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Limit Behavior of LQG Control Problem with Small Noise in Observation and Some Aerospace Application

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Abstract **-** A finite-horizon Linear-Quadratic-Gaussian (LQG) control problem is considered. Observation noise in this problem is assumed to be small. A limit behavior (as the noise intensity tends to zero) of the optimal value of the cost functional in this problem is analyzed. Theoretical results are applied to a planar interception problem of a maneuvering target with linearized first-order dynamics of both interceptor and target.

Keywords**:** Linear-quadratic-Gaussian problem, Small noise in observation, Singular perturbation techniques, Interception of a maneuvering target.

1. Introduction

Modern measurement devices, being accurate enough, yield a small measurement error (noise). A controlled system, equipped with such a type of measurement devices, is modeled mathematically by dynamic (differential or difference) equation and observation (output) equation with a small noise.

Continuous-time linear optimal filtering and estimation problems, as well as LQG control problems, with a small noise in observation (or noise free observation) were studied extensively in the literature. Most of the works in this topic deal with the infinite horizon case in either steady state or integral versions (see e.g. (Friedland 1971, Kwakernaak & Sivan 1972*b*, O'Reilly 1983, Soroka & Shaked 1988, Halevi, Haddad & Bernstein 1993, Aganovic, Gajic & Shen 1995, Braslavsky, Seron, Mayne & Kokotovic 1999, Hippe 2011) and references therein). The finite horizon case in this topic, although being of a considerable interest in theory and applications, was considered much less in the literature (see (Bryson & Johansen 1965, Glizer 1984)). In the present paper, a finite horizon LQG control problem with a small noise in the observation equation is considered. This problem is analyzed by using a singular perturbation technique. In (Glizer 1984), such a technique was developed for an asymptotic analysis of the covariance matrix of the filtering error in a finite horizon linear optimal filtering problem with a small noise in the observation. In the present paper, we analyze a limit behavior of the optimal value of the cost functional in the considered LQG problem.

2. Problem Statement

Consider the controlled system

$$\dot{X}(t) = \mathsf{A}X(t) + \mathsf{B}u(t) + v(t), \quad X(0) = X_0, \quad t \in [0, t_f], \tag{1}$$

$$z(t) = \mathsf{C}X(t) + \varepsilon w(t), \quad t \in [0, t_f], \tag{2}$$

where $t_f > 0$ is a given final time moment; $X(t) \in R^n$ is a state vector; $u(t) \in R^r$ is a control; $z(t) \in R^q$, $(q < n)$, is an observed output; $\varepsilon > 0$ is a small parameter, $(\varepsilon << 1)$; $\mathcal{A}$, $\mathcal{B}$, C are given constant matrices of corresponding dimensions; X_0 is a Gaussian random vector with the average $\overline{X}_0$ and the symmetric positive semidefinite covariance matrix F_0; $v(t)$ and $w(t)$, $t \in [0, t_f]$ are Gaussian white noise vectors with zero averages and the covariances $\mathsf{D}(t)\delta(t-\tau)$ and $\mathsf{R}(t)\delta(t-\tau)$, respectively; the matrix $\mathsf{D}(t)$ is symmetric positive semidefinite, while the matrix $\mathsf{R}(t)$ is symmetric positive definite on the interval $[0, t_f]$; the random vector X_0 and the vector stochastic processes $v(t)$ and $w(t)$ are independent of each other.

The cost functional, to be minimized by the control $u(t)$ basing on the knowledge of the output (2), is

$$\mathsf{J}(u) = E\left[X^T(t_f)\mathsf{G}X(t_f) + \int_0^{t_f} (X^T(t)\mathsf{P}(t)X(t) + u^T(t)\mathsf{Q}(t)u(t))dt \right], \tag{3}$$

where $E[\cdot]$ denotes the mathematical expectation; the matrix G is symmetric positive semidefinite; the matrix $\mathsf{P}(t)$ is symmetric positive semidefinite for all $t \in [0, t_f]$; the matrix $\mathsf{Q}(t)$ is symmetric positive definite for all $t \in [0, t_f]$; the superscript T denotes the transposition.

