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Risk Measure Based on ARMA-TGARCH-GED-Copula Model

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Abstract - Financial return series often show the characteristics of peak and thick tail, bias, and volatility aggregation effect. In this paper, ARMA-TGARCH is introduced to model each asset return series, the standard residual term of which is assumed to obey the generalized error distribution (GED). The joint distribution model with a multivariate copula function is used to characterize the dependence structure between high-dimensional asset variables. Combining the Monte Carlo simulation method, the return series of each asset is generated, and the VaR and CVaR of portfolio investment are calculated. The empirical research shows that there is obvious autocorrelation, heteroscedasticity effect, and asymmetric volatility in the return series of the representative stock indexes of China and the United States, which is suitable for ARMA-TGARCH-GED to fit marginal distribution. The failure frequency test of VaR prediction proves that ARMA-TGARCH-GED-Copula model can be better applied to the risk measure of portfolio investment.

Keywords**:** Peak and thick tail; Generalized error distribution; Risk measure; ARMA-TGARCH-GED-Copula.

1. Introduction

Nowadays, with the rapid development of the international financial market, the impact of market volatility on the economy cannot be ignored. Meanwhile, the correlation between financial assets has become more complex, which further aggravates the investment risk of the market. Thus, it is necessary to model the financial return series, which is the basis of risk management in portfolio investment.

The autoregressive moving average (ARMA) model is an important random time series analysis method, which is composed of AR and MA. The model uses the value of the historical series and the disturbance term to predict the future series value, better fitting the autocorrelation of the return series [1]. However, only using ARMA model cannot describe the volatility clustering of financial return series. The autoregressive conditional heteroscedasticity (ARCH) model proposed by Engle [2] in 1982 is widely used to fit the volatility equation of financial asset indexes and can eliminate the heteroscedasticity of the series to a certain extent. However, the ARCH model also has some limitations. For instance, it is only applicable to the heteroscedasticity function of short-term autocorrelation. Therefore, Bollerslev [3] proposed the generalized autoregressive conditional heteroscedasticity (GARCH) model on this basis, and derived TGARCH, EGARCH, IGARCH, etc. The ARMA-GARCH model is often used to analyze and predict the stock index in the field of financial assets. Meanwhile, the dependence between asset variables plays a decisive role in the analysis of financial system risk, but this dependence is often nonlinear and cannot be characterized by the Pearson correlation coefficient. The copula theory proposed by Sklar [4] in 1959, provides a good idea for this nonlinear dependence analysis. Sklar pointed out that any multivariate joint distribution can be decomposed into several edge distributions and a copula function. This theory was first applied to the problem of financial risk measurement by Embrechts [5] in 1999. Since then, the application of copula functions has developed rapidly in the financial field.

Some scholars have applied ARMA-GARCH-Copula model to the dependence analysis of various financial asset variables and the risk prediction of portfolio investment. AB Razak et al. [6] established an ARMA-GARCH-Copula model to calculate the value-at-risk (VaR) of various stock portfolio investment under different weights and compared the traditional VaR model with the Copula-VaR model to prove that the risk prediction based on copula is more accurate and reasonable. However, this study only applies the standard GARCH model and does not involve the asymmetry of volatility (i.e., the leverage effect of the stock market is not introduced into the model). Besides, VaR does not meet the subadditivity and convexity requirements, which causes certain limitations in risk prediction. Ortobell et al. [7] used this model to analyze the dependence between the return of short-term treasury bills and the U.S. stock index, carried out portfolio optimization, and reflected the asymmetric dependence structure in the model. However, only the asymmetric-t-copula function was applied, but other copula functions were not. Autchariyapanitkul et al. [8] used multivariate t-copula to explain the portfolio structure of high-dimensional asset allocation and calculated conditional value-at-risk (CVaR) based on the Monte Carlo method, but only assumed that the standard residual obeyed the t distribution in marginal fitting. Pan [9] applied the model to the study of the systemic risk spillover effect of listed commercial banks in China and introduced several common copula parameter estimation methods, but did not conduct empirical research on the established model. Wu et al. [10] analyzed the dependence of Shanghai and Shenzhen stock markets and measured the risk by constructing ARFIMA-GARCH-Copula model. This model successfully captured the long memory of stock asset return series and estimated conditional heteroscedasticity more accurately, but it also did not consider the asymmetric volatility of the stock market.

