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Analysis and Optimization of Actuator Forces in Hybrid Cable-driven Manipulators

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Abstract **–** This study discusses the force analysis of hybrid cable driven manipulators having both cables and rigid links with actuators installed on both of them. It is desired to operate the manipulator with minimum actuation forces to minimize cable elongation and reduce the operational cost. A minimization procedure is discussed here based on Dykstra's algorithm to minimize the 2-norm of the actuators forces. This procedure was implemented on an example 3-DOF manipulator with two redundant actuators.

Keywords**:** Wire-driven parallel robot manipulators, Optimization, Tension minimization, Hybrid layout.

1. Introduction

Cable-driven parallel manipulators (CPM) are parallel manipulators that are based on cables instead of rigid-links. They are known for their large workspace and high acceleration capability, compared to those of rigid-link parallel manipulators. There have been a number of CPM designs presented in the literature such as NIST Robocrane (Albus et al., 1993), Falcon-7 (Kawamura et al., 1995), WARP (Maeda et al., 1999), WiRo (Ferraresi et al., 2004) DeltaBot (Behzadipour and Khajepour, 2005), and the hybrid cable-actuated robot developed by Mroz et al (2004). Because cables can only be under tension, many researchers have studied the ability of CPM's to achieve static equilibrium in the workspace with taut cables (Kawamura and Ito, 1993), (Diao and Ma, 2007). Other studies (Fang et al., 2004), (Hassan and Khajepour, 2008, 2011) focused on the optimization of cable forces in the manipulator.

In order for spatial cable-driven manipulators to be fully constrained against any arbitrary externally applied wrenches, it is necessary to have at least seven cables, which can be costly, especially in applications not requiring all 6 degrees of freedom (DOF). Hassan and Khajepour (2009) discussed the analysis and optimization of the layout of a hybrid cable-driven parallel manipulator with a serial linkage consisting of conventional joints to constrain the task space (See Fig. 1). The advantage of the linkage is constraining the moving the platform in the taskspace, hence, reducing the number of cables required to drive the moving platform. A 5-DOF cable driven manipulator can be driven by five cables, while the redundant actuation is provided by the serial linkage. These types of manipulators can be applied in lower-DOF applications that require the high acceleration benefits provided by the cables. Mroz and Notash (2004) developed a 4-DOF cable-actuated manipulator that is constrained by a rigid-link mechanism. Zhang and Gosselin (2001) and Lu and Hu (2007) investigated the kinetostatic models of lower-DOF rigid-link parallel manipulators actuated by prismatic actuators and constrained by a passive constraining leg.

This paper discusses the force analysis and the force minimization in hybrid cable-driven manipulators in which cables and rigid links both operate the platform.

2. Manipulator Layout

The general layout of the hybrid cable-driven manipulator discussed in this paper consists of n_c driving cables and the serial linkage consists of n_p passive joints and n_a active joints. The total number of joints in the serial linkage is $n_s = n_p+n_a$, and the total number of active joints in the manipulator whose function is driving the motion of the manipulator and providing degrees of freedoms to the moving platform is n_c+n_a. In addition, the manipulator must have at least one redundant actuator whose function is generating internal forces in the manipulator to keep the n_c cables in tension. That redundant actuator can be redundant cable(s) or redundant actuator(s) mounted on one of the passive joints in the serial linkage. The location and direction of the redundant actuator(s) must be carefully designed to ensure that the internal redundant force generated will develop tension in all the cables.

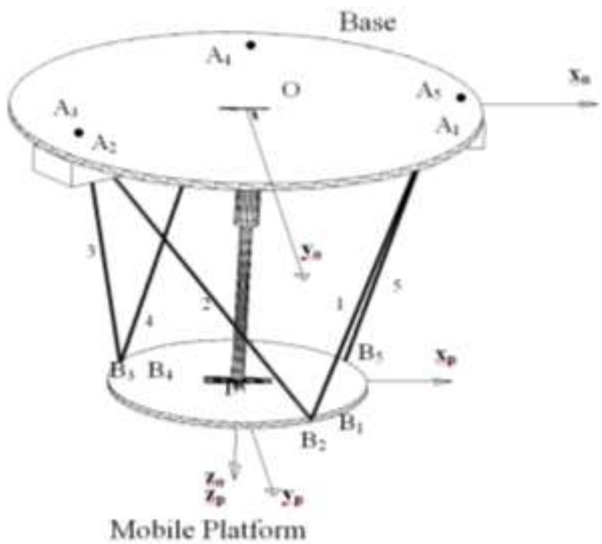

Fig. 1. Drawing of a 5-DOF cable-driven parallel manipulator constrained by universal joints.

