

# **Bearing Fault Detection by Resonant Frequency Band Pursuit Using a Synthesized Criterion**

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**Abstract** - Bearing fault can be detected using envelope demodulation of the bearing vibration signal in the resonant frequency band. Hence locating the correct resonant band is the key to successful application of the resonance envelope demodulation. A statistical criterion of the vibration signal can be used to guide the identification of the resonant band. However, a single criterion is often not robust in response to difference noises and interferences which are unavoidable for real bearing vibration signals. In this paper, we propose a bottom-up strategy to partition the frequency band of the vibration signal. The partitioning process is guided using a synthesized criterion that is formed by fusing multiple important criteria. The proposed technique is evaluated using both the simulated and real bearing vibration signals. The results show that the proposed technique can be used to detect bearing faults effectively.

**Keywords:** Bearings, Fault detection, Criterion fusion, Vibration signals, Resonance band pursuit.

## **1. Introduction**

Bearing fault detection is one of the most important tasks for the machinery health maintenance. Bearing fault can be reflected by such operation parameters as vibratory, acoustic, thermal and lubricating symptoms (Amerini, and Meo, 2011, Li and Liang 2011). Therefore the vibration signal analysis is a commonly-used parameter to detecting the bearing faults. Under real operations, the vibration signal of the bearing suffers from different noises and interferences. Therefore the fault feature is often buried too heavy to be identified. To reveal the vibration characteristics of the faulty bearing, researchers have developed different approaches, among them resonance demodulation is an effective method. As the fault feature may be modulated to high-frequency resonant band, the bearing defect can be identified through envelope demodulation of the bearing signal in the resonant band. However, the resonant band is usually unknown for real application. Hence the resonant band pursuit is the key to the resonance demodulation.

To locate the resonant frequency band for bearing fault detection, Antoni(2007) developed a kurtogram method using kurtosis as the criterion to detect the resonant band. Based on frequency domain kurtosis criterion, Li and Liang (2012) introduced a continuous-scale mathematical morphology approach for the optimal band demodulation of the bearing defect signature. In addition to employing the kurtosis as the criterion, some other statistical variables have also been used to guide the resonant band pursuit. Bozchalooi and Liang (2007) introduced a smoothness index criterion, the subband corresponding to the minimal smoothness index was chosen as the optimal resonant band. Gryllias and Antoniadis(2009) investigated the characteristics of the kurtosis and the smoothness index, and pointed out that the kurtosis

is prone to outliers, while the smoothness index is vague to small values. They proposed a peak energy criterion to determine the resonant band. However, the peak energy is sensitive to high-frequency noises.

In general, there are two problems in most of the popular techniques for the resonant band determination: a) subjectively dividing the whole band into evenly distributed sub-bands, and b) determining the resonant band by maximizing or minimizing a single statistical criterion (or cost function, e.g., kurtosis, smoothness index or peak energy. However, it is difficult to find a perfect criterion for different cases. For the above reasons, this paper presents a synthesized criterion incorporating with the fine spectral merge technique to identify the resonant frequency band. With the proposed technique, the spectral representation of the bearing vibration signal is partitioned using a bottom-up based fine spectral merge strategy. The spectral merge process is guided using the synthesized criterion that is the fuzzy fusion of multiple criteria. The present technique can improve both the accuracy of the resonant band boundary and the robustness of the criterion simultaneously, and therefore being capable of detecting the bearing fault from noisy background.

## 2. Spectral Merge Guided by the Synthesized Criterion

### 2.1. An improved Bottom-Up Strategy for Spectral Segmentation

Consider a temporal vibration signal  $x(t)$ . Suppose its Fourier transform is  $X(f)$ , which can be represented by a set of  $L$ -dimensional series, i.e.,