In view of the shortcomings of existing research, this paper will select six representative stock indexes for empirical research, establish an ARMA-GARCH model that is ranked according to AIC criteria, and eliminate the autocorrelation and volatility clustering of return series. The GED distribution is introduced to accurately fit the characteristics of peak and thick tail, and TGARCH is used to describe the asymmetry of return volatility. Besides, t-copula and three types of Archimedes copula functions are applied to describing the nonlinear dependence between asset variables, and marginal distribution models are connected into a multivariate distribution model. In the selection of risk measurement indicators, in addition to VaR, the conditional value-at-risk (CVaR) belonging to consistent risk measurement is introduced as an auxiliary index. Then the risk measure of portfolio investment is calculated by the Monte Carlo simulation method.

The rest of this paper is organized as follows. Section 2 introduces ARMA-TGARCH model with the GED distribution and copula functions. Section 3 presents the joint distribution model and the specific calculation process of risk measure. Section 4 describes empirical research on representative stock indexes. The conclusions drawn from this study are presented in Section 5.

2. Marginal Model and Copula Function

2.1. ARMA-TGARCH-GED Model

The ARMA model is widely applied in the economic field and can handle stationary time series well. Moreover, the GARCH model can eliminate the heteroskedasticity of time series. Therefore, the ARMA model is used to deal with the mean equation, and the TGARCH model, which can also effectively overcome the asymmetry of volatility, is used to deal with the variance equation.

Assume that the daily logarithmic return series after centralization is y_t, then the ARMA(p,q)-TGARCH(r,s) model can be expressed as follows:

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t \tag{1}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{r} \left(\alpha_i + \gamma_i \cdot I_{\{\varepsilon_{t-i} < 0\}} \right) \cdot \varepsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{2}$$

$$\omega > 0, \alpha_i \geqslant 0, \beta_i \geqslant 0, \alpha_i + \gamma_i \geqslant 0 \tag{3}$$

$$\sum_{i=1}^{r} \alpha_i + \sum_{j=1}^{s} \beta_j + \frac{1}{2} \sum_{i=1}^{r} \gamma_i < 1 \tag{4}$$

where ε_t is the perturbation term, σ_t is the conditional variance of ε_t, z_t is the standard residual sequence, ϕ_i and θ_j are the parameters of ARMA, and ω, α_i, β_j, and γ_i are the parameters of TGARCH. Eq (3) ensures the nonnegativity of σ_t^2 and Eq (4) ensures its stability. Moreover, the indicative function $I_{\{\varepsilon_{t-i}<0\}}$ is introduced into the model, and the specific form is as follows:

$$I_{\{\varepsilon_{t-i}<0\}}=\begin{cases}1 & \varepsilon_{t-i}<0\\ 0 & \varepsilon_{t-i}\geqslant 0\end{cases} \tag{5}$$

The TGARCH model shows that the coefficient of squared volatility is α_i when there is good news, and the coefficient of squared volatility is $\alpha_i+\gamma_i$ when there is bad news, which combines the leverage effect of the stock market (i.e., the stock market is more sensitive to bad news).

Through the analysis of actual data, it is concluded that the financial return series usually refuses to obey the normal distribution. Thus, it is assumed that the standard residual term z_t obeys the generalized error distribution (GED) so as to describe the peak and thick tail characteristics. The probability density function of the GED distribution can be described as

$$f(x;v)=\frac{v\,\mathrm{e}^{-\frac{1}{2}\left|\frac{x}{\lambda}\right|^{v}}}{\lambda 2^{\frac{1+v}{v}}\Gamma\left(\frac{1}{v}\right)},\quad \lambda=\left[\frac{2^{-\frac{2}{v}}\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}\right]^{\frac{1}{2}} \tag{6}$$

where $\Gamma(\cdot)$ is the gamma function and v is the parameter of the GED distribution.