In what follows, we assume:

(A1) the matrix C has full rank q;

(A2) the matrix-valued function $\mathsf{D}(t)$ is twice continuously differentiable on the interval $[0, t_f]$;

(A3) the matrix-valued functions $\mathsf{R}(t)$, $\mathsf{P}(t)$ and $\mathsf{Q}(t)$ are continuously differentiable on the interval $[0, t_f]$;

(A4) $\det(\mathsf{C}\mathsf{D}(t)\mathsf{C}^T) \neq 0$ for all $t \in [0, t_f]$;

(A5) $\mathsf{C}\mathsf{F}_0 = 0$.

Let C_c be a complement matrix to the matrix C, i.e., the dimension of C_c is $(n-q) \times n$ and the block matrix $(\mathsf{C}_c^T, \mathsf{C}^T)^T$ is nonsingular. Consider the matrices

$$\mathsf{L}(t) = \mathsf{C}_c - \mathsf{C}_c\mathsf{D}(t)\mathsf{C}^T(\mathsf{C}\mathsf{D}(t)\mathsf{C}^T)^{-1}\mathsf{C}, \quad \mathsf{M}(t) = (\mathsf{C}^T, \mathsf{L}^T(t))^T. \tag{4}$$

Due to results of Glizer, Fridman & Turetsky (2007), the matrix $\mathsf{M}(t)$ is nonsingular for all $t \in [0, t_f]$. The latter, along with the assumption (A2) and the equation (4), means that the matrix-valued functions $\mathsf{M}(t)$ and $\mathsf{M}^{-1}(t)$ are twice continuously differentiable on the interval $[0, t_f]$. Let us transform the state variable in the optimal control problem (1) – (3) as follows: $X(t) = \mathsf{M}^{-1}(t)x(t)$, where $x(t)$ is a new state variable. Due to this transformation, we obtain the following optimal control problem, equivalent to the one (1) – (3):

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + \mathsf{M}(t)v(t), \quad x(0) = x_0, \ t \in [0, t_f], \tag{5}$$

$$z(t) = Cx(t) + \varepsilon w(t), \tag{6}$$

$$J(u) = E\left[x^T(t_f)Gx(t_f) + \int_0^{t_f} (x^T(t)P(t)x(t) + u^T(t)Q(t)u(t))dt \right], \quad (7)$$

where $A(t) = \mathrm{M}(t)\mathrm{A}\mathrm{M}^{-1}(t) + \dot{\mathrm{M}}(t)\mathrm{M}^{-1}(t)$, $B(t) = \mathrm{M}(t)\mathrm{B}$, $C = \mathrm{C}\mathrm{M}^{-1}(t) = (I_q, 0)$, $G = (\mathrm{M}^{-1}(t_f))^T \mathrm{G}\mathrm{M}^{-1}(t_f)$, $P(t) = (\mathrm{M}^{-1}(t))^T \mathrm{P}(t)\mathrm{M}^{-1}(t)$, $Q(t) = \mathrm{Q}(t)$, $x_0 = \mathrm{M}(0)X_0$, I_q is the identity matrix of the dimension q; x_0 is a random Gaussian vector with the average $\bar{x}_0 = \mathrm{M}(0)\bar{X}_0$ and the covariance

$$F_0 = \mathrm{M}(0)\mathrm{F}_0\mathrm{M}^T(0) = \begin{pmatrix} 0 & 0 \\ 0 & \Phi \end{pmatrix}, \quad \Phi = \mathrm{L}(0)\mathrm{F}_0\mathrm{L}^T(0). \quad (8)$$

Since the matrices G, $\mathrm{P}(t)$ and F_0 are symmetric positive semidefinite, then the matrices G, $P(t)$ and Φ are symmetric positive semidefinite.

In the sequel, of the paper, we deal with the optimal control problem (5) – (7). We call this problem the Original Optimal Control Problem (OOCP). Our objective is to establish the existence of the limit (as $\varepsilon \to +0$) of the optimal value of the cost functional in the OOCP, and to calculate this limit value.

3. Control Optimality Conditions for the OOCP

By using results of Kwakernaak & Sivan (1972*a*), Bryson & Ho (1975), Sage & White (1977), we obtain that, for a given $\varepsilon > 0$, the optimal control $u(t) = u_\varepsilon^*(t, \hat{x}(t))$ in the OOCP has the form

$$u_\varepsilon^*(t, \hat{x}(t)) = -Q^{-1}(t)B^T(t)R(t)\hat{x}(t), \quad (9)$$

where the $n \times n$-matrix-valued function $R(t)$ is the unique solution of the terminal value problem for the Riccati equation

$$\dot{R}(t) = -R(t)A(t) - A^T(t)R(t) + R(t)B(t)Q^{-1}(t)B^T R(t) - P(t), \quad t \in [0, t_f], \; R(t_f) = G. \quad (10)$$

Note that the matrix $R(t)$ is symmetric positive semidefinite.