$$X(f) = \{0, X_1, X_2, \dots, X_k, \dots, X_{L-1}\}, k \in [1, L-1] \quad (1)$$

sorted by frequency points  $\{0, f_1, f_2, \dots, f_k, \dots, f_{L-1}\}$ , where  $X_k$  is the spectral value at the  $(k+1)$ th discrete frequency point. Suppose the resonance frequency of the bearing system is inside the frequency band  $f = \{0, f_1, f_2, \dots, f_k, \dots, f_{L-1}\}$ . There should be the optimal frequency subset  $\Phi$  ( $\Phi \in [0, f_{L-1}]$ ) where the fault feature dominates the spectrum, i.e., the signal-to-noise ratio (SNR) is the highest. A reasonable way to find out  $\Phi$  is to divide the frequency band of interest into segments, among which the one corresponding to the highest ratio between the fault feature and the noise is the optimal band  $\Phi$ . In our case, the  $i$ -th segment of the frequency spectrum  $X(f)$  is a set of consecutive frequency points:

$$S_i(a_i, b_i) = \{X_{a_i}, \dots, X_{b_i}\}, \quad (2)$$

where  $a_i, b_i$  represent left and right boundaries of segment  $i$  ( $i \in [1, L]$ ), respectively. In this way, the frequency spectrum  $X(f)$  can be split into  $l$  ( $l \leq L$ ) frequency intervals, i.e.,

$$X(f) = \{S_1(a_1, b_1), \dots, S_i(a_i, b_i), \dots, S_l(a_l, b_l)\}, \quad (3)$$

where  $a_1=0$ ,  $a_2=f_1$  and  $b_l=f_{L-1}$ . The segmentation objective is to find internally homogeneous segments from the given sequence. The internal homogeneity of segment  $S_i(a_i, b_i)$  can be defined using a cost function  $C(S_i)$ . The sum of the costs for all the individual segments is therefore given by

$$C(X) = \sum_{i=1}^l C(S_i) \quad (4)$$

For the conventional segmentation algorithms, in general, the optimal partition can be obtained by minimizing the sum of the costs of all the individual segments, i.e.,

$$X^*(f) = \arg \min_x C(X) \quad (5)$$

For computational efficiency, the above optimization problem can be heuristically solved by the traditional bottom-up method that begins with creating the finest possible approximation of the sequence, and then merge the segments until a stopping criterion is satisfied. However, this method requires that the number of the segments ( $l$ ) and the bandwidth be pre-determined without the “cost” consideration. This obviously will undermine the quality of the results due to the subjectivity and bias. Another drawback of this method is that it minimizes the sum of the costs of all segments as expressed by Eq. (5). This does not adequately reflect the true purpose of the vibration spectrum segmentation because minimizing the **total** cost does not necessarily lead to a band in which the fault feature dominates other signal components.

As such, we propose an improved bottom-up technique. In light of the above discussions, we begin with a more appropriate definition of an optimal partition, i.e.,

$$X^*(f) = \arg \min \left( \min_i (C(S_i)) \right). \quad (6)$$

Comparing Eq. (6) with Eq. (5), one can see that the objective of the improved bottom-up based spectral segmentation is to minimize the cost of one segment (i.e., the resonance band) among all the segments of  $X(f)$ . The reason is that, we are only interested in the optimal band that is correlated to  $\min(\min(C(S_i)))$ . Hence we propose an improved bottom-up method involving the following steps:

*Step 1.* Create initial fine approximation  $\{S_1(0, f_1), \dots, S_i(f_i, f_{i+1}), \dots, S_{L-1}(f_{L-2}, f_{L-1})\}$ .

*Step 2.* Calculate  $C(S_i)$ , and the cost of the attempted merge,  $MC(S_j)$ , using Eq. (6).

*Step 3.* Find the pair of segments,  $S_m(a_m, b_m)$  and  $S_{m+1}(a_{m+1}, b_{m+1})$ , corresponding to the minimum cost of the attempted merge, i.e.,  $m = \arg \min_j (MC(S_j))$ .

*Step 4.* Recalculate the costs of the merged pair  $m$  and of its immediate predecessor.

*Step 5.* Let  $L=L-1$ . If  $MC(S_{\min}) < +\infty$ , and  $L > 2$ , go to Step 3; Otherwise, output the segment with the minimum cost as the optimal band  $\Phi$ .

