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Diagonal Vector Autoregressive and Multivariate Autoregressive Distributed Lag Models and Their Variance Properties

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Abstract
When working with multivariate time series with a significant number of lag components, the presence of multicollinearity among predictor lagged variables is likely. This underscores the requirement for parsimonious models in time series models that allow for parameter reduction. Diagonal Vector Autoregressive (VAR) and Multivariate Autoregressive Distributed Lag (MARDL) models are subclasses of general multivariate time series models with a significant number of lagged variables that can be identified when the coefficient matrices' parameters are restricted to the diagonal elements. The upper and lower diagonal VAR and MARDL models, as well as their variances, are derived. The prerequisites for identifying the diagonal VAR and MARDL models were found in this paper, and the models' validity was shown. To compare the performances of the new classes of multivariate lag models, data from certain macroeconomic variables such as Nigeria Gross Domestic Product (GDP), Crude Oil Petroleum (C/PET), Agriculture (AGRIC), and Telecommunication (TELECOM) are used after the first order difference of the logarithm of the series to achieve stationarity. The models were estimated, and the variances of the processes and errors were determined using the model parameters. The results show that the two models have almost the same comparative advantage. As a result, the two models complement each other when modelling multivariate lag variables.

Keywords: Upper Diagonal VAR Models, Lower Diagonal VAR Models, Upper Diagonal MARDL Models, Lower Diagonal MARDL Models, Variable and Error Variances.

Introduction

Multivariate Autoregressive Distributed Lag Models (MARDLM) are models for multiple responses that are based on the lagged and non-lagged terms of the predictor variables and solely the lagged terms of the response. Each time variable in a multivariate time series is a linear mixture of its lagged terms and other lagged terms. Multivariate Autoregressive Distributed Lag Models are a combination of Multivariate Linear Regression Models (MLRM) and Vector Autoregressive Models (VARM). The Multivariate Linear Regression Models express a linear relationship between the current time of the response and the predictor variables. Vector Autoregressive Models are well-known multivariate time series models that are used to represent a variety of time series characterised by autoregressive processes. The models are an extension of a univariate time series model with the response variable determined by its lag components. The feed-forward and feed-back mechanism, as well as the interdependence established between the vectors of responses and predictors, are significant features of VAR models. In vector Autoregressive Models, each response variable is a linear combination of its lag terms, predictors, and an error term, accounting for the contributions of the response and predictors' past values, which are always represented in the form of multiple linear regression model. VAR models are a type of modelling and forecasting tool that focuses on several time series and frequently employs a multiple regression strategy based on [1]. [2] created Distributed Lag Models to express the present-time influence of predictor factors in a multiple linear relationship between the response and a group of predictor variables. MARDLM differs from VARM in that it incorporates the predictor variable's current time in each independent variable, whereas VARM limits the independent variables to predictor lagged terms. This is true in the sense that there is always a causal relationship between the predictor variables' present time and the response.

Multivariate diagonal time series models limit and restrict the number of parameters in the coefficient matrices. When the matrix coefficients are restricted to the major diagonal, the model becomes a pure diagonal multivariate time series model. If the parameters in the matrix of coefficients are limited to the upper or lower diagonal, the models are referred to as upper or lower diagonal multivariate time series models, respectively. Depending on the specification, it could be the principal, upper, or lower diagonal elements of the coefficient matrices. There is a special instance of multiple time series models with diagonal autoregressive and moving average parameter matrices, as proposed by [3]. The models are known as MTS-D models, and their flexibility and utility have been proved in the context of a comparative market situation with five sales series. In multivariate time series, the three diagonal models are pure diagonal, upper diagonal, and lower diagonal.

Aside from VAR or MARDL models, this paper discusses covariance analysis of the square of only diagonal bilinear time series models while investigating the features of bilinear time series models by [4]. The properties of the squares of the linear moving average process were compared to the squares of the exclusively diagonal bilinear process in the article. The stability analysis of the first-order periodic autoregressive diagonal bilinear model by [5] is also on diagonal models. The research presented a comprehensive framework of stability that incorporates the majority of the probability features examined for the pure diagonal bilinear model. [6] proposed a criterion for stationarity and invertibility for pure diagonal time series models. For any pure diagonal bilinear process, the covariance structure of pure diagonal bilinear models was determined and presented. [7] investigated the theoretical and actual differences in risk management between Diagonal and Full BEKK. They used simulated financial returns series to compare estimates of conditional variances and covariances from DBEKK and Full BEKK. Full BEKK conditional variance estimates are lower in the left tail and higher in the right tail than DBEKK estimates. Recent applications of multivariate time series models include [8], [9]. Diagonal Multivariate Generalised Autoregressive Conditional Heteroskedasticity Models were developed as a subclass of conventional MGARCH models by [10]. The models fared well in comparison to the Full MGARCH models.

Earlier research looked at diagonal VAR models. The diagonal VAR and MARDL models are required to compare the performance of the two models using variance properties. The objective of this work is the investigation of upper and lower diagonal VAR and MARDL models, as well as their variance properties.