3. Force Analysis

To balance arbitrary wrenches applied to the moving platform, redundant actuation is required to generate internal forces that keep all the cables in tension. Thus, we can write the total wrenches applied to the moving platform as:

$$\mathbf{w} = \mathbf{w}_e + \mathbf{w}_r \tag{1}$$

where $\mathbf{w} = \left[\mathbf{f}^T \mathbf{m}^T\right]^T$ is the sum of wrenches applied to the moving platform that consists of force **f** and moment **m**; $\mathbf{w}_e$ is the external wrench applied to the moving platform by the task; $\mathbf{w}_r$ is an internal wrench that is applied by the redundant actuation of the serial linkage or by a redundant cable in order to create tensile forces in the cables to keep them taut.

From the principle of virtual work, one can determine the mapping between the wrench applied to the moving platform and the cable forces as:

$$\begin{bmatrix} \boldsymbol{\tau}_c \\ \boldsymbol{\tau}_a \end{bmatrix} = \left[\mathbf{J}_p \left(\mathbf{J}_c \mathbf{J}_p\right)^{-1} \quad \left(\mathbf{J}_a - \mathbf{J}_p \left(\mathbf{J}_c \mathbf{J}_p\right)^{-1} \mathbf{J}_c \mathbf{J}_a\right)\right]^T \mathbf{w}$$

with $(\boldsymbol{\tau}_c)_i \geq 0 \quad \forall i$ (2)

where $\boldsymbol{\tau}_c$ is a vector of the cable tensions, which can only be under tension; and $\boldsymbol{\tau}_a$ is a vector of the forces of the active joints in the serial linkage; $\mathbf{J}_p$ is ($6\times n_p$) sub-matrix of $\mathbf{J}_s$ that corresponds to the passive joints in the serial linkage; $\mathbf{J}_a$ is ($6\times n_a$) sub-matrix of $\mathbf{J}_s$ that corresponds to the active joints in the serial linkage.

From (1) and (2), we get:

$$\begin{bmatrix} \boldsymbol{\tau}_c \\ \boldsymbol{\tau}_a \end{bmatrix} = \begin{bmatrix} \left(\mathbf{J}_c\mathbf{J}_p\right)^{-T}\mathbf{J}_P^T \\ \left(\mathbf{J}_a - \mathbf{J}_p\left(\mathbf{J}_c\mathbf{J}_p\right)^{-1}\mathbf{J}_c\mathbf{J}_a\right)^T \end{bmatrix}\left(\mathbf{w}_e + \mathbf{w}_r\right) \quad (\boldsymbol{\tau}_c)_i \geq 0 \ \ \forall i \tag{3}$$

We can write the cable tensions in (3) as:

$$\boldsymbol{\tau}_c = \left(\mathbf{J}_c\mathbf{J}_p\right)^{-T}\mathbf{J}_p^T\mathbf{w}_e + \left(\mathbf{J}_c\mathbf{J}_p\right)^{-T}\mathbf{J}_p^T\mathbf{w}_r \quad : (\boldsymbol{\tau}_c)_i \geq 0 \ \forall i \tag{4}$$

The redundant wrench $\mathbf{w}_r$ needed to keep the cables taut can be generated by a redundant actuator in the serial linkage or by a redundant cable. These two cases will be discussed separately in the following sections. Redundant actuation can be provided from redundant actuator(s) mounted on one (or more) of the n_p passive joints in the serial linkage. The redundant actuator(s) is used only for generating the internal redundant wrench $\mathbf{w}_r$ needed to keep the cables taut and is not for motion control. The redundant actuator forces in the serial linkage can be determined from the redundant wrench as (Tsai, 1999).

$$\boldsymbol{\tau}_r = \mathbf{J}_p^T\mathbf{w}_r \tag{5}$$

where $\boldsymbol{\tau}_r$ is n_r–dimensional vector of the forces of the n_r redundant actuators mounted on the passive joints in the serial linkage that are required to generate wrench $\mathbf{w}_r$ on the moving platform.