### 3. Synthesizing Criteria and Its Application

We now illustrate the method to obtain the cost function by fusing several complementary criteria. This cost function will be used in the proposed algorithm to guide the spectral segmentation process.

#### 3.1. Fuzzy Fusion towards a Synthesized Criterion

Kurtosis, smoothness index (SI) and crest factor (CF) are among the most widely used and effective criteria though each has its own limitations. For reliable fault detection, it is desirable to take advantage of their complementary features by fusing them into a single normalized non-dimensional “cost” criterion. Kurtosis, SI and CF are defined as follows (Antoni and Randall, 2006, Bozchalooi and Liang, 2007, Nikolaou and Antoniadis, 2002):

$$KU(x(t)) = \frac{E\{x(t) - E\{x(t)\}\}^4}{E\{x(t) - E\{x(t)\}\}^2}, \quad SI(x(t)) = \frac{G(x(t))}{A(x(t))}, \quad CF(x(t)) = \frac{\max(x(t)) - A(x(t))}{RMS(x(t))} \quad (7)$$

where  $E\{\cdot\}$  stands for the expectation operation of the signal  $x(t)$ ,  $A(x(t)) = \frac{1}{N} \sum_{t=1}^N x(t)$ ,

$G(x(t)) = \left( \prod_{t=1}^N x(t) \right)^{1/N}$ ,  $N$  is the length of the signal  $x(t)$ , and  $RMS(\cdot)$  denotes the root mean square of the

signal. The above three criteria can be fused as a single cost function:

$$C(S_i(f)) = F(CR_q(x_i(t))). \quad (8)$$

where  $F(\cdot)$  denote the fusion of the criteria,  $CR_q$  represent the  $q$ th criterion, with  $q=1, 2$  and  $3$  respectively representing kurtosis, SI and CF. The three criteria are expressed as

$$CR_1 = 1/KU, CR_2 = SI, \text{ and } CR_3 = 1/CF. \quad (9)$$

According to the information theory, the entropy is a measure of the degree of information disorder. High heterogeneity in the values of a criterion in different segments indicates low information entropy of the criterion. In other words, this criterion can provide more useful information, and therefore should be given greater weight (Zou, et al., 2006). In this way, the lower the entropy of a criterion is, the greater the weight should be. Hence, different criteria can be fused using a simple weighting approach detailed as follows. To perform the entropy weighting method, we normalize  $CR_q$  as follows

$$CR'_q(x_i(t)) = \frac{CR_q(x_i(t)) - \min_{i=1}^l(CR_q(x_i(t)))}{\max_{i=1}^l(CR_q(x_i(t))) - \min_{i=1}^l(CR_q(x_i(t)))} \quad (10)$$

where the normalized  $CR'_q(x_i(t))$  varies in the range  $[0, 1]$ . Its probability of appearing in sequence  $\{ CR'_q(x_1(t), x_2(t), \dots, x_i(t), \dots, x_l(t)) \}$  is  $P_q(x_i(t)) = CR'_q(x_i(t)) / \sum_{i=1}^l CR'_q(x_i(t))$ . If  $P_q(x_i(t)) \ln(P_q(x_i(t))) = 0$  when  $P_q(x_i(t)) = 0$ , the entropy  $H_q$  of the  $q$ th normalized criteria is given by (Zou, et al., 2006)

$$H_q = \frac{1}{\ln l} \sum_{i=1}^l P_q(x_i(t)) \ln(P_q(x_i(t))). \quad (11)$$

The weight  $W_q$  can therefore be calculated by  $W_q = (1 - H_q) / \sum_{q=1}^3 (1 - H_q)$ . Obviously  $\sum_{q=1}^3 W_q = 1$ . Having given the weight of each normalized criterion, Eq. (8) can be recast as

$$C(S_i(f)) = \sum_{q=1}^3 (W_q CR'_q(x_i(t))). \quad (12)$$

The above cost function will be used to guide the spectral segmentation procedure as illustrated earlier.

