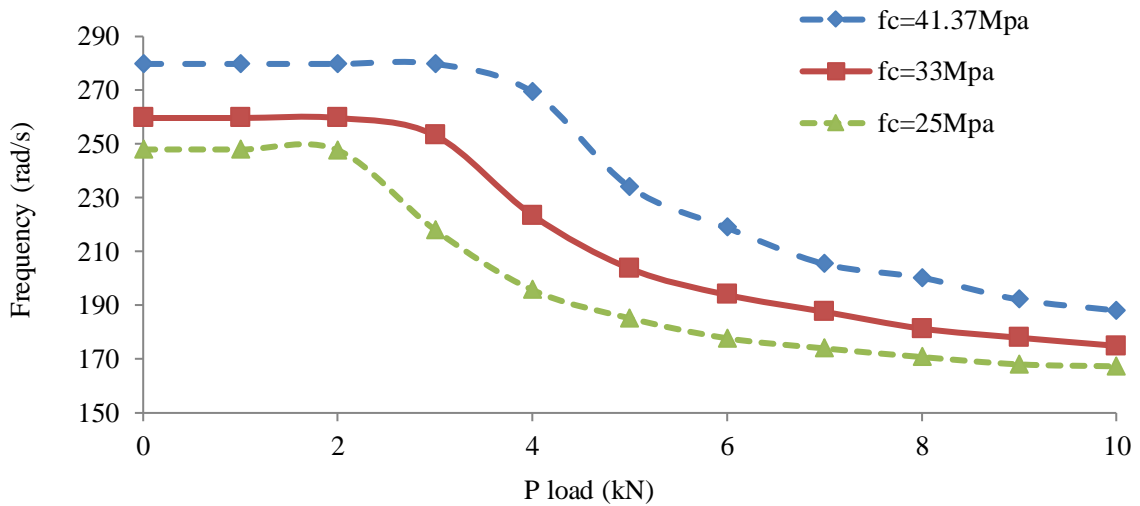


Fig. 7: Frequency versus beam load at different concrete compressive strength values.

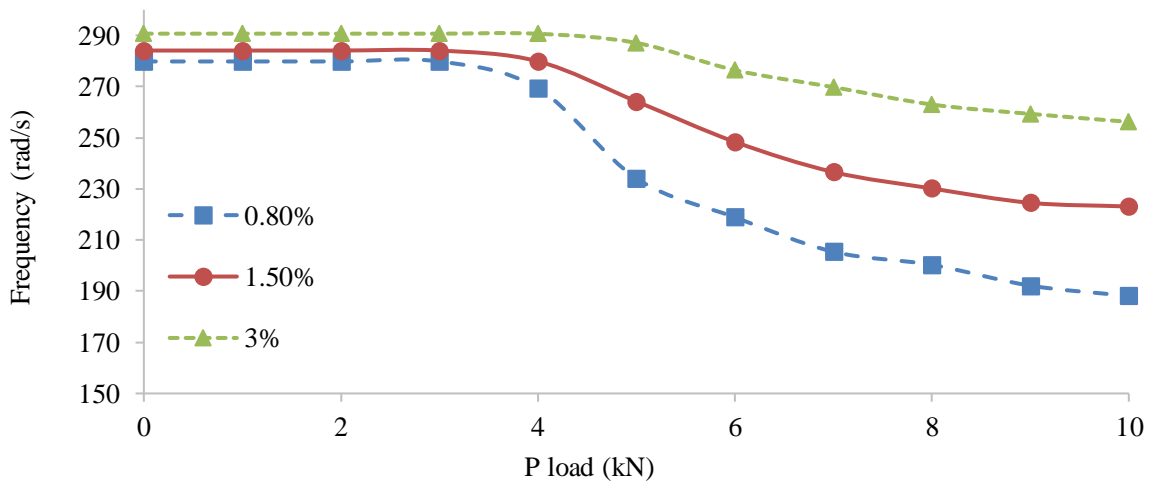


Fig. 8: Frequency versus beam load at different steel ratios.

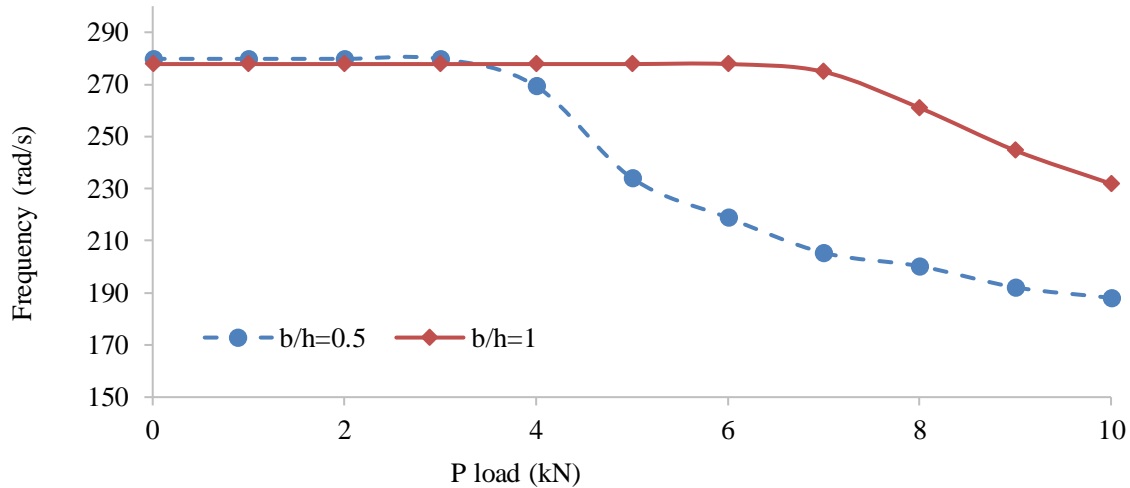


Fig. 9: Frequency versus beam load at different b/h ratios.

4 Conclusion

This study demonstrates that the Concrete Damaged Plasticity (CDP) model in ABAQUS provides a robust and reliable framework for simulating both the static and dynamic responses of reinforced concrete beams subjected to progressive damage. The finite element model accurately captures the nonlinear load-deflection behaviour and the onset of cracking, as validated by close agreement with experimental results, especially in the elastic and early nonlinear ranges. The model's ability to simulate stiffness degradation and crack propagation under flexural loading confirms its suitability for practical engineering applications.

A key finding is the strong correlation between damage progression and reductions in natural frequency. Parametric studies reveal that beams with lower concrete compressive strength or reduced reinforcement ratios exhibit earlier and more significant declines in frequency as damage develops. Additionally, beam geometry, particularly the width-to-depth ratio, influences the onset and rate of frequency reduction. These results establish that shifts in natural frequency are effective, non-destructive indicators of structural damage, supporting the use of vibration-based structural health monitoring for reinforced concrete structures.

From a practical standpoint, the findings highlight the importance of carefully selecting material properties and reinforcement detailing to enhance damage resistance and maintain dynamic performance. The modelling approach presented here enables engineers to predict the effects of various design parameters on both static load capacity and dynamic behaviour, offering a valuable tool for structural assessment, retrofitting, and health monitoring in real-world applications.

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