The vector-valued function $\hat{x}(t)$ satisfies the initial value problem

$$\dot{\hat{x}}(t) = (A(t) - K(t)S(t,\varepsilon))\hat{x}(t) + B(t)u_\varepsilon^*(t, \hat{x}(t)) + \frac{1}{\varepsilon^2}K(t)C^T(t)\mathrm{R}^{-1}(t)z(t), t \in [0, t_f],$$
$$\hat{x}(0) = \bar{x}_0 = \mathrm{M}(0)\bar{X}_0, \quad (11)$$

where $S(t,\varepsilon) = (1/\varepsilon^2)C^T(t)\mathrm{R}^{-1}(t)C(t)$; the $n \times n$-matrix-valued function $K(t)$ is the unique solution of the initial value problem for the Riccati equation

$$\dot{K}(t) = A(t)K(t) + K(t)A^T(t) - K(t)S(t,\varepsilon)K(t) + D(t), \quad t \in [0, t_f], \; K(0) = F_0. \quad (12)$$

The $n\times n$-matrix-valued function $D(t)$, appearing in (12), has the form

$$D(t) = \mathrm{M}(t)\mathrm{D}(t)\mathrm{M}^T(t) = \begin{pmatrix} D_1(t) & 0 \\ 0 & D_2(t) \end{pmatrix},\; D_1(t) = \mathrm{C}\mathrm{D}(t)\mathrm{C}^T,\; D_2(t) = \mathrm{L}(t)\mathrm{D}(t)\mathrm{L}^T(t). \tag{13}$$

Note that the matrix $K(t)$ is symmetric positive semidefinite. Note also, that the $n\times n$-matrix-valued function $S(t,\varepsilon)$ can be represented in the block form

$$S(t,\varepsilon) = \begin{pmatrix} \mathrm{R}^{-1}(t)/\varepsilon^2 & 0 \\ 0 & 0 \end{pmatrix}. \tag{14}$$

The optimal value of the cost functional in the OOCP has the form

$$J_\varepsilon^* = \bar{x}_0^T R(0)\bar{x}_0 + \mathrm{tr}\left[\int_0^{t_f} K(t)S(t,\varepsilon)K(t)R(t)dt + GK(t_f) + \int_0^{t_f} P(t)K(t)dt\right]. \tag{15}$$

In this expression, only the matrices $S(t,\varepsilon)$ and $K(t)$ depend on ε. Therefore, in order to study a behavior of J_ε^* for $\varepsilon \to +0$, one has to study such a behavior of the matrix $K(t)$.

4. Asymptotic Behavior of $K(t)$

Due to (14), the right-hand side of the equation (12) has singularity for $\varepsilon = 0$. In order to remove this singularity, we look for the solution $K(t) = K(t,\varepsilon)$ of the problem (12) in the form

$$K(t,\varepsilon) = \begin{pmatrix} \varepsilon K_1(t,\varepsilon) & \varepsilon K_2(t,\varepsilon) \\ \varepsilon K_2^T(t,\varepsilon) & K_3(t,\varepsilon) \end{pmatrix}, \tag{16}$$

where the matrices $K_1(t,\varepsilon)$, $K_2(t,\varepsilon)$ and $K_3(t,\varepsilon)$ have the dimensions $q\times q$, $q\times(n-q)$ and $(n-q)\times(n-q)$.

Let us also partition the matrix $A(t)$ into blocks as follows:

$$A(t) = \begin{pmatrix} A_1(t) & A_2(t) \\ A_3(t) & A_4(t) \end{pmatrix}, \tag{17}$$

where the blocks $A_1(t)$, $A_2(t)$, $A_3(t)$ and $A_4(t)$ have the dimensions $q\times q$, $q\times(n-q)$, $(n-q)\times q$ and $(n-q)\times(n-q)$, respectively.

Based on the singular perturbation techniques approach (Yackel & Kokotovic 1973, Shinar, Glizer, & Turetsky 2014), we obtain the following proposition.