2.2. Copula Function

In the financial field, multivariables often do not obey the multivariate normal distribution and are generally nonlinearly correlated. Through empirical analysis, financial assets often show significant tail dependence. This correlation cannot be described using the traditional Pearson correlation coefficient and covariance matrix. However, the appropriate copula function can capture the tail dependence between multivariables.

The copula function can connect various marginal distributions, and the dependence between variables can be studied as well. Thus, it is widely applied in multi-dimensional financial models.

Set

$$C(u_1,u_2,\ldots,u_d):[0,1]^d\rightarrow[0,1] \tag{7}$$

If the independent variable u_i of C obeys the uniform distribution of [0,1], it is called a copula function. According to Sklar's theorem [4], the following formula can be obtained:

$$C(F_1(x_1),F_2(x_2),\ldots,F_d(x_d))=F(x_1,x_2,\ldots,x_d) \tag{8}$$

where F is the multivariate joint distribution function, and $F_1,\ldots,F_d$ are the corresponding marginal distribution functions. It can be seen that a multivariate joint distribution function describing the dependence between financial variables can be constructed by selecting an appropriate copula function when the marginal distribution of each variable is known.

According to the definition, three basic properties of copula function can be obtained:

(1) $C(u_1,u_2,\ldots,u_d)$ monotonically increases with respect to any variable.

(2) If the value of $C(u_1,u_2,\ldots,u_d)$ is equal to zero, then if and only if there is $i\in\{1,2,\ldots,d\}$ such that the value of u_i is equal to zero.

(3) For any $i\in\{1,2,\ldots,d\}$ and $u_i\in[0,1]$, the value of $C(1,1,\ldots,u_i,\ldots,1)$ is equal to u_i.

2.3. Types of Copula Functions

The copula function family is mainly divided into two categories, one is the elliptic copula function that is usually used to describe the symmetric dependence, and the other is the Archimedes copula functions that can capture the asymmetric dependence. The elliptic copula function family mainly includes Gaussian Copula and Student t Copula, both of which show symmetrical tail dependence, while Student t Copula can describe stronger correlation and higher tail characteristics. In addition, the Archimedes Copula function mainly includes three types: Clayton Copula, Gumbel Copula, and Frank Copula, the multivariate forms of which are shown as

$$C_{Cl}(u_1,\ldots,u_d;\theta)=(u_1^{-\theta}+\ldots+u_d^{-\theta}-d+1)^{-1/\theta},\quad \theta>0 \tag{9}$$

$$C_{Gum}(u_1,\ldots,u_2;\delta)=\mathrm{e}^{-\left[\left(-\ln u_1\right)^{\delta}+\ldots+\left(-\ln u_d\right)^{\delta}\right]^{1/\delta}},\quad \delta\geqslant 1 \tag{10}$$

$$C_{Fr}(u_1,...,u_d;\gamma)=\left(-\frac{1}{\gamma}\right)\cdot\ln\left(1+\frac{\left(e^{-\gamma u_1}-1\right)\cdots\left(e^{-\gamma u_d}-1\right)}{\left(e^{-\gamma}-1\right)^{d-1}}\right),\ \gamma\neq 0 \tag{11}$$

where θ, δ, and γ are the parameters of the three copula functions, respectively. Different copula functions have different applications in the field of financial assets. Both Clayton Copula and Gumbel Copula show asymmetric distributions. The former is more sensitive to lower tail changes and can capture lower tail dependencies, while the latter is more sensitive to upper tail changes and can capture upper tail dependencies. Frank Copula is suitable for describing the coupling relationship between symmetric thick-tailed structural variables.

3. Joint Distribution Model and Risk Measure

3.1. ARMA-TGARCH-GED-Copula Model

On the basis of knowing each marginal distribution model and the copula function, the joint distribution model will also be uniquely determined. Furthermore, the estimation method of the established model adopts the two-stage maximum likelihood estimation method. The model construction and estimation process are as follows:

(1) Estimate the parameters of the marginal distribution model.
(2) Obtain the standard residual series.
(3) Perform probability integral transformation (PIT) on the standard residual series.
(4) Implement copula fitting on the transformed standard residual series to determine copula parameters.

Based on the above process, the ARMA-TGARCH-Copula joint model has been constructed. Besides, the Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC) can be used for the order determination of the ARMA-GARCH model.

