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Spin Wave Spectra of Two-Dimensional Magnetic Nanodots: Special Modes and Multimode Hybridization

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Abstract **-** The aim of this paper is to study spin-wave excitations in two-dimensional nanodots. We use a discrete dipole model taking into account the nearest-neighbour exchange and dipolar interactions. Magnetic configuration is assumed to form an in-plane vortex (circular magnetization). We examine the dependence of the frequencies and profiles of normal spin-wave modes versus the dipolar-to-exchange interaction ratio d, the size of the dot L, and the symmetry of the 2D lattice, from which the dot is cut. Special attention is paid to some particular modes, including the lowest-frequency mode and the fundamental mode, an analogue of the uniform excitation, the frequency of which proves almost independent of d. For the lowest mode different profiles are observed. Various types of localized spin waves prove responsible for the transition to a new magnetic configuration. Far from the critical value of d azimuthal modes are the lowest with azimuthal number being the compromise between exchange and dipolar interactions. Finally, we study the hybridization of the modes, including the multi-mode hybridization, and explain the selection rules.

Keywords**:** Magnetic nanodots, Magnetic vortices, Spin-waves, Multimode hybridization.

1. Introduction

In magnetic systems, taking into account both the short-range and long-range interactions results in a variety of interesting effects: causes the complete bandgap to open in magnonic crystals (Krawczyk et al., 2002; Kłos et al., 2012; Mamica et al., 2012a, b; Romero Vivas et al., 2012), is responsible for the splitting of the spectrum into subbands in patterned thin films (Krawczyk et al., 2011; Pal et al., 2012), or results in surface and subsurface localization in thin films with natural surface (Puszkarski et al., 1998; Mamica et al., 1998, 2000). The competition between the local exchange and long-range dipolar interactions has a significant influence on the spin-wave spectrum and leads to a variety of stable and metastable magnetic configurations. In magnetic nanodots with a thickness small enough with respect to their diameter one of these configurations is the vortex state (Metlov and Lee, 2008), the potential applications of which include microwave-frequency oscillators (Guslienko, 2012), frequency multiplication (Demidov et al., 2011), magnetic nanoparticles trapping (Donolato et al., 2009), or data storage and information processing (Cowburn and Welland, 2000). In all of these applications a crucial role is played by spin waves. They also have a significant influence on the stability of the magnetic configuration (Mozaffari and Esfarjani, 2007), even if the system is smaller than the characteristic exchange length (Rohart et al., 2010). In the present paper we study the spin-wave spectrum of the two-dimensional (2D) nanodot with particular attention paid on two special modes in the spectrum: the lowest-frequency mode and the fundamental mode, an analogue of the uniform excitation.

The lowest mode is especially important in metastable vortices, in which it plays a role of a soft mode responsible for the transition to a different magnetic configuration (Depondt, et al., 2013; Mamica et al., 2014). Both experimental studies and micromagnetic simulations reported the lowest spin-wave mode to be an azimuthal mode of different order (Buess et al., 2005), a localized mode (Zhu et al., 2005), or even the fundamental mode (Wang and Dong, 2012). In the present paper we demonstrate that, besides the vortex itself, also this richness of the observed spin waves results from a compromise between the dipolar and exchange interactions. We show that for a critical value of the dipolar-to-exchange interaction ratio, for which the vortex state loses its (meta)stability, the lowest mode is strongly localized at the dot centre. Beyond this critical situation, the lowest mode is an azimuthal mode of the order depending on the dipolar to exchange interaction ratio as well as the size of the dot.

We also analyze the multi-mode hybridization in which fundamental mode is involved. The latest studies show a major role of mode hybridization in plasmonic devices (Ju et al., 2013). The issue also affects significantly the profiles of the hybridizing modes, which is especially important in the context of the crucial role of the excitation profile for the effective switching of the vortex core (Bauer et al., 2014). Finally, we address the influence of the symmetry of the lattice the dot is cut out.

2. The Model

The object of our study is a circular dot cut out of a 2D lattice with elementary magnetic moments (spins) in its sites. A system thus defined is naturally discrete, without recourse to artificial discretization applied to continuous systems, e.g. in micromagnetic simulations. Obviously, the boundary of such dot is not perfectly circular, thus by 'circular' we understand a system cut out by means of circles. The edge cannot be smoothed; its smoothness is related to the size of the dot (measured in lattice constant units).

