

Exploring Chaotic Dynamics: Finding the Edge State in a Divergent Pipe Flow

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Extended Abstract

The scientific exploration of fluid flows in pipes, specifically the transition from laminar to turbulent regimes, has been an area of persistent fascination for researchers over multiple generations. These flow regimes are remarkable in their complexity; they exhibit a coexistence of laminar and turbulent states that are both spatially and temporally distributed. The governing dynamics of this system can be mathematically described through the Navier-Stokes equations. In this mathematical framework, the entire system evolves as an infinite-dimensional dynamical system, where the laminar flow functions as a stable fixed point, while turbulent flow manifests as a chaotic attractor that emerges from invariant solutions.

Over the past decade, considerable progress has been made in dissecting these invariant solutions, with a focus on a transitional state known as the 'edge of chaos.' This intriguing state, which has been extensively studied [1][5][6], exists at the boundary between laminar and turbulent flow regimes. It essentially marks the set of initial conditions that will either direct the flow back towards a laminar state or catapult it into a turbulent one [2][3]. Intriguingly, the 'edge of chaos' tends to inhabit the lower branch of a saddle-node bifurcation and requires only a fraction of the energy compared to fully turbulent states. This insight opens up the prospect of manoeuvring the flow towards this state through precise control measures, offering substantial energy-saving advantages by reducing drag in pipe flow systems.

In our research, we employed Nek5000, a DNS code [4], to simulate the fluid flow in a divergent pipe. One key characteristic of this configuration is the existence of a stationary turbulent spot. This unique feature simplifies the experimental investigation of such flows when compared to the canonical pipe flow where turbulent patches are advected downstream.

To identify the edge of chaos in divergent pipe flows, we implemented an edge-tracking algorithm. Adapted from the straightforward bisection methodology proposed by [2], this algorithm starts with a pair of initial conditions derived from a scaled turbulent field. One condition evolves into a turbulent state while the other relaxes into a laminar state without manifesting turbulence. Through iterative refinement, the algorithm is tuned to pinpoint a trajectory that remains stable, neither escalating into turbulence nor collapsing into the laminar state. The computational method was initially validated on a short, straight pipe with a length-to-diameter ratio of 5, serving as a validation step. Subsequent applications of the algorithm were executed on divergent pipes with various angles of divergence, extending the scope and applicability of our research.

References

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