3.2. Risk Metrics

Value-at-Risk (VaR) is often used to measure the downside risk of portfolio investment in the financial risk field, which refers to the maximum possible loss of portfolio assets within a certain holding period and under a certain confidence level. The loss function can be expressed as

$$L(\boldsymbol{r},\boldsymbol{w})=-P\sum_{i=1}^{d} r_i\cdot w_i \tag{12}$$

where P is the total investment, r_i is the return on various assets in the future holding period, and w_i is the corresponding investment weight. if the future holding period is m days, then

$$r_i=\exp\left(\sum_{j=1}^{m} R_{i,j}\right)-1 \tag{13}$$

where $R_{i,j}$ represents the predicted value of logarithmic return of the asset i on day j. Thus, VaR can be expressed as

$$\begin{aligned}\mathrm{VaR}_\alpha &\triangleq \inf\{l:P(L>l)\leqslant 1-\alpha\}\\ &=F_L^{-1}(\alpha)\end{aligned} \tag{14}$$

where $F_L(l)$ is the distribution function of the loss amount, $F_L^{-1}(l)$ is its inverse function, and α is the confidence level.

VaR has certain limitations, does not satisfy subadditivity, and is not representative in extreme cases such as stock market crashes, so CVaR is introduced as an additional risk measure, and its definition is as follows:

$$\mathrm{CVaR}_\alpha=\mathbf{E}(L|L>\mathrm{VaR}_\alpha) \tag{15}$$

Through the above equation, CVaR represents the expected value when the loss exceeds VaR. It satisfies subadditivity and convexity, and belongs to a consistent risk measure.

3.3. Simulation Process of Risk Measure

In this paper, the Monte Carlo simulation method is used to calculate the risk. The process is as follows:

(1) According to the fitted copula function, use the Monte Carlo method to generate a random number matrix **A** obeying the specified distribution, and use it as the transformed standard residual series of various assets.

(2) Based on the GED probability density function of each asset, the series generated by the simulation is inversely transformed to obtain the standard residual series.
(3) Through the determined ARMA-TGARCH marginal model, calculate the corresponding return series $R_{j,i}$ and $L(\boldsymbol{r}, \boldsymbol{w})$.
(4) By repeating the above process n times, the loss series L_k $(k = 1, \ldots, n)$ of the portfolio assets can be obtained, the α quantile of the series is VaR, and CVaR can be calculated on this basis.

Through the above four steps, the VaR and CVaR predicted can be calculated. As the stock market has the characteristics of immediate selling, the one-day-ahead risk measure has great reference value, and then this paper only considers the situation that the future holding period is one day ($m = 1$).

4. Empirical Research

4.1. Descriptive Statistical Analysis

The following stock indexes are used as research objects in this paper: Shanghai Composite Index (SZZZ), Shenzhen Component Index (SZCZ), ChiNext Composite Index (CYBZ), Dow Jones Index (DJI), NASDAQ Index (NASQ), and S&P 500 Index (SPX). The sample time range is from June 1, 2010 to December 30, 2016. In addition, the data comes from Wind database, and the data processing adopts Eviews10.0, R language, and MATLAB_R2020a.

Table 1: Basic statistics of six representative stock index return series.

Index	Mean	Std. Dev.	Skewness	Kurtosis	ARCH p-Value	JB p-Value
SZZZ	1.18e-4	0.014849	-0.9615	8.6470	0.000000	0.000000
SZCZ	4.85e-6	0.017438	-0.7220	6.4652	0.000000	0.000000
CYBZ	6.14e-4	0.021534	-0.6393	4.6869	0.000000	0.000000
DJI	4.24e-4	0.009113	-0.2541	7.1397	0.000000	0.000000
NASQ	5.53e-4	0.010928	-0.3333	6.8689	0.000000	0.000000
SPX	4.61e-4	0.009726	-0.3055	8.1034	0.000000	0.000000

The ADF test is carried out on the return series of each stock index, and the test result is that there is no unit root, indicating that they are all stationary time series. The autocorrelation test of the series shows that there is series autocorrelation, so it is reasonable to introduce the ARMA equation into the mean equation. Table 1 provides the corresponding basic statistics, and Fig. 1 shows the return of three stock indexes.