### 3.2. Defective Bearing Signature Demodulation Using the Proposed Approach

Once the optimal frequency band  $\Phi$  corresponding to  $s(a_{\text{opt}}, b_{\text{opt}})$  has been determined, the central frequency  $f_{cf}$  and the band width  $w_b$  of the optimal band can be calculated as

$$f_{cf} = (b_{\text{opt}} - a_{\text{opt}}) / 2, \text{ and } w_b = b_{\text{opt}} - a_{\text{opt}}. \quad (13)$$

The envelope signal of  $S(a_{\text{opt}}, b_{\text{opt}})$  is obtained using the Hilbert transform as (Hsu, et al., 2013)

$$e(t) = \text{mod} \left( \mathbf{H} \left( \mathbf{F}^{-1} (X(f)) \Big|_{a_{\text{opt}}}^{b_{\text{opt}}} \right) \right). \quad (14)$$

The bearing fault signature can be detected from either the temporal or the spectral representation of  $e(t)$ .

#### 4. Evaluation of the Proposed Approach Using Simulated Signals

##### 4. 1. Simulated Signal Analysis Using the Proposed Approach

The vibration signal  $x(t)$  of a defective rolling element bearing can be considered as a mixture of three signal components:

$$x(t) = u(t) + \theta(t) + \delta(t) \quad (15)$$

where  $u(t)$  denotes the transient component,  $\theta(t)$  represents the interference component, and  $\delta(t)$  stands for the white Gaussian noise component. The transient signature of the defective rolling element bearings can be modeled as (Li et al., 2012)

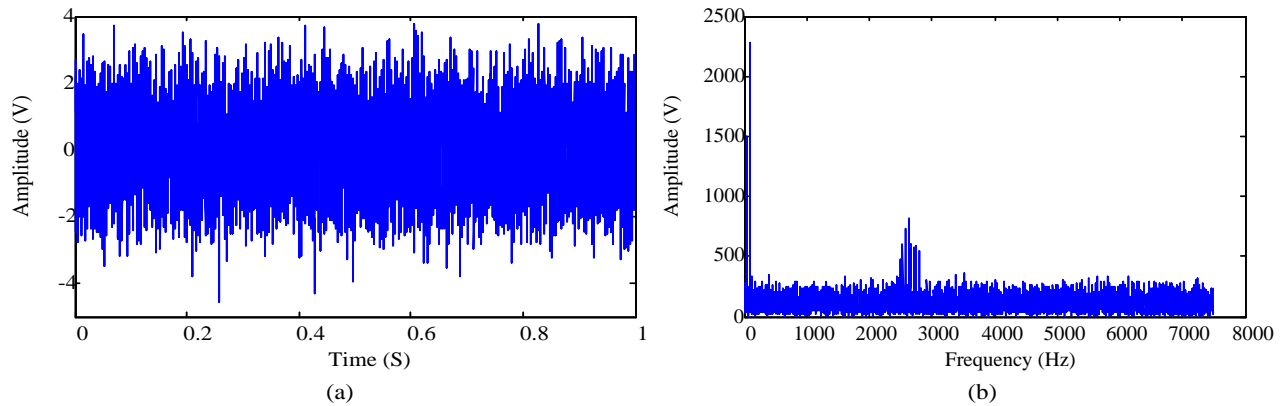
$$u(t) = B \sum_n p(t - n / f_c) \quad (16)$$

where  $B$  is the amplitude of the impulses,  $n$  the number of the impulses,  $f_c$  the defective characteristic frequency, and  $p(\cdot)$  the impulse response function that is given by

$$p(t) = \begin{cases} e^{-a_d t} \sin(2\pi f_0 t) & ; t > 0 \\ 0 & ; \text{otherwise} \end{cases} \quad (17)$$

where  $f_0$  is the resonance frequency and  $a_d$  the decay (or bandwidth) parameter.