Proposition 1. *Let the assumptions (A1) – (A5) be valid. Then, there exists a positive number* ε_0 *such that, for all* $\varepsilon \in (0,\varepsilon_0]$, *the blocks* $K_i(t,\varepsilon)$, $(i = 1,2,3)$ *of the solution (16) to the problem (12) satisfy the inequalities*

$$\left\|K_j(t,\varepsilon) - (K_j^o(t) + K_j^b(\tau))\right\| \le a\varepsilon,\; \tau = t/\varepsilon,\; j = 1,2;\;\; \left\|K_3(t,\varepsilon) - K_3^o(t)\right\| \le a\varepsilon,\; t \in [0,t_f], \tag{18}$$

where $\|\cdot\|$ denotes the Euclidean norm of a matrix; $a > 0$ is some constant independent of ε; the $q \times q$-matrix-valued function $K_1^o(t)$ has the form

$$K_1^o(t) = \mathrm{R}^{1/2}(t)\left(\mathrm{R}^{-1/2}(t)D_1(t)\mathrm{R}^{-1/2}(t)\right)^{1/2}\mathrm{R}^{1/2}(t), \tag{19}$$

and the superscript "1/2" denotes the unique symmetric positive definite square root of a symmetric positive definite matrix, the one "- 1/2" denotes the square root of an inverse matrix; the $(n-q)\times(n-q)$ -matrix-valued function $K_3^o(t)$ is the unique solution of the initial value problem for the Riccati equation

$$\dot{K}_3^o(t) = A_4(t)K_3^o(t) + K_3^o(t)A_4^T(t) - K_3^o(t)A_2^T(t)D_1^{-1}(t)A_2(t)K_3^o(t) + D_2(t),\ t \in [0, t_f],\quad K_3^o(0) = \Phi; \tag{20}$$

the $q \times (n-q)$-matrix-valued function $K_2^o(t)$ has the form

$$K_2^o(t) = \mathrm{R}(t)(K_1^o(t))^{-1}A_2(t)K_3^o(t); \tag{21}$$

the pair of the matrix-valued functions $\{K_1^b(\tau), K_2^b(\tau)\}$ is the unique solution of the initial value problem

$$dK_1^b(\tau)/d\tau = -K_1^b(\tau)\mathrm{R}^{-1}(0)K_1^o(0) - K_1^o(0)\mathrm{R}^{-1}(0)K_1^b(\tau) - K_1^b(\tau)\mathrm{R}^{-1}(0)K_1^b(\tau),\quad \tau \ge 0, \tag{22}$$

$$dK_2^b(\tau)/d\tau = -K_1^b(\tau)\mathrm{R}^{-1}(0)K_2^o(0) - K_1^o(0)\mathrm{R}^{-1}(0)K_2^b(\tau) - K_1^b(\tau)\mathrm{R}^{-1}(0)K_2^b(\tau),\quad \tau \ge 0, \tag{23}$$

$$K_1^b(0) = -K_1^o(0),\ K_2^b(0) = -K_2^o(0). \tag{24}$$

The solution of (22) – (24) satisfies the inequalities $\|K_j^b(\tau)\| \le c\exp(-\beta\tau)$, $\tau \ge 0$, $j = 1,2$, with some positive constants c and β.

5. Limit Behavior of J_ε^*

Let us partition the matrices G and $P(t)$ into blocks as follows:

$$G = \begin{pmatrix} G_1 & G_2 \\ G_2^T & G_3 \end{pmatrix},\quad P(t) = \begin{pmatrix} P_1(t) & P_2(t) \\ P_2^T(t) & P_3(t) \end{pmatrix}, \tag{25}$$

where the blocks G_1 and $P_1(t)$ are of the dimension $q \times q$; the blocks G_2 and $P_2(t)$ are of the dimension $q \times (n-q)$; the blocks G_3 and $P_3(t)$ are of the dimension $(n-q)\times(n-q)$.

Consider the block matrix $\Lambda_K^o(t) = (K_1^o(t),\ K_2^o(t))$. Now, by using the equations (15), (16) and Proposition 1, we obtain the following theorem.

