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Simulation of Filtration Flow Towards Hydraulically Fractured Vertical Well

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Abstract- This paper presents an approach to numerical simulation of fluid flow within oil formation towards hydraulically fractured wells. The fluid flow is described by simplified two-phase three-dimensional Darcy's law for incompressible fluid. The integral averaging along fracture width is employed to derive two-dimensional equation for flow in the fracture. Presented method comes to finding consistent formation and hydraulic fracture fluid pressure fields. Suggested solution technique requires fracture geometry and permeability distribution along the fracture. It could be applied for fractured well production calculation using either volumetric flow rate or well bore pressure as specified parameters.

Keywords: Hydraulic fracturing, Oil reservoir simulation, Superelements, Darcy's law.

1. Introduction

Hydraulic fracturing is used to enhance well production due to additional fluid flow through the fracture. Filtration properties of the fracture can be characterized by the absolute fracture permeability k_f, which is typically several orders of magnitude higher than the average rock permeability k (Cadet, Selyakov, 1988, Kanevskaya, 2002, Kanevskaya, 1988). Its direct determination, as well as determination of the fracture geometry is practically impossible, so hydrodynamic methods based on the analysis of well tests data by mathematical models are widely used (for example, Economides, Nolte, 2000). Such models exploit simplified models of well with a fracture. The most popular shapes of fracture representation are ellipse, wedge and plate (Economides, Nolte, 2000, Gringarten et al., 1974). Their sizes are associated with the volume of injected fracturing mixture and fracturing well bore pressure. It is believed that at reservoir depth greater than 1 km hydraulic fractures are oriented vertically. Mathematical models of filtration flow in a fracture towards the well are based on the diffusion equation and differ by assumed fracture shape and boundary conditions on its surface. In some cases of homogeneous layers and canonical fracture shapes analytical solution of the problem can be derived (Economides, Nolte, 2000, Gringarten et al., 1974). At present time different applied numerical approaches which use non-uniform locally refined meshes are used (Hairullin et al., 2009, Hisamov et al., 2010).

In this paper, we present a mathematical model of the filtration flow towards the fractured well. The fracture is presented as a vertical plate of finite size, which passes through the axis of a vertical well (fig. 1a). Corresponding filtration problems are formulated separately for the flow inside and outside fracture on the basis of Darcy's law for incompressible fluid. Pressure and filtration velocity equalities are used to combine inner and outer flow problems solution. Presented numerical algorithm is used in large scale (superelement) reservoir simulation method (Mazo, Bulygin, 2011) for fractured well performance estimation.

2. Mathematical Model

Superelement reservoir simulation technique uses large scale grids for calculating cell averaged values of fluid pressure p_a and water saturation s_a. Three-dimensional superelements grid is constructed on the basis of two-dimensional Voronoi diagram built by treating 2D projections of vertical wells as its cell sites.

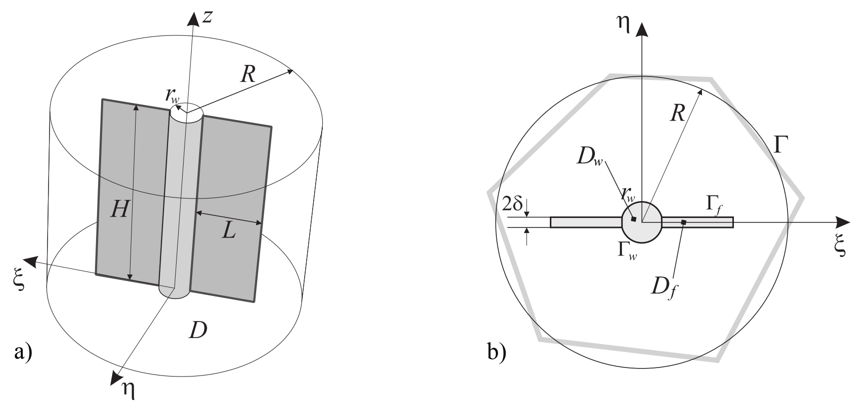

Fig. 1. 3D representation of the fractured well (a); 2D projection of the well within a superelement and cylindrical computational area (b)

2.1. Outer Problem Formulation

Consider the vertical well with defined radius r_w and volumetric rate q that is located in the center of superelement (fig. 1b). Suppose that at moment $t = t_0$ before fracturing procedures took place averaged cell pressure and saturation were calculated. Let us study the filtration flow within vertical cylinder $D = \{0 < r < R, 0 < z < H\}$ with lateral boundary Γ which contains the well and the vertical fracture of length $2L$, height H and width 2δ. The radius R of the cylinder is equal to the average size of the corresponding superelement (usually $R \ll L$), its top and bottom boundaries ($z = 0$ and $z = H$) are impermeable horizontal planes.