To validate the proposed approach, we generated a simulated vibration signal  $x(t)$  using the model given by Eqs. (15)-(17). The parameters are:  $B=[1.4, 1.7]$ ,  $a_d = 620$ ,  $f_c=43\text{Hz}$ ,  $f_0=2600\text{Hz}$ ,  $\theta(t)$  is composed of two sinusoidal interference components:  $0.3\sin(168\pi t)$  and  $0.2\cos(62\pi t)$ , and  $\delta(t)$  is the white Gaussian noise with signal-to-noise ratio (SNR) of -10dB. We further specify that the time span is  $[0, 1]$  sec and sampling frequency  $f_s$  is 15000Hz. Figs. 1(a) and 1(b) display the temporal and Fourier representations of the simulated signal  $x(t)$ . In Fig. 1(b) the two interferences (84Hz and 31Hz) dominate the spectrum. Neither the fault characteristic frequency nor its harmonics can be identified.



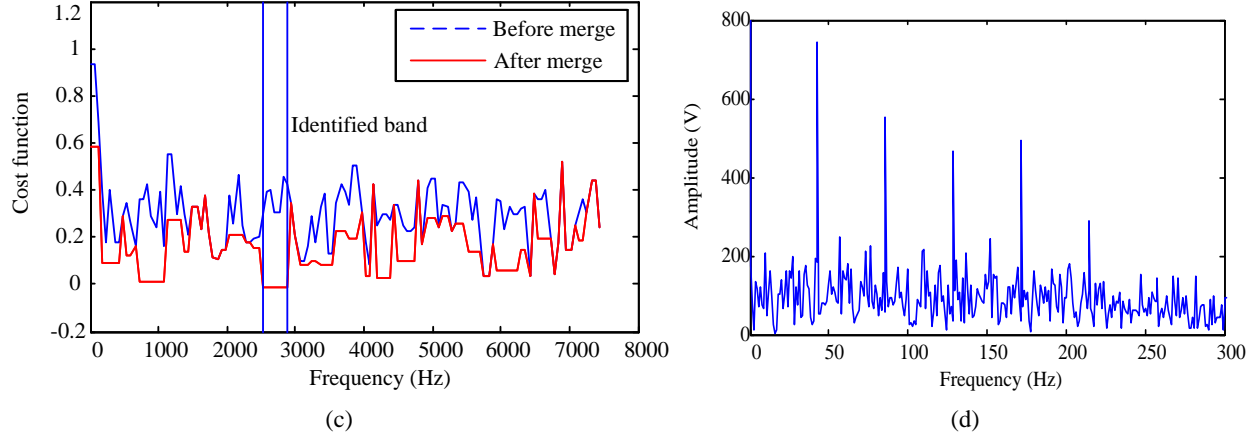


Fig. 1. Analysis results of the simulated signal using the proposed technique: (a) raw signal, (b) Fourier spectrum of the signal, (c) resonant band, and (d) envelope demodulation spectrum obtained using the resonant band.

We then employ the proposed technique to pursue the resonant frequency band. Fig. 1(c) displays the cost functions before and after the merge operation. It is shown that the minimal cost function after the merge occurs between [2460, 2940]Hz, with a corresponding cost of -0.01676. Hence [2460, 2940]Hz is identified as the resonant band, which correctly covers the real central frequency 2600Hz of the simulated signal. The envelope signal of the resonant frequency band can be calculated using Eq. (10) and its spectral representation is shown in Fig. 1(d). From the figure, the fault characteristic frequency (FCF)  $f_c=43\text{Hz}$  and its five harmonics can be observed.

#### 4. 2. Comparisons with Single-criterion Methods

To show the effectiveness of the proposed approach, the simulated vibration signal  $x(t)$  as shown in Fig. 1(a) was also analyzed using single-criterion methods. Three normalized criteria associated with kurtosis, SI and CF, were employed to segment the spectrum respectively. With the kurtosis as the criterion, the minimum cost 0 (i.e., maximum kurtosis) is found at [5730, 5850]Hz. However, the true resonant frequency is 2600 Hz which falls into the band of [2490, 2850] Hz. Due to the influence of the outlier, the real resonant band was ignored. The frequency representation of the envelope of the demodulated signal based on the identified band [5730, 5850]Hz is shown in Fig. 2 (a). Neither the damage characteristic frequency nor its harmonics can be identified from this figure.