The dynamics of a single magnetic moment $\boldsymbol{M_R}$, $\boldsymbol{R}$ being the position vector, is considered in the linear approximation, assuming $|\boldsymbol{m_R}| << |\boldsymbol{M_R}|$, $|\boldsymbol{M}_{0,\boldsymbol{R}}| \approx |\boldsymbol{M_R}|$ and $\boldsymbol{m_R} \perp \boldsymbol{M_R}$ where $\boldsymbol{M}_{0,\boldsymbol{R}}$ and $\boldsymbol{m_R}$ are the static and dynamic component of the magnetic moment, respectively. To describe the time evolution of $\boldsymbol{m_R}$, oscillating harmonically with a frequency ω, we use the damping-free Landau-Lifshitz (LL) equation taking into account the dipolar and exchange interactions. Since the Gilbert damping term introduces only second-order corrections to the spin-wave frequencies, we can expect that in systems based on Py dots, which are the most often studied experimentally, the Gilbert damping parameter is below 0.01 (Hiebert et al., 1997; Aliev et al., 2009). Thus, its influence on standing spin-wave excitations is negligible.

After linearization of the LL equations we obtain a system of equations for the in-plane and out-of-plane coordinates of the dynamic component of the magnetic moments. Numerical diagonalization of the corresponding eigenvalue problem yields the frequency spectrum of the spin-wave excitations, and the spin-wave profiles, i.e., the distribution of the in-plane (m_r) and out-of-plane (m_k) amplitudes of precession of the elementary magnetic moments. For more details please see our previous papers (Mamica et al., 2012c, 2014).

It is important to notice, that there are no simulations performed in our approach. The magnetic configuration is assumed to be an in-plane vortex: each magnetic moment in the system lies in the plane of the dot perpendicularly to its radius. Obviously, such a configuration may be unstable, in which case, however, zero-frequency modes, being nucleation modes responsible for magnetization reconfiguration, will occur in the spin-wave spectrum. The lack of zero-frequency mode implies the (meta)stability of the assumed magnetic configuration. On the other hand, if damping is neglected, only purely real solutions will be physical. Thus, frequencies with a nonzero imaginary part will indicate that the assumed magnetic configuration is unstable (Rivkin et al., 2005). These two criteria are equivalent, which is reflected in our results: only frequencies with zero real part have a nonzero imaginary part. In comparison with simulations this method allows a relatively quick exploration of different magnetic states. The advantage is also obtaining the spin-wave frequencies and profiles directly from diagonalization, without recourse to the Fourier transformation used in time-domain simulations.

Another advantage is that there is only one material parameter in the adopted model, namely the dipolar-to-exchange interaction ratio d, defined as: $d = (g\mu_B)^2\mu_0 / (8\pi J a_{NN}^3)$, where g is the g-factor, μ_B the Bohr magneton, μ_0 the vacuum permeability, a_{NN} the nearest-neighbour distance, and J the nearest-neighbour exchange integral. In the case of the square lattice considered in this paper a_{NN} is the lattice constant. By the above definition d only depends on microscopic parameters. Thus, the stability of the assumed magnetic configuration will depend not only on d, but also on the structure of the system, i.e. its size and shape as well as the lattice from which it has been cut out.

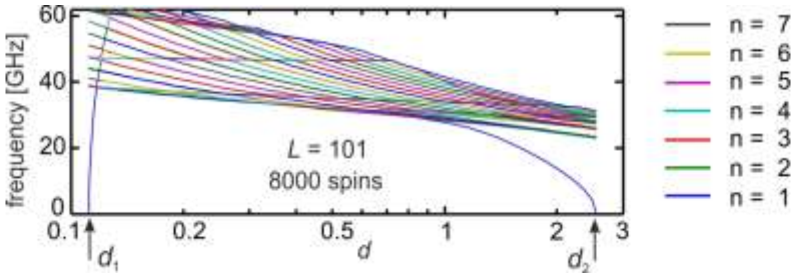

Fig. 1. Spin-wave frequency vs. the dipolar-to-exchange interaction ratio d (in logarithmic scale) for a 2D circular nanodot of diameter $L = 101$ lattice constants; the magnetic moments are assumed to form an in-plane vortex. The colour assignment of the first 7 mode lines is indicated at the left; the colours repeat cyclically for successive modes. The lack of zero-frequency modes for $d_1 < d < d_2$ indicates the (meta)stability of the assumed configuration.