Assuming that hydraulic fracturing at moment $t = t_0+$ doesn't impact fluid pressure at the distance R from the well bore and neglecting compressible and capillary effects, we can formulate boundary value problem for the filtration flow in the cylinder as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\sigma\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\sigma\frac{\partial p}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(\sigma\frac{\partial p}{\partial z}\right) = 0, \quad (r,z,\phi) \in D - D_f - D_w; \tag{1}$$

$$p = p_a, \quad (r,z,\phi) \in \Gamma; \qquad p = p_w, \quad (r,z,\phi) \in \Gamma_w; \qquad p = p_f, \quad (r,z,\phi) \in \Gamma_f; \tag{2}$$

$$\sigma\frac{\partial p}{\partial n} = 0, \quad z = 0, H, \tag{3}$$

where p and p_f are the pressure fields corresponding to outer and inner flow, r, z, ϕ are cylindrical coordinates, Γ_w, Γ_f are perforated section of the well and fracture boundaries. Coefficient σ is defined as function of absolute rock permeability k, relative permeabilities $k_w(s)$, $k_o(s)$ and fluid viscosities μ_w, μ_o where indices "w" and "o" correspond to water and oil properties respectively:

$$\sigma = k\left(\frac{k_w(s_a)}{\mu_w} + \frac{k_o(s_a)}{\mu_o}\right). \tag{4}$$

In boundary conditions (2) outer pressure p_a is defined, well bore pressure p_w could be defined or obtained from well rate q_w, pressure on the fracture surface p_f is unknown and should be calculated from inner problem solution. In the case of infinite fracture permeability ($k_f \to \infty$) we can assume that $p_f = p_w$ (see Economides, Nolte, 2000).

2.2. Inner Problem Formulation

To describe the surface of the fracture it is convenient to introduce local Cartesian coordinates ξ, η, z as shown on fig. 2. Let us assume that fracture surface sections at $\xi = \pm r_w \pm L$ are non-permeable, fracture pressure at $\xi = r_w$ is equal to well bore pressure, pressure and filtration velocities are continuous on $\eta = \pm\delta$ surfaces.

Assume the diffusion law as the governing equation for the fracture pressure p_f. For the right side of fracture area

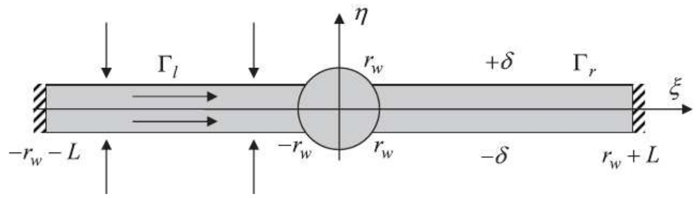

Fig. 2. Scheme of hydrodynamic interaction between reservoir, fracture and the well

it takes the form

$$\sigma_f \left(\frac{\partial^2 p_f}{\partial \xi^2} + \frac{\partial^2 p_f}{\partial \eta^2} + \frac{\partial^2 p_f}{\partial z^2} \right) = 0, \quad \xi \in [r_w, r_w + L], \eta \in [-\delta, \delta], z \in [0, H], \tag{5}$$

where

$$\sigma_f = k_f \left(\frac{k_w(s_a)}{\mu_w} + \frac{k_o(s_a)}{\mu_o} \right)$$

is constant under the assumption of constant fracture permeability k_f.