From (4) and (5), we get:

$$\boldsymbol{\tau}_c = \left(\mathbf{J}_c\mathbf{J}_s\right)^{-T}\mathbf{J}_s^T\mathbf{w}_e + \Delta\boldsymbol{\tau}_c \quad \text{with } (\boldsymbol{\tau}_c)_i \geq 0 \ \ \forall i \tag{6}$$

where

$$\Delta\boldsymbol{\tau}_c = \left(\mathbf{J}_c\mathbf{J}_p\right)^{-T}\boldsymbol{\tau}_r \tag{7}$$

4. Actuator Force Minimization

Assuming that the cables and actuators in the rigid links apply forces in one direction, the general solution for actuator forces in (2) can be written as:

$$\boldsymbol{\tau} = -\boldsymbol{A}^+\mathbf{w} + \boldsymbol{N}\,\mathbf{h} \quad : \tau_i \geq 0 \quad \forall i \tag{8}$$

where A is the matrix transpose in Eq.2; $\boldsymbol{A}^+$ is the Moore-Penrose inverse of the matrix transpose; $\boldsymbol{N}$ is a matrix whose columns form a basis of the null-space of matrix $\boldsymbol{A}$; and $\mathbf{h}$ is an n_r-dimensional vector of arbitrary real numbers.

Assuming that the manipulator is in a non-singular configuration, a sufficient condition for the existence of a solution to $\boldsymbol{\tau}$ is the existence of a null vector $\boldsymbol{N}\,\mathbf{h}$ whose components are all positive, indicating that the cable-based parallel manipulator can be fully constrained under any given $\mathbf{f}$ and $\mathbf{m}$. Assuming that the manipulator is fully constrained, this section will present the minimum-norm solution of $\boldsymbol{\tau}$ that will be denoted as $\boldsymbol{\tau}_{\min\|\boldsymbol{\tau}\|}$.

The condition that all components of $\boldsymbol{\tau}$ are non-negative means that $\boldsymbol{\tau}$ must belong to non-negative orthant R_+^n, i.e., $\boldsymbol{\tau} \in \mathrm{R}_+^n$, which can be expressed as:

$$\mathrm{R}_+^n = \left\{\boldsymbol{\tau} \in \mathrm{R}^n \,\middle|\, \tau_i \geq 0 \ \ \forall i\right\} \tag{9}$$

where $n = n_c + n_r$.

It is also evident that $\boldsymbol{\tau}$ must belong to an n_r-dimensional affine set in R^n that is a translation of the null space of $\boldsymbol{A}$ from the Origin "$\boldsymbol{\tau} = \mathbf{0}$" by vector $-\boldsymbol{A}^+\mathbf{w}$. This affine set denoted here as $\mathcal{A}$ can be expressed as:

$$\mathcal{A} = \left\{ \boldsymbol{\tau} \mid \ \boldsymbol{\tau} = -\boldsymbol{A}^{+}\mathbf{w} + \boldsymbol{N}\,\mathbf{h} \right\} \quad : \ \mathbf{h} \in R^{n_r} \tag{10}$$

Since $\boldsymbol{\tau} \in \mathcal{A}$ and R_{+}^{n},

$$\boldsymbol{\tau} \in \mathcal{C} \quad : \mathcal{C} = \mathrm{R}_{+}^{n} \cap \mathcal{A} \tag{11}$$

It should be noted here that the intersection set $\mathcal{C}$ is non-empty as a result of the assumption made in this paper that there exists a null-space vector whose components are all positive. This positive null-space vector intersects with the non-negative orthant. Figure 2 presents a geometrical illustration of the intersection between a 3-dimensional non-negative orthant and a 2-dimensional affine set.

