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Outdoor Acoustics: Range Estimation of Gunfire over an Acoustically Soft Impedance Ground in a Homogeneous Atmosphere

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Abstract **–** Predicting outdoor sound propagation in uncertain conditions remains a challenge. This increases the complexity of the inverse problem, e.g. parameter recovery in the presence of a particular sound source such as like gunfire. This paper investigates the use of maximum likelihood methods, both frequentist and Bayesian, in inverting true parameters from measured and simulated data. A simple source-receiver acoustic model is used which assumes; a homogeneous atmosphere, soft impedance ground and some medium range sound propagation to predict the deviation in sound pressure at the receiver. A blank firing pistol, Bruni Mod 92, is used to record a realistic sound source spectrum in an anechoic chamber. Gaussian noise is added to model predictions for this type of source to mimic uncertainty of real-life observations. Error analysis is performed by repeatedly generating observations and then evaluating the errors between the true range and recovered range estimate. This analysis is performed in broadband and octave frequency bands. It was found that the frequentist method greatly underestimates the range while the Bayesian method, even with a particularly flat prior, greatly reduces both over- and underestimations, significantly improving the range estimate to within $\pm 5m$ of the true value in the majority of cases. The inclusion of octave band filters in the infrasonic frequency showed these bands were mostly responsible for the accurate range estimates. This paper paves the way for applications of this class of statistical models to real-life acoustic data for source parameter recovery.

Keywords**:** Acoustics; Sound Propagation; Maximum Likelihood; Maximum A Posteriori; Inference; Error Analysis.

1. Introduction

Unlike other acoustical disciplines, outdoor sound propagation is not well understood in the presence of uncertainty. This complicates the application of the inverse process in which experimental data are used to infer sound source and environmental parameters. Improving the statistical understanding of this problem at the fundamental level will help the development of more robust inversion models and their practical applications.

This paper makes use of the likelihood function, frequentist and Bayesian methodologies to infer an unknown gunshot source range. This is done in the scenario where a simple homogenous atmosphere and soft impedance ground are present, and the remaining key parameters are known. Simulations are done by generating a small set of observations from an established acoustic model with gaussian noise added to simulate uncertainty in experimental data. Parameter estimates are then obtained from the frequentist *Maximum Log-likelihood* [1] and Bayesian *Maximum a Posteriori* methods [2]. The error of the inferred parameters against the true value is studied. This process repeated for over sets of generated observations. The performance of statistical methods is compared for broadband data and data filtered in octave band frequency windows. It is believed that the results obtained from this work will improve current inversion techniques and industrial practices which rely on outdoor sound propagation with uncertainties.

2. Acoustical Methods

2.1. Acoustic Foundations

A typical sound source produces a collection of sound waves, composed of different *frequencies* that propagate through some medium i.e. air in the outdoor case. A *homogenous*, atmosphere removes possible interferences from wind, turbulence and/or temperature gradients. Following this assumption, the sound pressure generated by this source can then be measured at the receiver position as a combination of the direct wave and wave reflected from the ground. These two waves interfere in constructive/destructive way resulting a complicated spectrum of the sound observed at the receiver position. The key parameters that need quantifying to make accurate predictions for this source/receiver/ground configuration are: sound frequency (ω), source height (s_h), range (r), receiver height (r_h) and impedance of the ground (σ_g). Figure 1 explains this problem schematically.

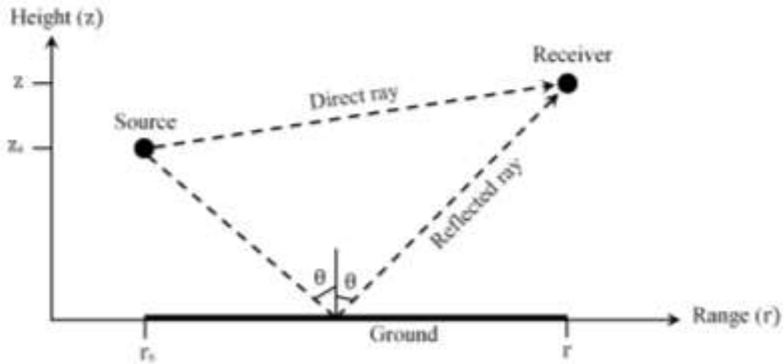

Fig. 1: Acoustic scenario in the (r, z) geometry.