3. Results

A sample dependence of the spin-wave spectrum on the dipolar-to-exchange interaction ratio d is shown in Fig. 1. The results are obtained for an in-plane vortex in the dot of diameter $L = 101$ lattice constants (8000 spins within the dot). The dependence clearly indicates the existence of three ranges of d. In two of them, i.e. below d_1 and above d_2, the frequency of the lowest mode is zero and according to the above discussion assumed magnetic configuration in unstable. On the other hand, for $d_1 < d < d_2$ the absence of the zero-frequency modes indicates that the in-plane vortex is (meta)stable. This picture reflects the nature of the magnetic in-plane vortex, which appears as a compromise between the exchange and dipolar interactions. When the exchange interaction is strong enough (d is too low) the out-of-plane component of the static magnetic moment rises at the vortex centre and the core-vortex is formed (Mozaffari and Esfarjani, 2007). This kind of spin reorientation we will refer to as exchange-driven reorientation. If the exchange interaction is too weak (d is too large) the dipolar interaction lead to a multi-domain or multi-vortex state and a dipolar-driven reorientation occurs (Depondt et al., 2011). The stability of the in-plane vortex vs. the size and the shape of the dot has been studied in our previous papers (Mamica et al., 2012c, 2013a, b). There are also two special modes in Fig. 1 the behaviour of which differs from the rest of the spectrum: the lowest-frequency mode and the fundamental mode.

3. 1. The Lowest-frequency Mode

For the majority of modes in Fig. 1 the frequency decreases with increasing d and this decreasing slows down for higher d. For the lowest mode the frequency vs. d dependence can be divided into three ranges. In the vicinity of the critical value d_1 the frequency increases very rapidly until the mode crossing at $d \approx 0.1237$. Then its behaviour is similar to other modes up to $d \approx 1.0$. Starting from this point the rate of the frequency decreasing is growing and near the critical value d_2 is very high again.

In Fig. 2a we show the lowest mode frequency vs. d dependence in dots of different size. The character of presented curves is very similar. Moreover, the first part of this dependence is exactly the same regardless the size of the dot; the only difference is the point of the mode crossing. To explain this result in Figs. 2b and 2c we show the evolution of the spin wave profile for the lowest mode in two dots: $L = 51$ and $L = 101$. In both cases for $d \approx d_1$ the profile is strongly localized at the dot centre. In the vicinity of the critical value d_1 this localized mode plays the role of the soft mode. Its frequency reaches zero for $d = d_1$, and the mode becomes a nucleation mode, responsible for the magnetic reconfiguration of the system. The strong localization at the centre manifests the tendency of the system to form a vortex core with a nonzero out-of-plane static component of the magnetization. As d diverges from the critical value the in-plane vortex regains stability, and the mode in question is excited at increasing cost. This, in turn, results in a steep increase in its frequency. Additionally, the reorientation for $d = d_1$ is due to exchange interactions, which are restricted to nearest neighbours. This means that strongly localized mode does not feel the borders and consequently its behaviour does not depend on the size of the dot.