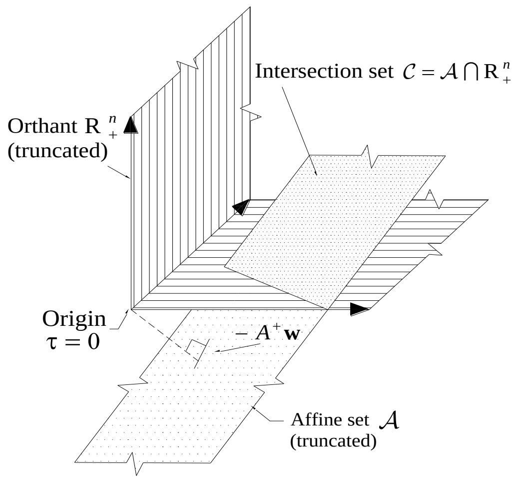

Fig. 2. Geometrical illustration of the intersection between a 2-dim affine set and a 3-dim non-negative orthant

The 2-norm of vector $\boldsymbol{\tau}$ can be represented as $\|\mathbf{0} - \boldsymbol{\tau}\|$, which is the Euclidean distance between the Origin "$\boldsymbol{\tau} = \mathbf{0}$" and point $\boldsymbol{\tau} \in \mathcal{C}$. Minimizing the 2-norm of $\boldsymbol{\tau}$ will be achieved by minimizing $\|\mathbf{0} - \boldsymbol{\tau}\|$. Hence, the problem can be stated as:

$$\min_{\boldsymbol{\tau} \in \mathcal{C}} \|\mathbf{0} - \boldsymbol{\tau}\| \tag{12}$$

The solution to this problem is the minimum-Euclidean-distance projection of the Origin "$\boldsymbol{\tau} = \mathbf{0}$" onto the intersection set $\mathcal{C}$ and, will be expressed as $\boldsymbol{\tau}_{\min\|\boldsymbol{\tau}\|} = proj_{\mathcal{C}}(\mathbf{0})$. This solution can be easily found using the Dykstra's alternating projection algorithm, which is a well-known algorithm for finding the minimum-Euclidean-distance projection of a point onto the intersection of a number of convex sets. It was first presented by Dykstra (1983) and reintroduced later by Han (1988) who provided a proof of its convergence. This algorithm is a simple and easy-to-apply minimization approach because it is based on cyclic projections that automatically converge to the solution. Also, the fact that the algorithm requires defining the variables in convex sets makes this approach geometrically intuitive and easy to visualize.

The Dykstra's Algorithm

Let point $\mathbf{b} \in \mathrm{R}^{m}$ and $\mathcal{S}_1, \ldots, \mathcal{S}_L$ are L convex sets in R^{m} and their intersection is the non-empty set $\mathcal{S}_{\cap} = \bigcap_{k=1}^{L}(\mathcal{S}_k)$. The Dykstra's algorithm can be used to find the minimum-Euclidean-distance projection of $\mathbf{b}$ onto set $\mathcal{S}_{\cap}$. This projection is denoted as $proj_{\mathcal{S}_{\cap}}(\mathbf{b})$ and is the solution to the minimization problem $\min_{\mathbf{v} \in \mathcal{S}_{\cap}} \|\mathbf{b} - \mathbf{v}\|$. To illustrate the application of Dykstra's algorithm in finding

$proj_{\mathcal{S}\cap}(\mathbf{b})$, let $\mathbf{x}_{k,i}$ and $\mathbf{y}_{k,i} \in \mathrm{R}^m$ where $\mathbf{x}_{k,i}$ is the minimum-Euclidean-distance projection of $\mathbf{t} \in \mathrm{R}^m$ onto convex set *k* at iteration *i*; and $\mathbf{y}_{k,i} = (\mathbf{x}_{k,i} - \mathbf{t})$. The term *k* is an index referring to convex set $\mathcal{S}_k$. The algorithm initializes at *i* = 0, where $\mathbf{x}_{1,0} = \mathbf{b}$, which is the point to be projected, and $\mathbf{y}_{k,0} = \mathbf{0}$ for all *k*.

for i = 1, 2, …, until convergence
$\mathbf{x}_{L+1,i} = \mathbf{x}_{1,i-1}$
for k = L, …, 1 (*L, …, 1 are indices for* $\mathcal{S}_L, \ldots, \mathcal{S}_1$)
$\mathbf{t} = \mathbf{x}_{k+1,i} - \mathbf{y}_{k,i-1}$
$\mathbf{x}_{k,i} = proj_k(\mathbf{t})$
$\mathbf{y}_{k,i} = (\mathbf{x}_{k,i} - \mathbf{t})$
loop end
loop end

This version of the algorithm is reported in (Han, 1988). At each iteration, the algorithm makes successive projections onto convex sets $\mathcal{S}_1, \ldots, \mathcal{S}_L$ until all sequences $\mathbf{x}_{k,i}$ converge to a unique point $\mathbf{x}_{k,\infty} = proj_{\mathcal{S}\cap}(\mathbf{b})$ when $\mathcal{S}_\cap$ is non-empty. Vector $\mathbf{y}_{k,i}$, which is normal to set $\mathcal{S}_k$ at iteration *i*, is part of the general algorithm and its role is to ensure that the algorithm converges at the minimum-Euclidean-distance solution as explained in (Dykstra, 1983). The calculation of this vector is unnecessary when all the intersecting sets are affine.