The *excess attenuation* spectrum is a common characteristic of the sound field predicted by this acoustical model. The excess attenuation, ΔL, represents the deviation from the free pressure field now due to the influence of the ground, frequency and geometry, taking positive and negative values due to the constructive or destructive interference between the direct and reflected rays, respectively [3]. ΔL is equated as

$$\Delta L = 10\log_{10}\left|1 + Q\frac{R_1}{R_2}\exp(ikR_2 - ikR_1)\right|^2, \tag{1}$$

where

$$R_1 = \sqrt{r^2 + (z - z_s)^2}, \tag{2}$$

$$R_2 = \sqrt{r^2 + (z + z_s)^2}. \tag{3}$$

Parameters k and i are the *wavenumber* and imaginary number, respectively. The parameter Q in eq. (1) is the *spherical wave reflection co-efficient*, describing the relative pressure in the spherical wave reflected by the ground. This is a combination of the *incident angle*, θ, and *normalised impedance* of the ground, Z. Q is calculated as

$$Q = \left(\frac{Z\cos\theta - 1}{Z\cos\theta + 1}\right) + \left(1 - \left(\frac{Z\cos\theta - 1}{Z\cos\theta + 1}\right)\right)F(w), \tag{4}$$

with the *boundary loss factor*, $F(w)$, as

$$F(w) = 1 + iw\sqrt{\pi}\exp(-w)\,\mathrm{erfc}(-iw), \tag{5}$$

and *error function*, $\mathrm{erfc}(z)$,

$$\mathrm{erfc}(z) = \frac{1}{\sqrt{2\pi}}\int_z^\infty \exp(-t^2)\,dt. \tag{6}$$

Calculating the acoustic impedance of the ground, Z, can be done be various methods, however assuming the ground to be porous, the method proposed by Horoshenkov et al is used [4] in this paper. This model makes use of the median pore size which relates to the ground impedance. It considers the ground as a porous media with circular pores of non-uniform cross-section. It can be noted however; recent findings suggested the impedance of the ground was not a strongly significant factor inside simple acoustic scenarios while uncertainty was present in the geometry [5].

2.2. Gunshot Evaluation

Acoustical characterisation work on gunfire show that the sound generated can be categorised into three parts; muzzle blast, mechanical action and supersonic projectile [6]. The paper uses data collected from a *Bruni mod 92 blank pistol*, meaning that no supersonic projectile is produced leaving only the muzzle blast and mechanical action of the pistol. Sound recordings were taken of the pistol shots in an *anechoic chamber* at the University of Sheffield. The source and receiver were placed on the hard ground and separated by 3 m. The *Fast Fourier Transform* algorithm was applied to the time data (left-hand side of Fig. 2) to determine the frequency spectrum of each firing event (right-hand side of Fig. 2).