We then examine the performance of the SI-guided spectral segmentation. The minimum cost -0.07291 (i.e., minimum SI) occurs at [690, 1110]Hz. Again the true target band [2490, 2910] Hz is missed. The spectrum of the envelope of the demodulated signal obtained based on the identified band [690, 1110]Hz is shown in Fig. 2(b). Again, the fault characteristic frequency and its harmonics cannot be observed in this result.

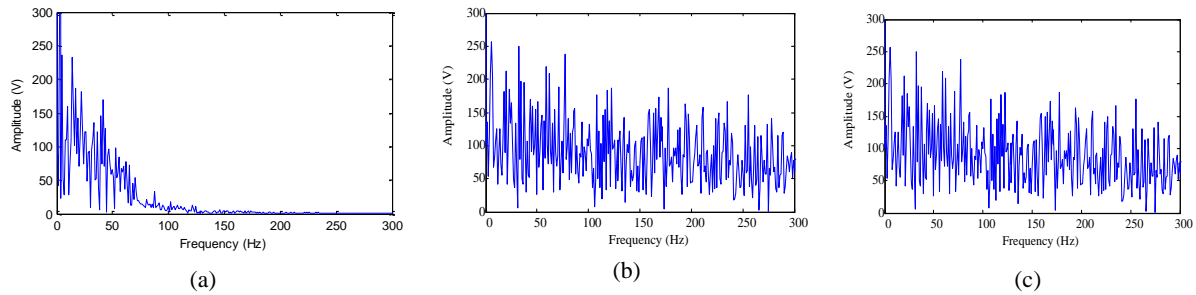


Fig. 2 Analysis results of the simulated signal obtained using: (a) the kurtosis based method, (b) the smoothness index based method, and (c) the CF based method.

Similarly, the CF was used to guide the spectral merge. The minimum cost (i.e., maximum CF) is -0.1182 corresponding to the identified band [690, 1110]Hz. Fig. 2 (c) displays the frequency representation of the envelope of the demodulated result obtained using the CF-guided band [690, 1110]Hz. Neither the fault characteristic frequency nor its harmonics can be identified from this figure.

The above comparison clearly indicates that the synthesized criterion yields much better result comparing to those obtained using the three criteria individually.

## 5. Experimental Evaluations

The proposed spectral merge technique was further evaluated using experimental vibration signals collected from a machinery fault simulator (SpectraQues, MFS-Magnum). The machinery fault simulator was driven by a 3-hp AC motor. To adjust the rotational frequency ( $f_r$ ) of the motor (and also the shaft), a PWM inverter was applied to control the motor. Two rolling element bearings were used to support the two ends of the shaft, respectively. An accelerometer was mounted on the housing of the left bearing to collect the vibration signals, which were fed to a computer through a data acquisition card.

Two bearings of the same type (MB, ER-16K), one with an inner race fault, and the other with an outer race fault, were used for the experiments, separately. The inside, pitch and ball diameters of the ER-16K bearings are 1", 1.548" and 0.3125", respectively, and number of balls is 9. All the faults of the bearings were pre-planted by the manufacturer. The sampling frequency of the vibration signals was set at 1.2 kHz, and the data length was chosen as 1 sec. The ER-16K rolling element bearing with an inner race fault was installed on the left end of the shaft. The rotational speed was set at 500 rpm. The faulty characteristic frequency of the inner race fault is  $5.408f_c$  ( $=44.9\text{Hz}$ ). Fig. 3 (a) presents the temporal signal collected from this bearing. The proposed method was used to analyze the signal, and the result is shown in Fig. 3(b). From Fig. 3(b) one can identify up to 9 harmonics of the fault characteristic frequency.

