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Numerical Modelling of Large Elastic-plastic Deformations

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Abstract - The paper is devoted to development and implementation for a numerical method for investigation of stress-strain state of the solids with large elastic-plastic deformations. Calculation algorithm is based on the linearized equation of virtual work, defined to actual state. The arc-length method is used. A spatial discretization is based on the finite element method. The developed algorithm of investigation of large elastic-plastic deformations is tested on the solution of the necking of circular bar problem and a cylindrical shell subjecting to a torque.

***Keywords*:** Large deformations, Nonlinear elasticity, Plasticity, Finite element method.

1. Introduction

In this paper the algorithm of numerical solution of the problem of large elastic-plastic deformations is considered. There are many publications are devoted to solving problem of finite deformations, for example, Eidel et al. (2002), Schröder et al. (2002) or Berezhnoi D.V. et al. (2011). They differ by the difference in the formulation and solution obtained algebraic problems.

In this paper a solution algorithm is based on an Update Lagrange formulation. The principle of virtual work in terms of the virtual velocity is used. Kinematics of a medium is described by the left Cauchy–Green tensor, the stress state is determined by the Cauchy stress tensor. The theory of flow is used for describing plastic deformation. The total deformation rate is represented as a sum of elastic and plastic parts. The linearized constitutive equations of elastic deformation are obtained as a function of the derivative of the Truesdell stress rate. The von Mises yield criterion with isotropic hardening is used. The arc-length method is used for solving general equations. And the radial return method with an iterative refinement of the current mode of deformation is applied for dividing of the elastic and plastic deformations. The numerical implementation is based on the method of finite elements (FEM) (Golovanov et al. (2005a, 2005b, 2008)). The developed algorithm of investigation of large elastic-plastic deformations is tested on the solution of the necking of circular bar problem and a conical shell subjecting to a torque. The results of solutions and comparison with results obtained by other authors are presented.

2. Kinematics

The deformation gradient **F**, the left Cauchy–Green tensor $\mathbf{B} = \mathbf{F} \cdot \mathbf{F}^{\mathrm{T}}$, the velocity gradient $\mathbf{h} = \dot{\mathbf{F}} \cdot \mathbf{F}^{-1}$ and the deformation rate $\mathbf{d} = \mathrm{sym}(\mathbf{h})$ are used for describing of kinematics of a continuum (Bonet et al., 1997). The total deformation rate is represented as a sum of elastic and plastic parts: $\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p$.

3. Constitutive Equations

The constitutive equations are obtained using the free energy function and yield function. The Cauchy stress tensor is defined as

$$\boldsymbol{\Sigma} = \frac{2}{J} \mathbf{B} \cdot \frac{\partial \psi}{\partial \mathbf{B}}, \tag{1}$$

where $J = \det(\mathbf{F})$ is a changing of volume, ψ is the free energy per unit volume in the reference configuration. For isotropic material the free energy is defined as

$$\psi = \psi\left(I_{1\mathbf{B}}, I_{2\mathbf{B}}, I_{3\mathbf{B}}\right),$$

where $I_{i\mathbf{B}}$ is the corresponding invariants of $\mathbf{B}$.

After linearization (1) the rate of Cauchy stress is defined as

$$\dot{\boldsymbol{\Sigma}} = \boldsymbol{\Lambda}_{\boldsymbol{\Sigma}} \cdot\cdot \mathbf{d} + \mathbf{h} \cdot \boldsymbol{\Sigma} + \boldsymbol{\Sigma} \cdot \mathbf{h}^{\mathbf{T}} - \boldsymbol{\Sigma} I_{1\mathbf{d}},$$

where $\boldsymbol{\Lambda}_{\boldsymbol{\Sigma}} = \frac{4}{J}\mathbf{B} \cdot \frac{\partial^2 W}{\partial \mathbf{B} \partial \mathbf{B}} \cdot \mathbf{B}$.

Or

$$\boldsymbol{\Sigma}^{\mathrm{Tr}} = \boldsymbol{\Lambda}_{\boldsymbol{\Sigma}} : \mathbf{d}, \tag{2}$$

where $\boldsymbol{\Sigma}^{\mathrm{Tr}} = \dot{\boldsymbol{\Sigma}} + \mathbf{h} \cdot \boldsymbol{\Sigma} + \boldsymbol{\Sigma} \cdot \mathbf{h}^{\mathbf{T}} - I_{1\mathbf{d}}\boldsymbol{\Sigma}$ is the Truesdell stress rate.