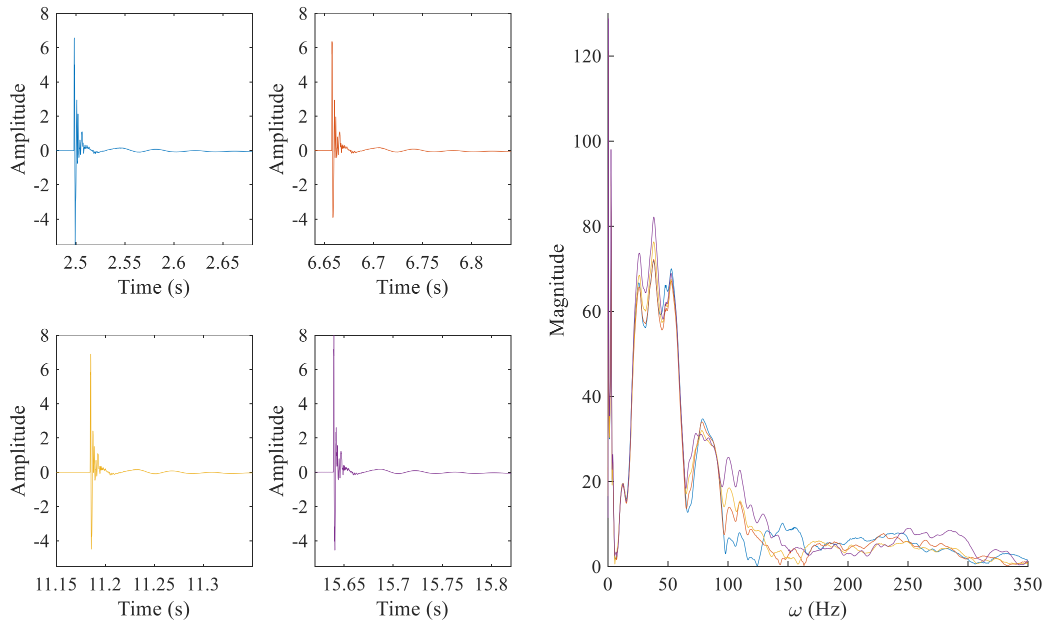

Fig. 2: Selection of gunshot recordings (left) with FFT of each (right).

Problematic mechanical action i.e. the casing being ejected, ricocheting etc is assumed to be the causes of the variation in the higher frequencies in the frequency plot (Fig. 2). It is also logical to assume that at long ranges, the smaller intensity of such sounds would likely dissipate before reaching the receiver. The measured frequency spectra were then averaged, using a cut-off point to remove small amplitudes, to leave a singular array of values which created a broadband frequency spectrum between a minimum of 0.24Hz and maximum of 93.02Hz. These frequencies correspond to the lower and upper octave bands used in this paper as it is common in acoustical practices to study outdoor sound propagation in individual octave bands. Octave filtered bands used in our analyses (Table 1) follow the current international standard set [7]. Bands with an asterisk (*) are described but are not used in analysis since the spectrum recovered did not contain frequencies inside that respective window.

Table 1: Octave band limits, in accordance to ISO 266.

Octave 1/1 Band	Lower Limit (Hz)	Centre Frequency (Hz)	Upper Limit (Hz)
Band 0	0.24	1	1.41
Band 3	1.41	2	2.82
Band 6	2.82	4	5.62
Band 9*	5.62	8	11.2
Band 12	11.2	16	22.4
Band 15	22.4	31	44.7
Band 18	44.7	63	89.1
Band 21	89.1	125	177

2.3. Parameter Selection and Observations

The *known* source and receiver heights were set to $2m$, as higher geometries begin to be highly subject to atmospheric effects, thus keeping our current model fit for use. The ground was considered as *acoustically soft*, which is typical for a field with low growing vegetation with an experimentally measured flow resistivity of at $500Pasm^{-2}$ corresponding to the median pore size of $530\mu m$ [8]. Range, being the *unknown* parameter, will have three true values being; $100m$, $200m$ and $300m$. The excess attenuation spectrum, ΔL for the each set of true values are shown in Fig. 3.

Before collecting observations, it is helpful to rewrite the acoustical model (eq. (1)) as a function of the parameters

$$Y = f\left(\omega, h_s, r, h_r, \sigma_g\right). \tag{7}$$

It is assumed that the acoustic model is *perfect* which means that it will predict the exact sound spectra for the input parameters given. To generate observations, the function (eq. (7)) has some gaussian noise added to predicted values, via an error term of $\varepsilon_s \sim N(0, \sigma_\varepsilon^2)$ with the variance σ_ε^2 fixed at 3dB. Thus, dropping constant terms from the notation, observations can be generated by the rewritten function

$$y_s = f(\omega, r) + \varepsilon_s . \tag{8}$$

In relation to the true excess attenuation spectrum, the possible values of the observed excess attenuation are shown to exist between the dashed limits depicted in Fig. 3.