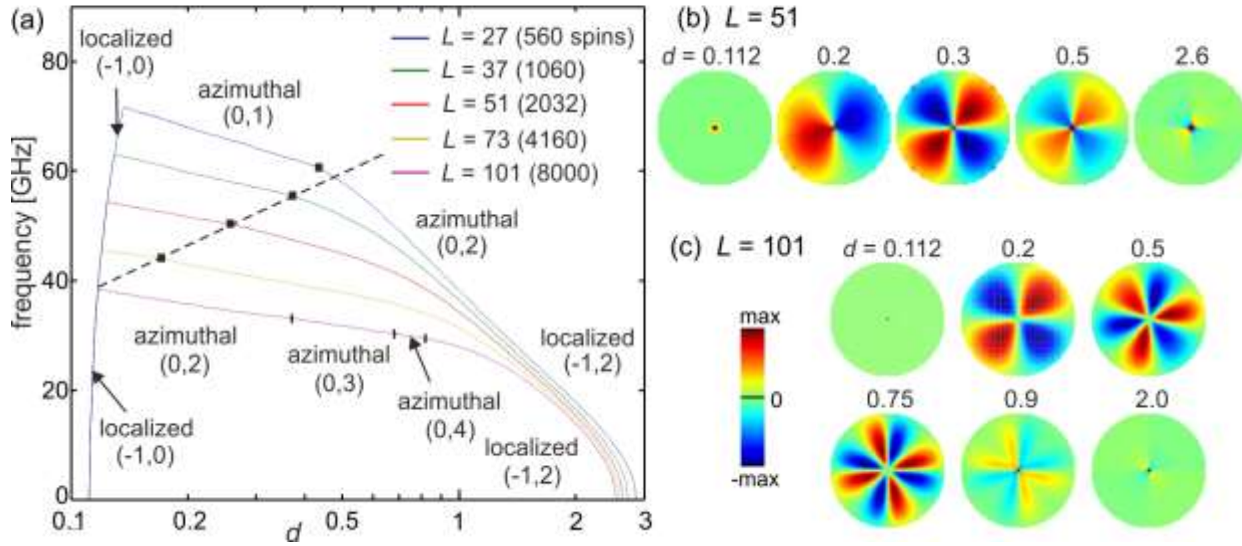

Fig. 2. (a) Frequency of the lowest mode vs. dipolar-to-exchange interaction ratio *d* (in logarithmic scale) in dots of different diameter *L*. From the top down, successive curves correspond to L = 27, 37, 51, 73, and 101 lattice constants. Black squares mark the points of intersection of first- and second-order azimuthal modes. (b, c) Evolution of the lowest mode profile with *d* in dots of diameter 51 and 101, respectively.

In the second range of *d* an azimuthal mode has the lowest frequency (Fig. 2). In this case the dependence of the frequency *f* of the lowest mode vs. the diameter *L* of the dot is approximately $f \sim L^{1/2}$, which is in good agreement with analytical studies (Ivanov and Zaspel, 2005; Zivieri and Nizzoli, 2005). The azimuthal number of the lowest mode depends on both *d* and *L*. For the diameter $L < 100$ first order azimuthal mode (0,1) crosses localized mode and becomes the lowest. (We use common notation for the mode profile type with the radial number at first position and the azimuthal number at the second. In this labelling the localized mode is a (-1,0) mode, where -1 refers to a complex wave number in the radial direction.) For higher *d* next azimuthal mode (0,2) has the lowest frequency and smoothly becomes localized at the centre with growing *d*. Higher order azimuthal modes are not the lowest for any value of *d* in dots smaller than 100 lattice constants.

The crossing points between azimuthal modes (0,1) and (0,2) are marked with black squares in Fig. 2. Except for the smallest dot, these points lie in a straight line which crosses the localized mode just above the lowest-frequency mode for $L = 101$, which means that for bigger dots (0,1) mode is not the lowest for any value of *d*. On the other hand, in the dot of $L = 101$ higher order azimuthal modes becomes the lowest in the spectrum with growing *d*: (0,3) at $d \approx 0.37$ and (0,4) at 0.69. Above $d \approx 0.82$ the lowest-frequency mode has the azimuthal number 2 again, but its profile is concentrated near the high spin density lines (Fig. 2c, $d = 0.9$) and further increase in the value of *d* results in progressive localization of this mode at the centre of the dot (Fig. 2c, $d = 2.0$), similarly to the smaller dots.

For a given value of *d* (fixed material) increasing the size of the dot results in increasing the order of the lowest azimuthal mode; such effect was observed experimentally by Buess et al. (2005) and in analytical calculations by Zivieri and Nizzoli (2005).

In the light of the results presented above, modes with increasing azimuthal number *m* fall successively to the bottom of the spectrum as the dipolar interaction gains in importance with increasing *d* or growing diameter of the dot. The exchange interaction favours modes with $m = 1$. Thus, the competition between the dipolar and exchange interactions manifests itself in the profile of the lowest-frequency mode.

