

Interaction of Non-Isothermal Flow and Solidification

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Extended Abstract

The interaction of flow and solidification emerged as a modelling problem in the context of additive manufacturing where the key process is the impact, spreading and solidification of liquid drops on top of each other to form complex 3D structures which would be difficult or impossible to produce in any other way. From the modelling perspective, this process poses two fundamental challenges: (i) the famous moving contact-line problem of dynamic wetting, where one has to remove the shear-stress singularity at the contact line and, more importantly, describe the behaviour of the contact angle formed by the free surface with the solid boundary, and (ii) the description of solidification dynamics from the onset all the way to the Stefan regime. As discovered experimentally [1], dynamic wetting and solidification occur initially as two independent processes and then they interact so that the contact-line motion becomes arrested. This fact poses a third challenge: dynamic wetting and solidification have to be described in the same conceptual framework, i.e., as elements of an embracing physical phenomenon which has to be identified and adequately modelled. Finally, given that conditions on the solidification front in the Stefan regime and hence, by continuity, in the regime that precedes it involve heat fluxes, one has to remove the heat flux singularity appearing at the contact line in non-isothermal flows [2].

The present work addresses the above challenges. Its key idea is that both dynamic wetting and solidification are actually particular cases in a wide class of flows where interfaces form and/or disappear. An indication of this comes from the fact that, as predicted in [3], discovered experimentally in [4] and explained in [5], in dynamic wetting there is no unique relationship between the dynamic contact angle and the contact-line speed. This fact invalidates a multitude of ad hoc models proposed in the past 50 years to describe dynamic wetting and leaves intact only the model of this process as an interface formation phenomenon [6], where the value of the dynamic contact angle is 'negotiated' by the dynamic surface tensions of the contacting interfaces. An extension of this model to include thermal effects and phase transitions makes it possible to describe the propagation of the solidification fronts in the same modelling framework. The numerical analysis of the model shows that the onset of solidification is followed by pure interface formation and then solidification in a non-equilibrium regime which tends to the Stefan regime as the process slows down. Curiously, the evolution of temperature at the front in this process is non-monotone with a surprising loop in the phase diagram. The spreading of the solidification front along the solid substrate occurs with zero angle (measured through the solidifying phase) and there are no singularities at its edge. The conditions of the arrest of the moving contact line appear to be in agreement with available experimental data.

References

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