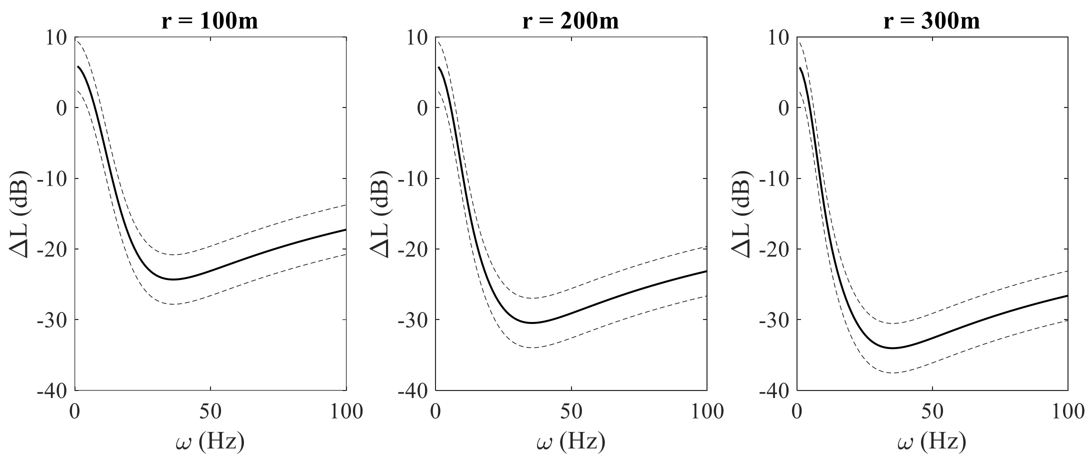

Fig. 3: Excess attenuation spectrum (solid) for true values, for range at 100m (left), 200m (middle) and 300m (right). Limits to observed values due to noise (dashed) are superimposed.

3. Statistical Methods

3.2. Maximum Log-likelihood Estimation (MLE)

The first method will be of the frequentist approach, maximising the likelihood function to estimate a given parameter. It is assumed that the observations generated can be described by some normal distribution, of some give mean and variance $X_s \sim N(\mu, \sigma^2)$, then it has the likelihood function $\mathcal{L}(\theta|X) = \prod_s^n f_N(x_s; \mu, \sigma^2)$, which can be log transformed to the log-likelihood as

$$\log\left(\mathcal{L}(\theta|X)\right) = \ell(\theta|X) = -\frac{n}{2}\log 2\pi - \frac{n}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum_{s=1}^{n}(x_s - \mu)^2 . \tag{9}$$

The above is the log-likelihood [1]. The function in eq. (9) can then, using eq. (7) and eq. (8), be rewritten as

$$\ell(\theta|Y, \omega, r) = -\frac{n}{2}\log 2\pi - \frac{n}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum_{j=1}^{q}\sum_{k=1}^{m}\sum_{s=1}^{n}\left(y_s - f\left(\omega_j, r_k\right)\right)^2 . \tag{10}$$

This log-likelihood, eq. (10), uses an n number of generated observations (y_s), to find the best estimate of the range from the source (r_k), from a parameter space of m points, by locating the point in which the function is maximised. This is done for each frequency (ω_j) from an array of q frequency points. Apart from finding the maximised value for r for all the complete frequency spectrum, maximised results inside frequency bandwidths like the defined octave bands (Table. 1), and thus a related estimate for r, can be located to be explored.

3.3. Bayesian Maximum a Posteriori (MAP)

The second method adheres to the Bayesian perspective. Bayes' theorem allows for the likelihood function to be combined with prior beliefs, giving such knowledge statistical weighting in the predictive process. The renown theorem can be written as

$$P(\theta|X) = \frac{P(X|\theta) \times P(\theta)}{P(X)} = \frac{\mathcal{L}(\theta|X) \times P(\theta)}{P(X)} \, , \tag{11}$$

where $P(\theta|X)$ is *posterior*, $P(X)$ is the *evidence*, $P(\theta)$ is the *prior* and $\mathcal{L}(\theta|X)$ the likelihood described earlier. The posterior probability is computed as a probability distribution of θ given the observed data X. Since the peak of the posterior distribution is the only value of interest to us, as this is the *most likely* estimate of the parameter/s, the normalising constant of $P(X)$ can be dropped greatly reducing computational effort. This results in eq. (3) being modified to

$$P(\theta|X) \propto \mathcal{L}(\theta|X) \times P(\theta) \, . \tag{12}$$

Eq. (12) allows for the MAP (Maximum a posterior) estimate, or the most likely value given the combination of prior beliefs and observed data, to be found. Like the earlier likelihood function, eq. (12) can be log transformed to

$$\log\big(P(\theta|X)\big) \propto \ell(\theta|X) + \log P(\theta) \ . \tag{13}$$

Priors can be used to import knowledge, or lack of, around the true value. Furthermore, the log of probability is a negative number, thus the value of $\log P(\theta)$ can be interpreted as a penalty term. When the estimated parameters fall outside the interval prior, the penalty becomes $\log(0)$, thus reducing the likelihood to $-\infty$. [2]. The prior probabilities applied in this work are assumed to be proportional to some normal distribution, $P(\theta) \sim N(\mu^*, \sigma^*)$, where the value of $\mu^*$is taken to be equal to the true parameter in each scenario. The standard deviation in the source range is fixed at $\sigma^* = 15m$, as this gives a possible error of up to $\sim \pm 50m$. This is analogous to the observer having an idea where the gunshot was fired yet giving themselves a large window of error, that is also similar to the parameter space of r that is numerically calculated from.