Theorem 1. *Let the assumptions (A1) – (A5) be valid. Then, there exists a finite limit (as $\varepsilon \to +0$) of the optimal value of the cost functional in the OOCP. This limit has the form*

$$\tilde{J}^* = \lim_{\varepsilon \to +0} J_\varepsilon^* = \bar{x}_0^T R(0)\bar{x}_0 + \mathrm{tr}\left[\int_0^{t_f} (\Lambda_K^o(t))^T \mathrm{R}^{-1}(t)\Lambda_K^o(t)R(t)dt + G_3 K_3^o(t_f) + \int_0^{t_f} P_3(t)K_3^o(t)dt\right]. \qquad (26)$$

6. Interception Problem Example

In this section, the theoretical results are applied to a planar interception problem where both vehicles, the interceptor (pursuer) and the target (evader), have the first-order dynamics. In Fig. 1, the schematic engagement geometry is depicted. The points $P(t) = (x_P, y_P)$ and $E(t) = (x_E, y_E)$ are current coordinates of the pursuer and the evader, respectively; a_P, a_E are their lateral accelerations; ϕ_P, ϕ_E are the respective angles between the velocity vectors and the x-axis (initial line of sight); $\lambda(t)$ is the line-of-sight angle.

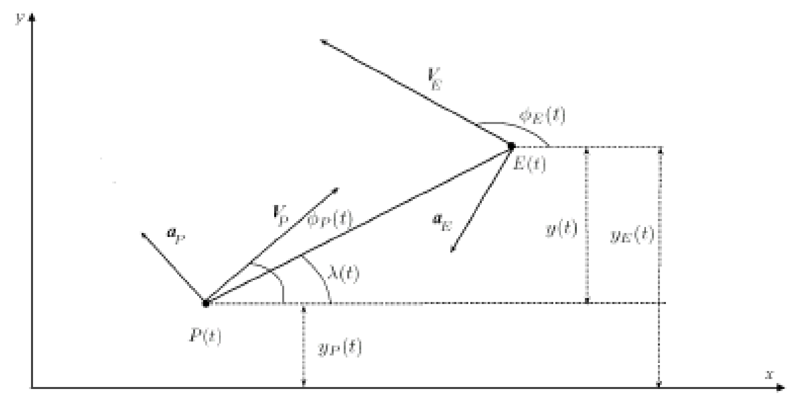

Fig. 1. Interception geometry.

Based on the small angles assumption (Shinar, Glizer & Turetsky 2013), the trajectories of the pursuer and the evader can be linearized with respect to the nominal collision geometry, leading to a constant closing velocity V_c. The final interception time t_f can be easily calculated for given initial range r_0 and interception lethality radius r_i: $t_f = (r_0 - r_i)/V_c$. This leads to the following linear model for $0 \le t \le t_f$:

$$\begin{aligned}
\dot{x}_1 &= x_2 + v_1, & x_1(0) &= 0,\\
\dot{x}_2 &= x_3 - x_4 + v_2, & x_2(0) &= x_{20},\\
\dot{x}_3 &= (v_3 - x_3)/\tau_E, & x_3(0) &= 0,\\
\dot{x}_4 &= (u - x_4)/\tau_P, & x_4(0) &= 0,
\end{aligned} \qquad (27)$$

where $x_1 = y_E - y_P$ is the relative separation normal to the initial line of sight; x_2 is the relative normal velocity; x_3 and x_4 are the lateral accelerations of the evader and the pursuer, respectively, both normal to the initial line of sight; τ_E, τ_P are the respective time constants; $x_{20} = V_E\phi_E^0 - V_P\phi_P^0$; ϕ_E^0 and ϕ_P^0 are initial values of ϕ_E and ϕ_P, respectively.

The controls v_3 and u of the evader and the pursuer, respectively, are the vehicles' acceleration commands in the y-direction. Since the behavior of the evader is unknown to the pursuer, the control v_3 can be modeled by a Gaussian white noise with zero average and the covariance $d_3(t)\delta(t-\tau)$, ($d_3(t) \geq 0$), (Fitzgerald & Zarchan 1978). The functions v_1 and v_2 model the process noise in the equations for the relative separation and the relative velocity, respectively. They are assumed to be Gaussian white noises with zero averages and the covariances $d_1(t)\delta(t-\tau)$, ($d_1(t) > 0$), and $d_2(t)\delta(t-\tau)$, ($d_2(t) \geq 0$), respectively. The functions $d_i(t)$, $i = 1,2,3$, are continuously differentiable for $t \in [0, t_f]$. It is assumed that v_1, v_2 and v_3 are independent stochastic processes.