Integration of (5) over the fracture width with boundary conditions taken into account gives us

$$2\delta\sigma_f \left(\frac{\partial^2 \langle p \rangle}{\partial \xi^2} + \frac{\partial^2 \langle p \rangle}{\partial z^2} \right) + \sigma \left. \frac{\partial p}{\partial \eta} \right|_{\eta=-\delta-0}^{\eta=\delta+0} = 0, \tag{6}$$

where $\langle p \rangle$ is the pressure averaged over the pressure width

$$\langle p \rangle = \frac{1}{2\delta} \int_{-\delta}^{+\delta} p_f \, d\eta.$$

Boundary conditions for equation (6) are

$$\langle p \rangle = p_w, \quad \xi = r_w; \qquad \frac{\partial \langle p \rangle}{\partial \xi} = 0, \quad \xi = r_w + L; \qquad \frac{\partial \langle p \rangle}{\partial z} = 0, \quad z = 0, H. \tag{7}$$

So the boundary value problem for the right side of the fracture is (6),(7); left side of the fracture is treated in the same way.

For thin fractures $\delta \ll L$ we can assume that

$$p(\delta) = p(-\delta) = \langle p \rangle. \tag{8}$$

Condition (8) is used to combine inner and outer solutions.

2.3. Non-dimensional Formulation

For further discussion the following non-dimensional variables are introduced:

$$\bar{r}, \bar{r}_w, \bar{\delta}, \bar{\xi}, \bar{R}, \bar{z}, \bar{H} = \frac{r, r_w, \delta, \xi, R, z, H}{l}; \quad \bar{\sigma} = \frac{\sigma}{\sigma_0}; \quad l = 100 r_w,$$
$$\bar{u} = \frac{ul}{\sigma_0 (p_a - p_w)}; \quad \bar{p} = \frac{p - p_w}{p_a - p_w}; \quad M = \frac{2\delta\sigma_f}{l\sigma_0},$$

where l is specific length, σ_0 is specific value of reservoir σ coefficient which is computed according to 4 using specific rock absolute permeability k_0, constant M describes the relation between permeability of rock and permeability of fracture.

Assume that the specific sizes of superelement ($R \approx 200\,m$), well ($r_w \approx 0.1\,m$) and fracture ($L \approx 10\,m$) provide inequalities

$$\bar{\delta} \ll \bar{r}_w \ll 1, \quad \bar{R} \gg 1.$$

Therefore we can represent the fracture as the zero width area within the outer flow problem computational domain. For the sake of convenience of obtaining numerical solution let us introduce logarithmic radial coordinate ρ as

$$\rho(\bar{r}) = \ln \frac{\bar{r}}{\bar{r}_w}, \quad 0 \le \rho \le \tilde{R} \equiv \ln \frac{\bar{R}}{\bar{r}_w}.$$

Uniform meshing along logarithmic radial coordinate ρ leads to a grid that reflects logarithmic origin of the pressure distribution. In the subsequent text the bar above non-dimensional variables will be omitted.

Non-dimensional formulations for outer (1)–(3) and inner (6), 7 flow problems takes the form:

- outer problem

$$\frac{\partial}{\partial \rho}\left(\sigma \frac{\partial p}{\partial \rho}\right) + \frac{\partial}{\partial \phi}\left(\sigma \frac{\partial p}{\partial \phi}\right) + r^2 \frac{\partial}{\partial z}\left(\sigma \frac{\partial p}{\partial z}\right) = 0, \quad (r, z, \phi) \in D - D_f - D_w; \tag{9}$$

$$p = p_a, \quad \rho = \tilde{R}; \qquad p = p_w, \quad \rho = 0; \qquad p = \langle p \rangle, \quad (\rho, z, \phi) \in \Gamma_f; \qquad \sigma \frac{\partial p}{\partial n} = 0, \quad z = 0, H, \tag{10}$$

- inner problem (for the left side of the fracture)

$$-M\left(\frac{\partial^2 \langle p \rangle}{\partial \rho^2} + r^2 \frac{\partial^2 \langle p \rangle}{\partial z^2}\right) = r\sigma \left.\frac{\partial p}{\partial \phi}\right|_{\phi=\pi+}^{\phi=\pi-}, \qquad 0 < \rho < \tilde{L}, \quad 0 < z < H, \tag{11}$$

$$\langle p \rangle = p_w, \quad \rho = 0; \quad \frac{\partial \langle p \rangle}{\partial \rho} = 0, \quad \rho = \tilde{L}, \quad \frac{\partial \langle p \rangle}{\partial z} = 0, \quad z = 0, H. \tag{12}$$

2.4. Numerical Solution

Spatial approximation of boundary value problems (9)–(12) was built on a regular mesh using finite volume method. Obtained systems of linear equations for inner and outer flow problems were used to construct coupled solver matrix, which was solved using algebraic multigrid method. So there was no need to introduce iterative process to combine inner and outer solutions.