The theory of flow is used for describing plastic deformation (Golovanov et al. (2005a, 2005b, 2008), Davydov et al. (2013)). The total deformation rate is represented as a sum of elastic and plastic parts: $\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p$. For the plastic deformation rate must hold association flow rule:

$$\mathbf{d}^p = \dot{\gamma}\frac{\partial \Phi}{\partial \boldsymbol{\Sigma}}, \tag{3}$$

where $\dot{\gamma}$ is the consistency parameter, Φ is a yield function.

The plastic flow $\dot{\gamma}$ can be computed from equation

$$\Phi\left(\boldsymbol{\Sigma}\right) = 0.$$

4. Variational Formulation: Integration Algorithm of the Flow Rules

The research algorithm is based on an Update Lagrange formulation. The principle of virtual work in terms of the virtual velocity as applied by Golovanov et al. (2005a, 2005b, 2008) is used:

$$\int_{\Omega} \boldsymbol{\Sigma} : \boldsymbol{\delta}\mathbf{d} d\Omega = \int_{\Omega} \mathbf{f} \cdot \boldsymbol{\delta}\mathbf{v} d\Omega + \int_{S^\sigma} \mathbf{p} \cdot \boldsymbol{\delta}\mathbf{v} dS,$$

where Ω is the current volume, S^σ is the surface on which the force $\mathbf{p}$ is applied, $\mathbf{f}$ is the body force vector, $\mathbf{v}$ is a velocity vector. After linearization the system of linear equations is obtained, where the unknown is the increment of displacement in the current state $\boldsymbol{\Delta}^{k+1}\mathbf{u}$. As previously used by Golovanov et al. (2008) for solving general system of equations the arc-length method is applied. The current state and trial stress are defined as ${}^{k+1}\mathbf{R} = {}^{k}\mathbf{R} + \Delta^{k+1}\mathbf{u}$ and ${}^{k+1}\tilde{\boldsymbol{\Sigma}} = {}^{k}\boldsymbol{\Sigma} + {}^{k+1}\Delta\boldsymbol{\Sigma}$. And if $\Phi\left({}^{\mathbf{k+1}}\tilde{\boldsymbol{\Sigma}}\right) \leq 0$ then the Cauchy stress ${}^{k+1}\boldsymbol{\Sigma} = {}^{k+1}\tilde{\boldsymbol{\Sigma}}$, else the radial return method with an iterative refinement of the current mode of deformation is applied, see Davydov et al. (2013).

5. Numerical Example

As an example the potential of elastic deformation is considered:

$$W = \frac{\lambda + 2\mu}{8}\left(I_{1\mathbf{B}} - 3\right)^2 + \mu\left(I_{1\mathbf{B}} - 3\right) - \frac{\mu}{2}\left(I_{2\mathbf{B}} - 3\right),$$

where λ, μ are Lame parameters. The von Mises yield criterion with isotropic hardening is used (e.g. Eidel et al. (2002) or Schröder (2002)). The numerical implementation is based on the finite element method. An 8-node brick element is used.

5.1. Necking of a Circular Bar

The necking of a circular bar is an example widely investigated in the literature; see e.g. Eidel et al. (2002), Schröder et al. (2002) or Berezhnoi D.V. et al. (2011). To initialize the necking process radius in the center is reduced by 1.8 %. Fig. 1 displays the final deformed structure and the equivalent plastic strain, which concentrates in the necking zone, Fig. 2 – the comparison of results. The results are in very good agreement with the computational reference solutions of Eidel et al. (2002) and Schröder (2002).