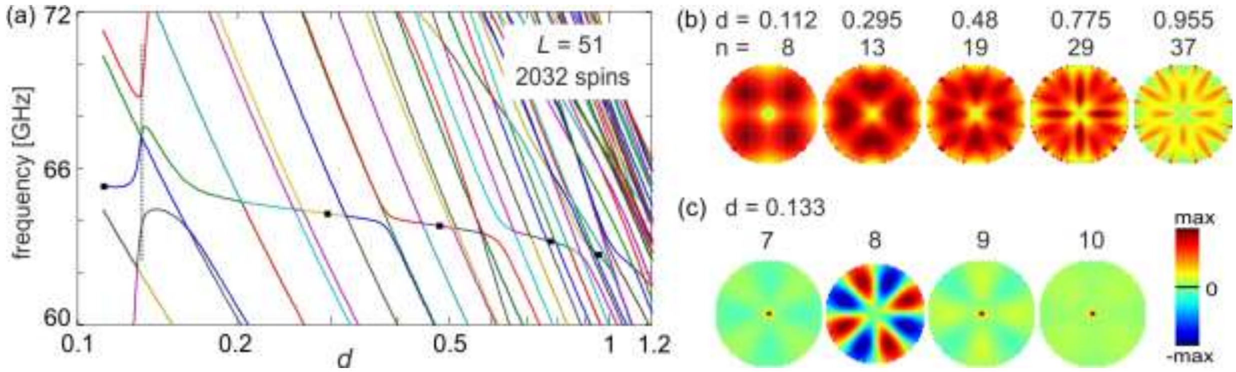

Fig. 3. (a) Evolution of the fundamental mode vs. the dipolar-to-exchange interaction ratio *d* (in logarithmic scale) for the dot of the diameter *L* = 51. (b) Profiles of the fundamental mode for points marked with black squares in (a). (c) Profiles of three hybridized modes for *d* = 0.133 (dotted line in (a)). Additionally, the profile of the second (0,4) mode which is not involved in the hybridization is shown.

3. 2. The Fundamental Mode

The second mode different than the others in the spectrum in Fig. 1 is the fundamental mode with the frequency only slightly dependent on *d*. In Fig. 3 we show the evolution of its frequency (panel a) and profile (panel b) for the dot of diameter *L* = 51 (2032 spins). The profile has not nodal lines and therefore will be labelled as (0,0), but it is not uniform as well what results in the dependence of the frequency on *d* (for the uniform mode the frequency does not depend on *d* at all). Similar effect occurs in micromagnetic simulations due to the artificial discretization of the sample (Wang and Dong, 2012) however in our case it is natural consequence of the discreteness of the lattice.

For growing *d* the fundamental mode frequency is almost constant while for other modes decreases thus several mode crossings and anticrossings occurs. The very special is the first anticrossing for *d* around 0.133 where three modes hybridize: these are fundamental, localized and one of two four-order azimuthal modes. Their profiles are shown in Fig. 3c together with the profile of the second (0,4) mode which is not involved in the hybridization. Fundamental mode hybridize with (0,4) azimuthal mode because of the same symmetry of the profile (compare Fig 3b, *d* = 0.112) but the choice of only one (0,4) mode means there is some additional condition for hybridization to occur. The second (0,4) mode is 'ignored' because the nodal lines in its profile match the amplitude maximums of the fundamental mode. In the case of the azimuthal mode involved in hybridization its maxima of the same phase match the maxima of the (0,0) mode profile. Thus, the hybridization requires both the same symmetry of the modes and the matching of the in-phase anti-nodes in the azimuthal mode with the maximums in the fundamental mode. The same selection rule we found valid for other cases of the hybridization at *d* around 0.38, 0.65, and 0.9.

While *d* continues to grow the maxima of the fundamental mode profile splits and its symmetry is doubled (Fig 3b, *d* = 0.295). Meanwhile, the fundamental mode crosses azimuthal modes of the symmetry which does not match its own symmetry: (0,5), (0,6) and (0,7). (Modes of fifth and seventh order are degenerated in pairs while frequencies of modes (0,6) are split. Please see next subsection for the explanation of this effect.) Then the symmetry of the fundamental mode is doubled and match the (0,8) azimuthal modes; next hybridization occurs for *d* = 0.35 – 0.42. Again, one of these modes is ignored and the other hybridize with (0,0) mode. Afterwards situation is repeated until $d \approx 1.0$: fundamental mode hybridizes with one of (0,12) modes for *d* = 0.60 – 0.70 and one of (0,16) for *d* = 0.85 – 0.95. For *d* > 1.0 the mode under the question loses its fundamental character; its profile has pronounced maximums and minimums, and its frequency noticeably depends on *d*.