3. Numerical Examples

A series of computations of the homogeneous ($\sigma = 1$) problems in a unit height reservoir with $p_w = 0$, $p_a = 1$ were performed for different M and L parameters. Fig. 3a shows the improvement of fractured well productivity due to enhancement of fracture length L and parameter M which reflects fracture permeability. In fig. 3b the relation between well flow rate q_w, fracture flow rate q_f, total flow rate q and fracture permeability is shown. Naturally for the low permeable fractures the main part of total flow rate goes through the well bore. For $M > 0.05$ the significant increase in the total flow rate is observed. In this case the fracture flow turns dominant, and the well bore flow rate goes to zero. The latter follows from the fact that high fracture flow rate decreases pressure gradient in a reservoir and therefore the filtration velocity towards the well in the direction parallel to fracture orientation decreases.

The pressure and velocity fields near the fractured wellbore and the pressure distribution in the direction parallel and perpendicular to fracture orientation are shown in fig. 4. It could be observed, that pressure distribution in a direction perpendicular to fracture is logarithmic, while in a direction parallel to fracture it has typical breakpoint at $r = L$. This is consistent with theoretical analysis provided by Kanevskaya (1998) and Badertdinova et. al (2010).

The results of 3D modelling of filtration flow in a non-homogeneous reservoir is shown in fig. 5. The dimensional sizes of computational domain were chosen as $R = 100\,m$, $H = 15\,m$, $r_w = 0.1\,m$. Well drainage area contained three separated permeable layers, the second of which was perforated. In the case of no fracturing (fig. 5b) pressure drop doesn't propagate outside perforated layer due to impermeable barrier above and below it, but in case of permeable fracture (fig. 5c) one can see that perforation impacts the whole fractured reservoir. In the latter example the total liquid flow rate increases by about 15 times.

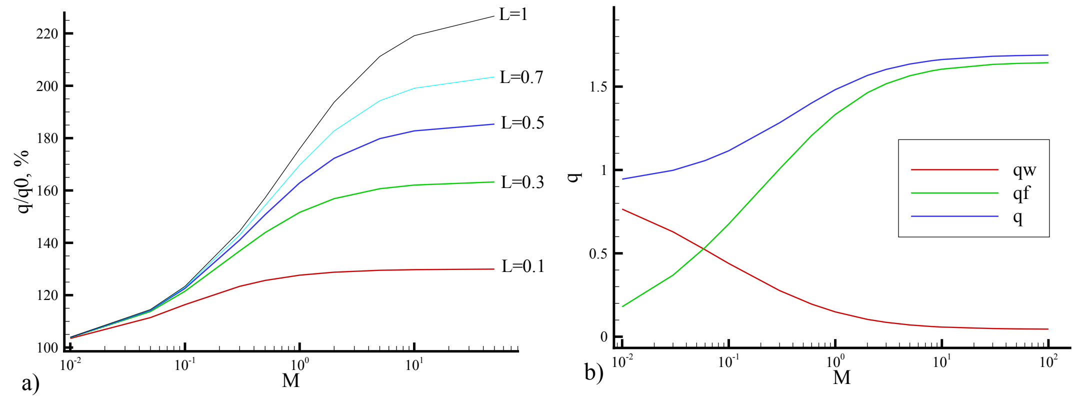

Fig. 3. Relation between volumetric flow rate enhancement due to fracturing (a), well bore and fracture flow rates for $L = 0.5$(b) from fracture parameters for homogeneous reservoir