It is assumed that only the line-of-sight angle $\lambda(t) \approx x_1(t)/r(t)$, where $r(t)$ is the current distance between the vehicles, is measured, and the measurement noise is small. Thus, the measured variable is $\lambda(t) = x_1(t)/r(t) + \varepsilon\eta(t)$, where $\eta(t)$ is a Gaussian white noise with zero average and the covariance $d_\eta(t)\delta(t-\tau)$, ($d_\eta(t) > 0$). The function $d_\eta(t)$ is continuously differentiable for $t \in [0, t_f]$. By multiplying $\lambda(t)$ by $r(t)$ and denoting $z(t) = r(t)\lambda(t)$, $w(t) = r(t)\eta(t)$, the observation equation becomes

$$z(t) = x_1(t) + \varepsilon w(t). \tag{28}$$

The interception process is evaluated by the performance index

$$J(u) = E\left[x_1^2(t_f) + \alpha \int_0^{t_f} u^2(t)dt \right], \tag{29}$$

where $\alpha > 0$ is a control penalty coefficient.

Thus, this interception problem can be described in the form of the OOCP (5) – (7) for

$$A(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1/\tau_E & 0 \\ 0 & 0 & 0 & -1/\tau_P \end{pmatrix}, \ B^T(t) = (0,0,0,1/\tau_P), \ v^T(t) = (v_1(t), v_2(t), v_3(t), 0), \tag{30}$$

$$\mathrm{M}(t) = \mathrm{diag}\{1,1,1/\tau_E,1\}, \ C(t) = (1,0,0,0), \ G(t) = \mathrm{diag}\{1,0,0,0\}, \ P(t) = 0, \ Q(t) = \alpha. \tag{31}$$

In Table 1, the values J_ε^*, $J(u_\varepsilon^*)$, $E[|x_1(t_f)|]$ and $\tilde{J}^*$ are presented for decreasing ε and for $\tau_P = \tau_E = 0.2$ s, $\alpha = 0.01$ s^3, $r_0 = 2000$ m, $r_i = 5$ m, $d_1(t) \equiv 0.1$ m^2/s^2, $d_2(t) \equiv 0.1$ m^2/s^4, $d_3(t) \equiv 50$ m^2/s^4, $x_{20} = 5$ m/s. The calculation was carried out for two values of the closing velocity: $V_{c1} = 500$ m/s and $V_{c2} = 250$ m/s, yielding $t_{f1} = 3.99$ s and $t_{f1} = 7.98$ s, respectively. The values of J_ε^* and $\tilde{J}^*$ are calculated by (15) and (26), respectively. The values of $E[|x_1(t_f)|]$ and $J(u_\varepsilon^*)$ are obtained by (27) and (29) as the average of the corresponding values in $N = 100$ Monte Carlo simulation runs. It is seen that for both closing velocities, $J_\varepsilon^* \to \tilde{J}^*$ and $J(u_\varepsilon^*) \to \tilde{J}^*$ for $\varepsilon \to 0$. It is also seen that the average miss distance $E[|x_1(t_f)|]$ decreases for decreasing ε. Moreover, its value is smaller than the lethality radius r_i.

Table 1. Comparison of J_ε^*, $J(u_\varepsilon^*)$ and $\tilde{J}^*$.

	$t_f = 3.99$				$t_f = 7.98$			
ε	J_ε^*	$J(u_\varepsilon^*)$	$E[\|x_1(t_f)\|]$	$\tilde{J}^*$	J_ε^*	$J(u_\varepsilon^*)$	$E[\|x_1(t_f)\|]$	$\tilde{J}^*$
10^{-3}	15.64	12.06	1.79	5.76	13.09	11.71	1.61	6.82
10^{-4}	7.48	7.02	1.13		7.83	7.96	1.14	
$5\cdot10^{-5}$	6.73	6.51	1.05		7.33	7.61	1.09	
10^{-5}	5.99	6.25	0.91		6.97	6.77	0.95	
$5\cdot10^{-6}$	5.89	5.62	0.93		6.90	6.29	0.83	

7. Conclusions

In this paper, the finite-horizon LQG control problem with small observation noise was considered. The convergence of the optimal value of its cost functional was established for the observation noise intensity tending to zero. The limit value was derived. Based on these results, the example of the interception of a maneuvering target was solved.

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