To apply this algorithm in finding the solution to minimize the 2-norm of the actuator forces in (8), i.e., finding $\boldsymbol{\tau}_{\min\|\boldsymbol{\tau}\|} = proj_{\mathcal{C}}(\mathbf{0})$, the terms in the general algorithm above are specified as follows: *m* = *n* (dimension of $\boldsymbol{\tau}$); $\mathbf{b} = \mathbf{0}$ (the Origin); sets $\mathcal{S}_1, \ldots, \mathcal{S}_L = \mathrm{R}^n_+, \mathcal{A}$ (*L*=2); and $\mathcal{S}_\cap = \mathcal{C} = \mathcal{A} \cap \mathrm{R}^n_+$; and *k = 2,1* (indices for $\mathcal{A}$ and R^n_+, respectively). At each iteration, this algorithm makes successive projections of point $\mathbf{t}$ onto sets $\mathcal{A}$ and R^n_+, denoted as $proj_{\mathcal{A}}(\mathbf{t})$ and $proj_{\mathrm{R}^n_+}(\mathbf{t})$, respectively, until sequences $\mathbf{x}_{k,i}$ converge to $\mathbf{x}_{k,\infty} = \boldsymbol{\tau}_{\min\|\boldsymbol{\tau}\|} = proj_{\mathcal{C}}(\mathbf{0})$.

This algorithm requires the calculation of projections $proj_{\mathrm{R}^n_+}(\mathbf{t})$ and $proj_{\mathcal{A}}(\mathbf{t})$. Projection $proj_{\mathrm{R}^n_+}(\mathbf{t})$ is determined as (Dattorro, 2005)**Error! Reference source not found.**:

$$proj_{\mathrm{R}^n_+}(\mathbf{t}) = \bar{\mathbf{t}} \quad : \quad \bar{t}_i = \max\{t_i, 0\} \quad \text{for } i = 1, \ldots, n \tag{13}$$

This projection is basically clipping all the negative coordinates of point $\mathbf{t}$ to zero. Projection $proj_{\mathcal{A}}(\mathbf{t})$ can be determined as:

$$proj_{\mathcal{A}}(\mathbf{t}) = \left(\boldsymbol{I} - \boldsymbol{A}^+\boldsymbol{A}\right)\mathbf{t} - \boldsymbol{A}^+\mathbf{w} \tag{14}$$

This projection is obtained by projecting $\mathbf{t}$ onto the null space of $\boldsymbol{A}$ and translating the result by $-\boldsymbol{A}^+\mathbf{w}$.

5. Example

The procedure explained in this paper is applied to the 3-DOF cable-based parallel manipulator shown in Fig. 3. It consists of three cables (n_c = 3) connected to the fixed base on the top at points A_i (*i* =

1, 2, 3) that form an equilateral triangle, where subscript i is an index identifying each cable. The corners of the triangle lie on a circle of a 300 mm radius with its centre located at O_A. The manipulator contains two redundant limbs (n_r = 2): redundant limbs 1 and 2 (compressive-only cylinders) which are connected to the base at [129.9 75.0 0] mm and [−129.9 75.0 0] mm, respectively. The position of P (end-effector) is [0 0 300] mm from O_A. The external force applied to P is $\mathbf{w} = [-10\ 5\ -6]^T$ N. All vectors are expressed in the reference frame x_A-y_A-z_A.