3.4. Computational Error Analysis

Investigating how efficient an estimate of the range is achieved computationally (using MATLAB™) by comparing every simulated estimate to the true value of r in question. This is done by creating 3 observations, using eq. (8), maximising likelihood function for the observations over the (m, q) space detailed in eq. (10) with, and without, a prior belief applied and comparing the related estimate of r to the true range for all the three different true ranges. The process is repeated 1000 times, for each true value of r, so an adequate amount of errors can be investigated. Errors are compared across combinations of each true value for range, statistical method applied and frequency windows.

4. Results

4.1. Broadband Analysis

Comapring the errors from using MLE to the MAP methods (Fig. 4) reveals clear differences in how well the true range of the source was recovered from the simulated observations. The MLE method is not very effective at any range, actually estimating each posible value of r in the parameter space used at least once. There is also a large tendency to underestimate by the minimum possible value. The shortest range $(100m)$ was least sucecptble to this but the remaining ranges $(200m, 300m)$ had over a 50% chance of being out by $-50m$, which is a 50% error in the $100m$ case. In only a small perecetnage of simulations was the true vlaue recovered, but this was as likely as recoving any parameter from the parameter space for r. There was also a recurrent overestimation $(+{\sim}15m)$ in the case were $r = 100m$, being the most estimated value, yet this slight overestimation dissappeared once the range was increased.

Bayesian MAP method showed that the application of a prior, even the flat one used, greatly reduced the margins of error with $60\% +$ of the simulations aprroximating the true value of r $(\pm 2m)$. Some underestimations of $\sim -50m$ remained present in the shortest range $(100m)$, but the occurrence of this was greatly reduced than to their MLE counterparts. For the ranges greater than $100m$, the variance in the error decreases greatly. Some errors of up to$+{\sim}20m$ for $r = 200m$ and

$+\sim 10m$ for $r = 300m$, yet these results were extremely uncommon. The majority of the results fell within $\pm 5m$, a far more reliable window of error than that of the MLE.

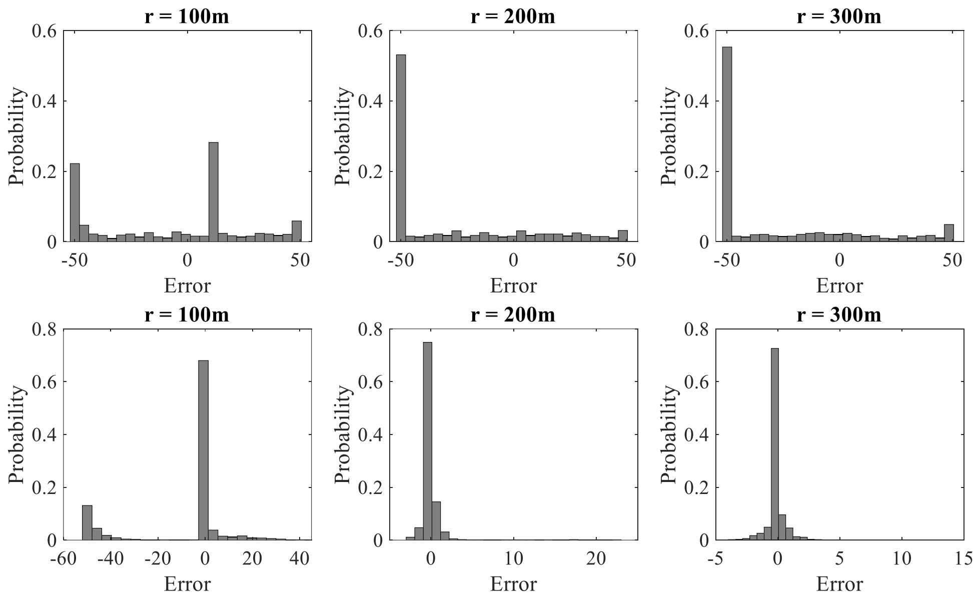

Fig. 4: Errors between estimate and true range across simulations using MLE (top row) and Bayesian MAP (bottom row) methods.