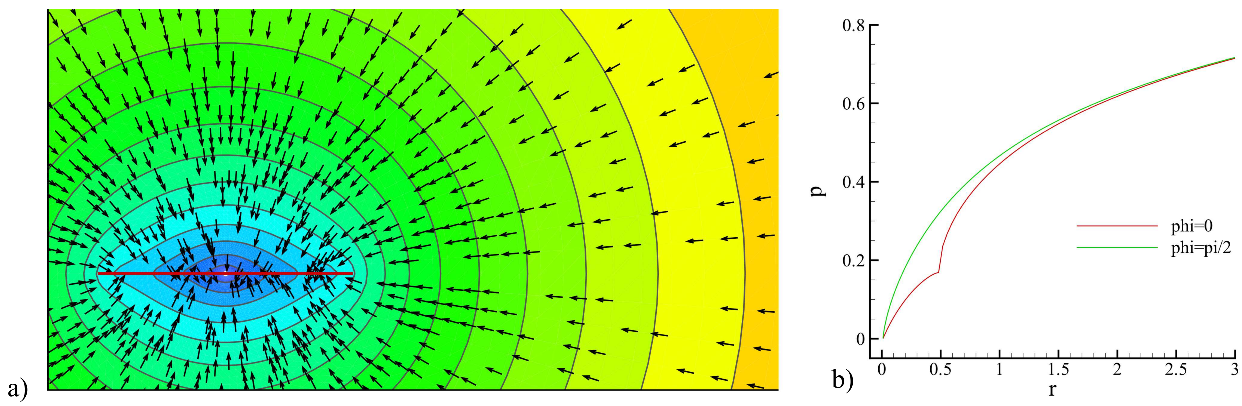

Fig. 4. Calculated pressure and filtration velocity for $L = 0.5$, $M = 5$ and homogeneous reservoir: pressure and velocity fields (a), pressure distribution along the axes parallel and perpendicular to fracture (b)

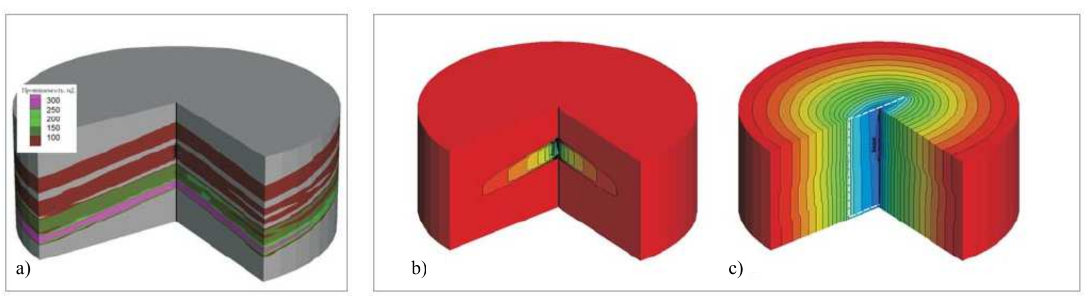

Fig. 5. An example of 3D flow calculation: absolute permeability of the drainage area (a), pressure field in the case of no fractures (b), pressure field around fracured well (c). Perforated well section is shown by marker, fracture is denoted by dashed white line

4. Conclusion

In present paper a numerical approach to obtain hydraulically fractured vertical well performance has been developed. The filtration flow fields are calculated within the well drainage area which is approximated by a vertical cylinder. Solution inside and outside fracture are searched separately and then combined using the equality of pressure and filtration velocities on the boundary of the fracture. Corresponding government equations are gained on the basis of simplified Darcy's law for incompressible fluid. Their non-dimensional formulations involve two parameters characterizing fracture properties: non-dimensional fracture length L and constant M which describes the relation between fracture and rock permeabilities, fracture width and well radius.

Large scale superelement reservoir solution is used to set Dirichlet boundary conditions on the lateral boundary of the cylinder. Presented method can deal with either well bore pressure or well volumetric flow rate as the specified parameter. In the latter case a special algorithm of decomposition of governing equation is used to treat non-local boundary condition on the well bore. The numerical solution of boundary value problems are obtained by finite volume method using regular grid with logarithmic refinement towards the well bore.

Presented numerical algorithm was used to obtain the set of solutions of model problems with different fracture properties. On the basis of these solutions the nomogram was built. It could be used to estimate the flow rate enhancement due to fracturing under the assumption of homogeneous reservoir.

In this paper only vertical wells with vertical fractures have been considered. However presented solution approach could be exploited to develop similar algorithm that deals with arbitrary shape of wells and fractures.

Large scale reservoir simulation which involves presented algorithm for fractured well performance calculation allows performing fast reservoir flow modelling with well interference effects taken into account and could be an effective tool for numerical estimation of hydraulic fracturing procedures benefits.

Acknowledgments

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