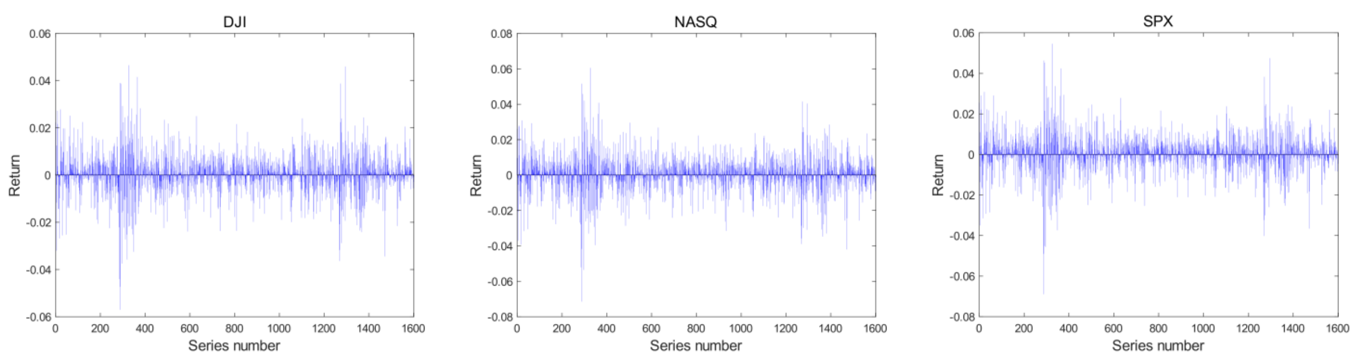


Fig. 1: Return of some representative stock indexes.

On the basis of the statistical values in Table 1, the average values of various stock index return series are all close to zero. Besides, they all obey the peak left-skewed distribution, and the ARCH effect is significant. The JB test rejects the assumption of normal distribution, indicating that the return series does not obey normal distribution. Meanwhile, Fig. 1 shows that the volatility of stock index return has obvious aggregation, showing that there is a heteroscedasticity effect in the residual series. Based on the above characteristics, it is necessary to model the stock index return series with TGARCH.

4.2. Marginal Distribution Fitting

In accordance with the AIC criterion, this paper selects ARMA (1,1)-TGARCH (1,1)-GED to fit the return series for SZZZ, SZCZ, and CYBZ. In addition, ARMA (2,1)-TGARCH (1,1)-GED is used to fit the return series for DJI, NASQ, and SPX. The estimation results of parameters are shown in Table 2.

It can be seen from Table 2 that there is continuous volatility in the returns of various stock indexes. In addition, the leverage effect of the three stock indexes DJI, NASQ, and SPX is relatively significant, and the asymmetric coefficient γ in the corresponding GARCH model is all positive, indicating that the stock market is generally more sensitive to bad news. The GED distribution parameters of the six stock indexes are all less than two, illustrating that the return series shows sharp peaks and thick tails.

Table 2: Parameter estimation results of ARMA-TGARCH model.

	ϕ_1	ϕ_2	θ_1	ω	α_1	β_1	γ_1	v
SZZZ	-0.9752	\	0.9814	1.69e-6	0.0497	0.9413	3.84e-4	1.1317
SZCZ	-0.8144	\	0.8444	3.34e-6	0.0418	0.9365	0.0190	1.2042
CYBZ	-0.5282	\	0.6198	3.46e-6	0.0526	0.9454	-0.0120	1.5272
DJI	0.5356	0.0403	-0.5508	3.52e-6	0.0000	0.8187	0.3094	1.2030
NASQ	-0.4980	0.0704	0.5244	5.40e-6	0.0000	0.8375	0.2424	1.3244
SPX	-0.8079	0.0259	0.8110	4.09e-6	0.0000	0.8191	0.2993	1.1802

The standard residual series of each stock index basically has no autocorrelation and heteroscedasticity effect after processing, and the K-S test shows that the transformed standard residual series obeys the uniform distribution, which means that the marginal distribution model is accurate and reasonable.

4.3. Joint Distribution Fitting

The marginal distribution of the six stock indexes has been determined, and the correlation between the transformed standard residual series of each stock index is shown in Fig. 2.