4.2. Octave Filtering

Filtering the excess attenuation data into octave bands and applying the MLE method to data presented in individual octave bands resulted in almost identical errors as in the case of broadband data. However, the application of the MAP method to octave band data proved more successful. Frequncy bands deep into the infrasonic range (< 6Hz) were most succesful in estimating the true value, with the full variance in errors greatly decreasing as the centre freqeuncy decreased. Application of the MAP method to Band 6 revealed that rarely overstimations were made, and when the exact value wasn't recovered the error was spread between $0m$ and $-50m$. Decreasing to Octave band 3 reduced the underestimation, to approximately $-20m$, with a small likelihood of overstimating by up to $\sim 20m$. Band 0 was the most effecient, with little to no understimation, but could overstimate by up to $10m$, while it had recovered the true value ($\pm 2m$) in over 50% of simulations. There was consistent effects of the range, apart from the increase in range to $300m$ inside Octave band 6 pushed up the maximal understimations to be the most persistent.

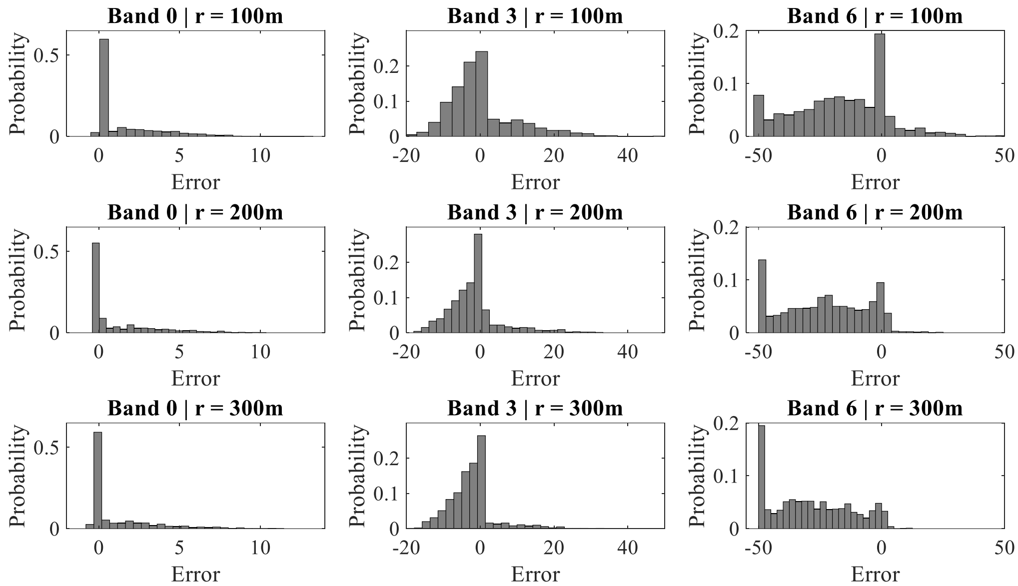

Fig. 5: Error between estimate and true range across simulations for the Bayesian MAP method for each true range (rows) and the lowest three Octaves (columns).

5. Conclusion

Estimating the sound source range solely form the maximising likelihood function does not recover its true value effectively being susceptible to large underestimations. The Bayesian use of a prior was significantly more effective. In the case of a *flatter* prior, it greatly improved the ability of the model to recover the true parameter value within ±2m. Most other simulations resulted in an error of $\pm 10m$, particularly at longer ranges. At the shorter range of $100m$ the method underestimated the true range value, but the occurrence of this reduced by half. This supports the case of using Bayesian techniques with data that has: a small sample size, not easily replicable or when time constraints may be present around the inference result.

Octave filtering using only the maximisation of the likelihood revealed no significant differences in the error analysis than to the broadband spectrum. However, the application of the Bayesian MAP method to octave band data was shown more successful when some particular frequency bands were adopted. In particular, the infrasonic (< 20Hz) frequency bands were found to produce less error. Octave bands higher than Band 6 consistently underestimated the range by $\sim -50m$, akin to that observed with the MLE method. Thus, it is recommended to use a combination of the lower frequency bands and MAP method with priors that allow for more accurate estimations. This supports the idea of relying on infrasonic measurements for a gunfire source when the impedance ground is soft to make accurate predictions while also and may prove useful when trying to invert parameters from higher dimensional problems i.e. inhomogeneous atmosphere.

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