Proceedings of the 4th International Conference on Statistics: Theory and Applications (ICSTA'22)
Prague, Czech Republic – July 28- 30, 2022
Paper No. 156
DOI: 10.11159/icsta22.156

On the Estimation of Empirical Copulas and Joint Densities

Yishan Zang[1]
[1]The University of Western Ontario
1151 Richmond Street, London, Canada
yzang8@uwo.ca

Extended Abstract

Copulas are principally utilized for modelling dependencies in multivariate distributions. The key idea behind copulas is that the joint distribution of two or more variables can be represented in terms of their marginal distributions and a specific correlation structure. As a measure of dependence, they have for instance found applications in reliability theory, signal processing, geodesy, hydrology and medicine. Results involving empirical bivariate copula densities are discussed in this presentation.

In the proposed methodology, kernel density estimates are utilized to determine the marginal distributions. Then, a moment-based approximation technique for estimating a copula density from bivariate observations is introduced. This approach relies on the estimated joint density, the marginal densities and polynomial representations of the inverse marginal distribution functions. The joint density can be determined by multiplying a base density by a bivariate polynomial whose coefficients are obtained from the joint moments of the distributions. The degrees of the bivariate polynomial adjustment will be selected according to a certain goodness-of-fit criterion. The resulting simple representation of this copula density is suitable for reporting purposes or carrying out further algebraic manipulation.

A new approach for obtaining an initial copula density estimate from Deheuvels' empirical copula is also proposed. For a given bivariate data set, Deheuvels' copula is evaluated and approximated by means of a bivariate least-squares polynomial. On differentiating this polynomial estimate with respect to both variables, one can obtain a preliminary estimate of the copula density function.

Additionally, it is found that the domain of a joint density function can be determined from the support of a density estimate of the standardized vector, which is taken to be a rectangle. It turns out that the domain of the original distribution can be delimited by a parallelogram once the back transformation is applied.

From another perspective, one can estimate a joint density function from a given copula density estimate and the marginal density estimates. A Bernstein's copula density of high order which closely approximate the underlying copula density can be utilized in this case.

The results are applied to two stocks' closing prices as well as a stock's price and its running maximum. In the former case, the copula density estimates which are secured by resorting to different approaches, are seen to share similar features; they also happen to be consistent with Bernstein's empirical copula density. In the latter case, since the model is related to a Brownian motion process, an appropriate transformation is applied to the data so that it possesses the characteristics of such a process.