In this sub-section, the minimum-norm solution $\boldsymbol{\tau}_{\min\|\boldsymbol{\tau}\|}$ will be calculated. At this configuration, matrix $\boldsymbol{A}$ is calculated as:

$$\boldsymbol{A} = \begin{bmatrix} 0.707 & -0.354 & -0.354 & -0.387 & 0.387 \\ 0 & -0.612 & 0.612 & -0.224 & -0.224 \\ -0.707 & -0.707 & -0.707 & 0.894 & 0.894 \end{bmatrix}$$

The first three columns are the directions of the cables forces and the last two columns are the directions of the redundant-limb forces. These redundant limbs are cylinders that apply compressive forces to the mobile platform. In this example, $\boldsymbol{\tau}$ belongs to the intersection of 5-dimensional (n = 5) non-negative orthant $\mathbb{R}^5_+$ and 2-dimensional (n_r = 2) affine set $\mathcal{A}$, i.e., $\boldsymbol{\tau} \in \mathcal{C}$, where $\mathcal{C} = \mathbb{R}^5_+ \cap \mathcal{A}$. The Dykstra's alternating projection algorithm is applied to determine the minimum-Euclidean-distance projection of the Origin onto the intersection set $\mathcal{C} = \mathbb{R}^5_+ \cap \mathcal{A}$ using the projection formulas in (13) and (14). After 31 iterations within 0.031 second of CPU time in MATLab software each projection vector converged when the difference between the norms of each vector in two successive iterations became less than a specified acceptable convergence error of (1e-5 N). Table I lists the successive projections (i.e., values for $\mathbf{x}_{k,i}$) and shows that they converge to $\boldsymbol{\tau}_{\min\|\boldsymbol{\tau}\|} = proj_{\mathcal{C}}(\mathbf{0})$ = [8.54 2.52 0.00 1.46 13.99]^T N. The norms of the total actuator forces, $\|\boldsymbol{\tau}\|$, and that of the cable forces, $\|\boldsymbol{\tau}_C\|$, are 16.65 N and 8.90 N, respectively.

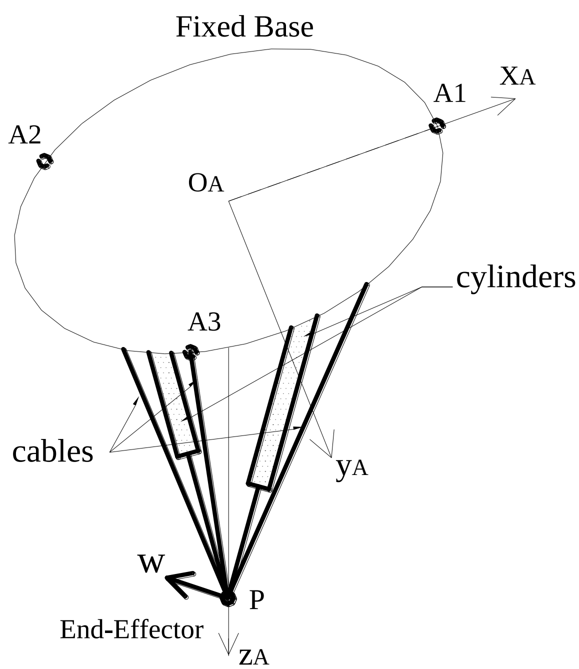

Fig. 3. A schematic diagram of the 3-DOF example manipulator.

Table 1. Convergence of Dykstra's algorithm to the minimum norm solution of the cable tensions.

Iteration	Projections on $\mathcal{A}$					Projections on $\mathbf{R}^5_+$				
	τ_{c1}	τ_{c2}	τ_{c3}	τ_{r1}	τ_{r2}	τ_{c1}	τ_{c2}	τ_{c3}	τ_{r1}	τ_{r2}
1	5.85	-1.01	-7.49	-1.38	5.99	5.85	0.00	0.00	0.00	5.99
2	7.49	0.40	-4.27	0.48	9.09	7.49	0.00	0.00	0.00	9.09
3	7.86	1.13	-2.79	0.83	10.79	7.86	0.52	0.00	0.00	10.79
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30	8.54	2.52	0.00	1.46	13.99	8.54	2.52	0.00	1.46	13.99
31	8.54	2.52	0.00	1.46	13.99	8.54	2.52	0.00	1.46	13.99

6. Conclusion

The paper discusses the force analysis of hybrid cable-driven manipulators where both cables and rigid links operate the platform. A procedure to minimize the 2-norm of the actuator forces was also discussed using Dykstra's algorithm. This procedure was successfully demonstrated on a 3-DOF example having three cables and two rigid links.

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