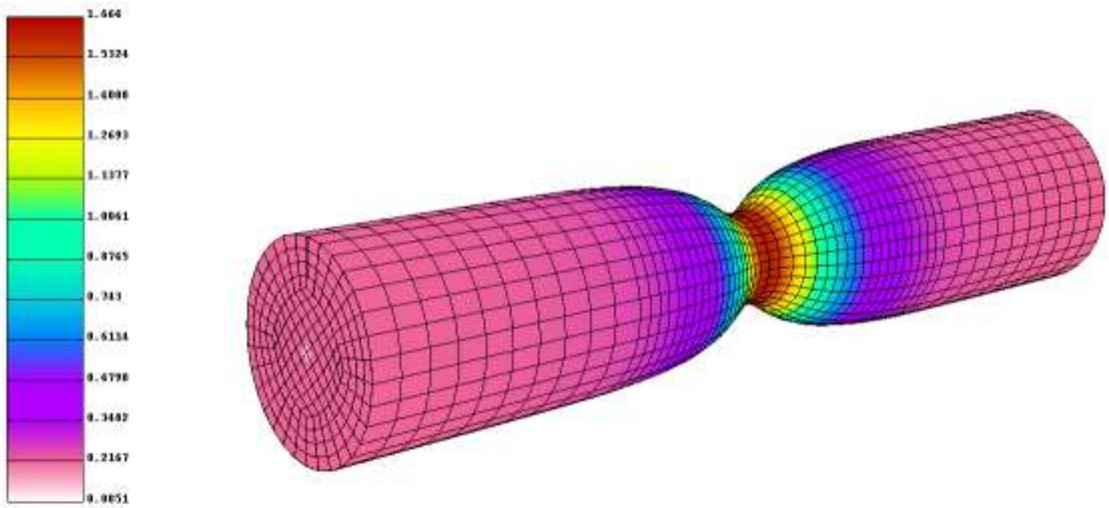

Fig. 1. Equivalent plastic strain at final structure

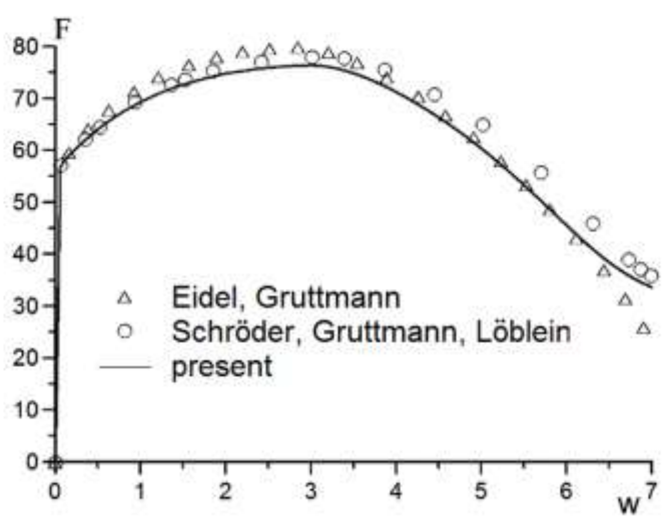

Fig. 2. Computational results of applied force F [kN] versus axial elongation w [mm].

5.2. Torsion of a Cylindrical Shell

In the second example of isotropic elastoplasticity a cilindrical shell subjecting to a torque. Fig. 3 displays the final deformed structure and the equivalent plastic strain.

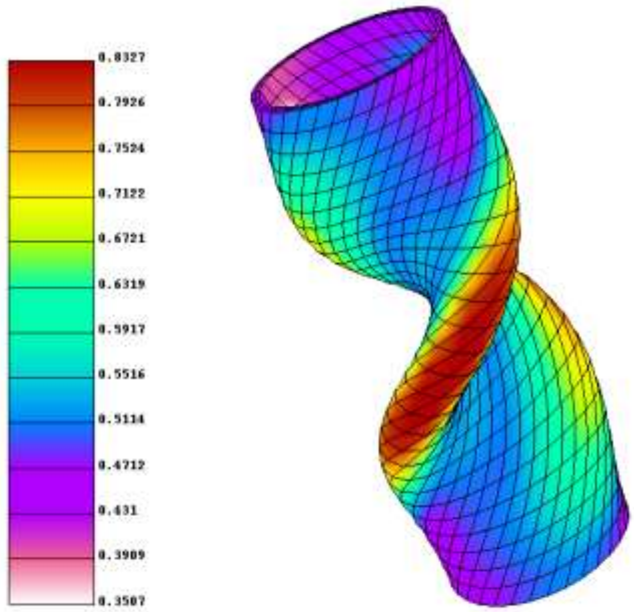

Fig. 3. Equivalent plastic strain at final structure.

6. Conclusion

In this paper a finite element model for isotropic elastic-plastic material behavior at large deformations has been presented. Physical relations are defined by the free energy function. The general radial return method is applied. Calculation algorithm is based on the linearized equation of virtual work in terms of the virtual velocity. The arc-length method is used. The results of the solutions of the necking of circular bar problem and a cylindrical shell subjecting to a torque are presented. The computed results have a good agreement with available solutions from the literature.

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