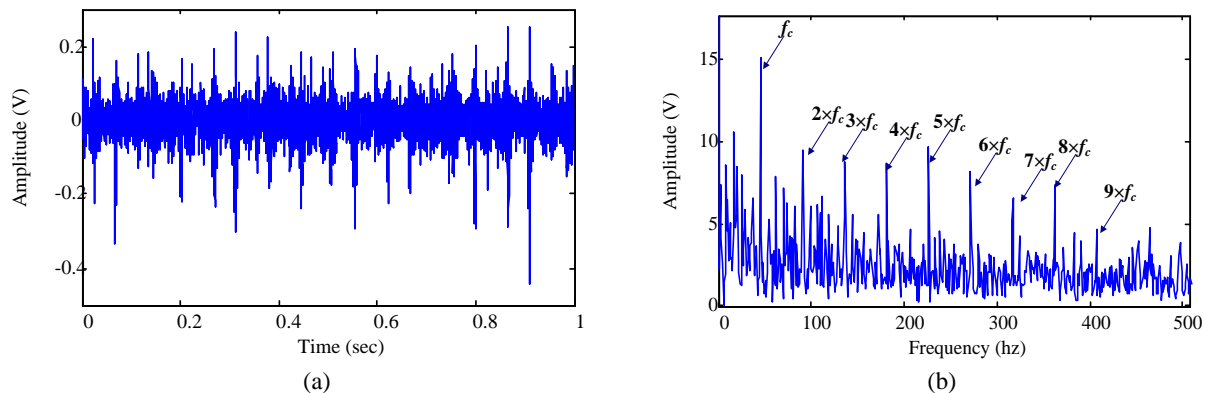


Fig.3. Inner race fault bearing diagnosis: (a)temporal waveform of the collected raw signal, and (b)envelope demodulation result using the proposed method.

The inner race fault bearing was then replaced by the one with an outer race fault. The shaft speed was again set at 500 rpm (i.e.  $f_r=8.3\text{Hz}$ ). The fault characteristic frequency is  $3.592f_r$  ( $=29.9\text{Hz}$ ). Fig. 4(a) plots the raw vibration signal. The proposed approach was applied to detect the possible bearing fault. The envelope of the extracted signal using the proposed method was shown in Fig. 4(b). Up to six orders of  $f_c$  can be observed clearly from this figure.

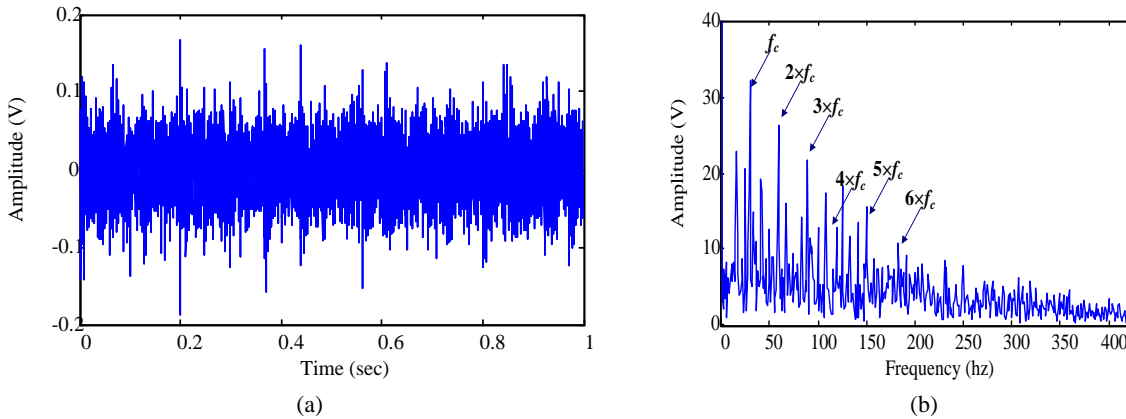


Fig.4. Outer race fault bearing diagnosis: (a) Vibration signal collected from the inner race fault bearing, and (b) Envelope demodulation result using the proposed method.

## 6. Conclusion

In this paper, a synthesized criterion guided spectral merge technique is developed to pursue resonant band for bearing fault detection. Three commonly used criteria, namely, kurtosis, smoothness index and crest factor, were synthesized into a single criterion. The spectral merge process is performed using an improved bottom-up strategy. Among all the merged subbands, the one with the minimal cost function (the value of the synthesized criterion) was identified as the resonant frequency band. Both the simulated and the real bearing signals were collected to evaluate the proposed technique. The results show that the proposed method can reliably detect bearing faults.

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