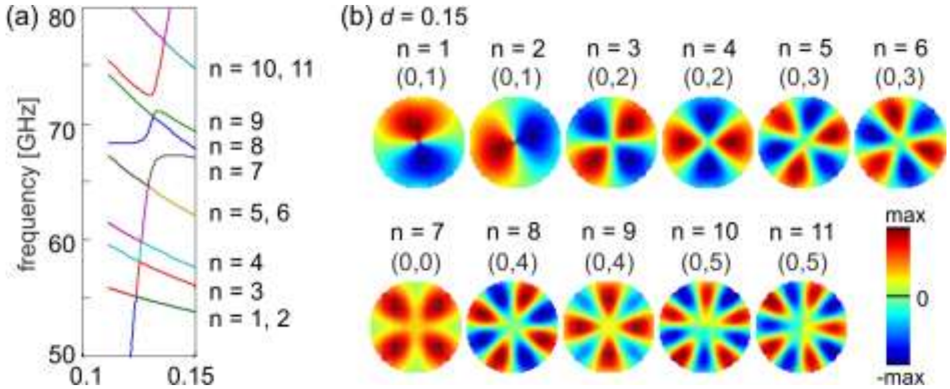

Fig. 4. (a) Enlarged part of the spectrum shown in figure 3 with mode numbers given to the right. (b) Profiles of the eleven lowest modes for $d = 0.15$. For n = 9 the profile is modified a bit due to the hybridization with (0,0) mode. Frequencies of modes with even azimuthal number are split as a consequence of the symmetry of the square lattice the dot is cut out.

3. 3. Influence of the Lattice Symmetry

As it is already mentioned, modes of fifth and seventh order are degenerated in pairs, while these of sixth order are split. In Fig. 4 we examine this frequency splitting in more detail. Among the azimuthal modes shown in this figure those of odd order are degenerate in pairs. In contrast, the pairs of modes with an even azimuthal number are not degenerate. Their non-degeneracy is related to their symmetry, the same as that of the lattice from which the dot has been cut out. For example, one of the (0,2) modes (n = 3) has nodal lines along the high spin density lines while in the other (0,2) mode (n = 4) the high spin density lines coincide with anti-nodal lines. An analogical situation occurs in periodic structures, in which a band gap forms between two states at the boundary of the Brillouin zone if one state has nodes and the other anti-nodes in the potential wells. Moreover, as in the case of states at the boundary of the Brillouin zone, the frequency difference between two non-degenerate modes in a pair decreases with increasing azimuthal number. In the literature there are reports of lifted degeneracy of azimuthal modes in core vortices due to the coupling of the spin waves with the gyroscopic motion of the core (Hoffmann et al., 2007; Guslienko et al., 2008). Since in my study I consider coreless vortices, coupling with the motion of the core is out of the question. The non-degeneracy is due to the fact that the dot has been cut out from a discrete lattice. It is the symmetry of the lattice that determines which modes have lifted degeneracy.

Also the profile of the fundamental mode, since its nonuniformity is due to the discreteness of the lattice, reflects the lattice symmetry. This fact has a consequence in hybridization. In the case of square lattice considered in this paper fundamental mode hybridizes with azimuthal modes of fourth, eighth and twelfth order, i.e. with the azimuthal number divisible by four. On the other hand, for hexagonal lattice the fundamental mode profile has six-fold symmetry thus it hybridizes with azimuthal modes with azimuthal number divisible by six, what is shown in our paper devoted to hexagonal lattice (Mamica, 2013b). Of course, the additional condition holds also for hexagonal lattice and only one azimuthal mode of proper symmetry is involved in hybridization.

4. Conclusions

The lowest-frequency mode has different character depending on the stability of the vortex state. Far from the critical value of the dipolar-to-exchange interaction ratio d azimuthal modes are the lowest with azimuthal number m being the compromise between exchange and dipolar interactions: exchange interactions prefer lower m while dipolar interactions favour higher m, regardless of whether their predomination is due to the material (d) or size (L) of the dot. Close to the critical value of d the localized mode is the lowest in dots and the uniform one in rings (see our papers devoted to rings (Mamica, 2013a, b). Thus the profile of the lowest mode carry information on the stability of the vortex.

The profile of the fundamental mode, an analogue of the uniform mode, has the same symmetry as the discrete lattice the dot is based on (compare our results concerning hexagonal lattice (Mamica, 2013b)). As a consequence, with growing *d*, it hybridize with descending azimuthal modes of the same symmetry. Additional condition is coincidence of the maxima of the fundamental mode with the anti-nodal lines of the same phase of the azimuthal mode. Also, the nonuniformity of the profile results in a slight dependence of the fundamental mode frequency on *d*.

The symmetry of the lattice the dot is cut out is reflected in the symmetry of the fundamental mode and therefore has an influence on the order of azimuthal modes involved in hybridization. It also affects splitting of the frequency of azimuthal modes; for square lattice the degeneracy is lifted for modes with azimuthal number divisible by two, while for hexagonal one for those divisible by three.

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