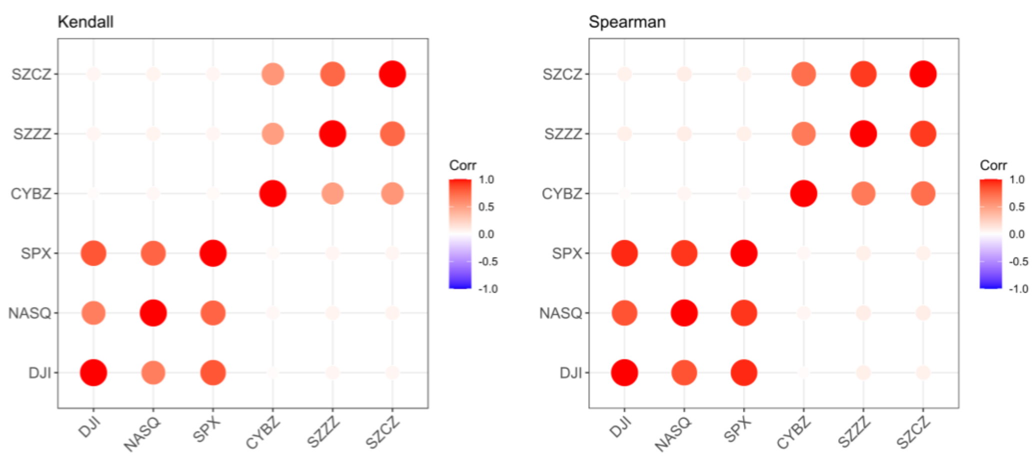


Fig. 2: Correlation between six stock indexes (Kendall and Spearman)

The figure shows that there is a strong correlation between the Chinese representative stock indexes SZZZ, SZCZ, and CYBZ. Meanwhile, there is also a strong correlation between the American representative stock indexes DJI, NASQ, and SPX, but the correlation between the two types of stock indexes is very weak.

Table 3: Parameter estimation results of various copula functions

	Clayton	Gumbel	Frank	t
China	1.739	2.246	7.395	4.817

	(1.717)	(0.039)	(0.157)	
U.S.	3.538	3.529	12.55	6.417
	(0.177)	(0.071)	(0.302)	

Note: (*) is the standard error of the estimated value, and only the degree of freedom estimation for t-Copula is listed.

Therefore, this paper implements coupling for Chinese stock indexes and American stock indexes respectively, investigating the fitting effect of Clayton Copula, Gumbel Copula, Frank Copula, and t Copula in turn. The parameter estimation results are shown in Table 3.

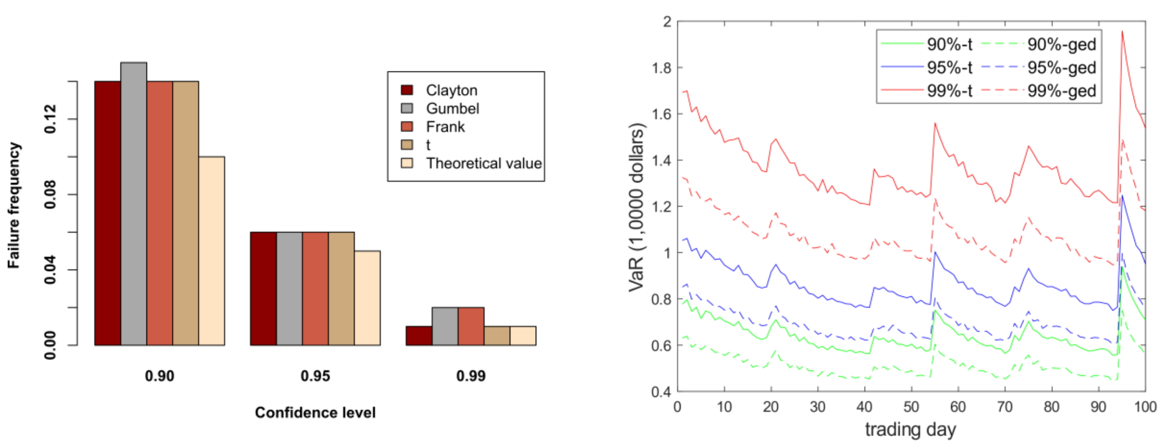
4.4. Results of Risk Measure

Assuming that the total investment is 900,000 dollars and the six stock indexes are invested in equal proportion, the VaR and CVaR of the next trading day are calculated by the Monte Carlo simulation method when the confidence level is 90%, 95%, and 99%, respectively. Then the results calculated by various copula functions are shown in Table 4.

Table 4: VaR and CVaR of portfolio investment

level		Clayton	Gumbel	Frank	t
90%	VaR	0.619	0.613	0.636	0.618
	CVaR	0.937	0.883	0.882	0.923
95%	VaR	0.850	0.817	0.829	0.838
	CVaR	1.151	1.061	1.039	1.129
99%	VaR	1.337	1.212	1.176	1.326
	CVaR	1.592	1.421	1.331	1.583

Based on Table 4, it is found that the risk measures calculated by Clayton Copula and t Copula at each confidence level are almost the same. In addition, the results calculated by Gumbel Copula and Frank Copula are generally close to the former two, but there are slight differences.



(a) Failure frequency test (b) Comparison of VaR forecasts

Fig. 3: VaR test and forcast for the next 100 trading days.

In order to further test the validity of the model, the failure frequency test is implemented according to the closing price data of the above six stock indexes in the first 100 trading days of 2017. On the basis of the established prediction model, the number of VaR failures (i.e., the number of days when the loss exceeds the VaR) and the corresponding failure frequency are calculated within 100 trading days. The test results are shown in Fig. 3 (a).

It can be seen from Fig. 3 (a), the joint distribution model established by each copula function predicts a similar VaR failure frequency. Besides, the sum of the confidence level and the corresponding failure frequency is basically close to one for various copula functions. In terms of the overall test effect, Clayton Copula and t Copula are the best.

In addition to the empirical analysis based on the established ARMA-TGACH-GED-Copula model, this study also considers other stock asset distribution models. According to the ARMA-TGARCH-t marginal model applied by Autchariyapanitkul et al. [8], assuming that the standard residuals obey the t distribution, the marginal and joint distribution models are re-fitted so as to calculate the risk measure for the next trading day for the new model. Fig. 3 (b) shows the VaR forecast comparison curve for the next 100 trading days (taking t-Copula as an example).

As shown in Fig. 3 (b) that the VaR predicted by the t-distribution model under various confidence levels is greater than the prediction result of the model in this paper. Through calculation, it is found that the failure frequency at 90%, 95%, and 99% confidence levels is 0.06, 0.02, and 0.00 respectively. The value is significantly lower than the theoretical value, indicating that the t distribution is too conservative and is not suitable for risk management of stock portfolio investment. It also indirectly proves that the GED distribution is more reasonable.

5. Conclusion

The relationship between financial assets is quite complex, and the tail dependence between variables is significant. The traditional assumption of multivariate normal distribution has great limitations on the prediction of the actual stock market. Besides, the financial return series presents the characteristics of sharp peak, thick tail, and left deviation, and the volatility series shows a strong heteroscedasticity effect. In order to better describle these characteristics, in this paper, the ARMA-TGARCH-GED model is used to fit the stock market return series, and then various copula functions are introduced to capture the tail dependence between assets. On this basis, combining the Monte Carlo stochastic simulation method, this paper calculates the VaR and CVaR of portfolio investment.

On the basis of the empirical research, stock index returns have leverage effects, as well as autocorrelation and ARCH effect. Therefore, the return series is suitable for ARMA-TGARCH to fit marginal distribution. The four types of copula Functions applied in this paper can couple the marginal distribution model well, and their predictions of VaR and CVaR are basically similar. According to the failure frequency test, the prediction of VaR is effective. As SZZZ, SZCZ, and CYBZ are representative in China's stock market, and DJI, NASQ, and SPX are typical in the U.S. stock market, focusing on the portfolio investment of these six representative stock indexes can effectively help understand the statistical characteristics of stock market return. Meanwhile, the model can be extended to portfolio investment of other financial assets, providing investors with risk prediction under different investment strategies.

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