2. Model Derivations

(a) Generalized VAR Models

Definition

Let $\underline{Z}_t = (Z_{1t}, Z_{2t}, \dots, Z_{mt})^I$ be the vector of response time variables, $\underline{\emptyset} = \left(\emptyset_{k.ij}\right)$ is the coefficients vector, $\underline{Z}_{t-k} = (Z_{1t-k}, Z_{2t-k}, \dots, Z_{nt-k})^I$ is defined as the vector of the predictive lag time variables, $\underline{\delta} = (\delta_1, \delta_2, \dots, \delta_m)^I$ is the vector of constants and $\underline{\varepsilon}_t = (\varepsilon_{t1}, \varepsilon_{2t}, \dots, \varepsilon_{mt})^I$ is the vector of error terms associated with the vector of response time variables. The above definition is reduced to the following model,

$$Z_{it} = \delta_i + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.ij} Z_{jt-k} + \varepsilon_{it}, i = 1, \dots, m \tag{1}$$

$\emptyset_{k.ij}$ are parameters of contribution of $j's\ predictors\ to\ i's\ respnses\ at\ k\ lags.$ $\delta_{i(i=1,\dots,m)}$ are constants.
Equation (1) is the general Vector Autoregressive Models (VARM).

i. Upper Diagonal VAR Models and Their Variances

This section considers the conditions for identification of the upper diagonal VAR models from the general form. From Equation (1), the following set of models is obtained

$$Z_{it} = \begin{cases} \delta_1 + \emptyset_{k.1j} Z_{jt-k} + \varepsilon_{1t}, i = 1; j = 1, \dots, n; k = 1, \dots, p \\ \delta_2 + \emptyset_{k.2j} Z_{jt-k} + \varepsilon_{2t}, i = 2; j = 2, \dots, n; k = 1, \dots, p \\ \delta_3 + \emptyset_{k.3j} Z_{jt-k} + \varepsilon_{3t}, i = 3; j = 3, \dots, n; k = 1, \dots, p \\ \vdots \\ \delta_m + \emptyset_{k.mj} Z_{jt-k} + \varepsilon_{mt}, i = m; j = n; k = 1, \dots, p \end{cases} \tag{2}$$

The parameters are as defined in Equation (1).

Equation (2) defines a set of Upper Diagonal Vector Autoregressive Models.

Proof:
Given Equation (1)

$$Z_{it} = \delta_i + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.ij} Z_{jt-k} + \varepsilon_{it}, i = 1, \dots, m$$

Case 1: $if\ i = 1; j = 1, \dots, n; k = 1, \dots, p$

$$Z_{1t} = \delta_1 + \emptyset_{1.11}Z_{1t-1} + \emptyset_{1.12}Z_{2t-1} + \cdots + \emptyset_{1.1n}Z_{nt-1} + \emptyset_{2.11}Z_{1t-2} + \emptyset_{2.12}Z_{2t-2} + \cdots + \emptyset_{2.1n}Z_{nt-2} + \cdots + \emptyset_{p.11}Z_{1t-p} + \emptyset_{p.12}Z_{2t-p} + \cdots + \emptyset_{p.1n}Z_{nt-p} + \varepsilon_{1t} \quad (3)$$

$$Z_{1t} = \delta_1 + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.1j} Z_{jt-k} + \varepsilon_{1t} \quad (4)$$

Case 2: $if\ i = 2; j = 2, \dots, n; k = 1, \dots, p$

$$Z_{2t} = \delta_2 + \emptyset_{1.22}Z_{2t-1} + \emptyset_{1.23}Z_{3t-1} + \cdots + \emptyset_{1.2n}Z_{nt-1} + \emptyset_{2.22}Z_{2t-2} + \emptyset_{2.23}Z_{3t-2} + \cdots + \emptyset_{2.2n}Z_{nt-2} + \cdots + \emptyset_{p.22}Z_{2t-p} + \emptyset_{p.23}Z_{3t-p} + \cdots + \emptyset_{p.2n}Z_{nt-p} + \varepsilon_{2t} \quad (5)$$

$$Z_{2t} = \delta_2 + \sum_{k=1}^{p}\sum_{j=2}^{n} \emptyset_{k.2j} Z_{jt-k} + \varepsilon_{2t} \quad (6)$$

Case 3: $if\ i = m; j = n; k = 1, \dots, p$

$$Z_{mt} = \delta_m + \emptyset_{1.mn}Z_{nt-1} + \emptyset_{2.mn}Z_{nt-2} + \cdots + \emptyset_{p.mn}Z_{nt-p} + \varepsilon_{mt} \quad (7)$$

$$Z_{mt} = \delta_m + \sum_{k=1}^{p} \emptyset_{k.3j} Z_{jt-k} + \varepsilon_{mt} \quad (8)$$

Equations (4), (6) and (8) are a set of Upper Diagonal VAR Models, and these complete the proof.

Variances of Upper Diagonal VAR Models

Variances of Z_{it}:

Let Z_{it} in Equation (1) be a stationary process that is distributed about the origin such that $E(Z_{it}) = 0, => \delta_1 = \delta_2 = \cdots = \delta_m = 0$.

Case1: $for\ i = 1,$ multiply Equation (3) by Z_{1t} and take the expectations.

$$E(Z_{1t}Z_{1t}) = E[Z_{1t}(\emptyset_{1.11}Z_{1t-1} + \emptyset_{1.12}Z_{2t-1} + \cdots + \emptyset_{1.1n}Z_{nt-1} + \emptyset_{2.11}Z_{1t-2} + \emptyset_{2.12}Z_{2t-2} + \cdots + \emptyset_{2.1n}Z_{nt-2} + \cdots + \emptyset_{p.11}Z_{1t-p} + \emptyset_{p.12}Z_{2t-p} + \cdots + \emptyset_{p.1n}Z_{nt-p} + \varepsilon_{1t})]$$

$$\xi_{1t,1t} = \emptyset_{1.11}\xi_{1t,1t(1)} + \emptyset_{1.12}\xi_{1t,2t(1)} + \cdots + \emptyset_{1.1n}\xi_{1t,nt(1)} + \emptyset_{2.11}\xi_{1t,1t(2)} + \emptyset_{2.12}\xi_{1t,2t(2)} + \cdots + \emptyset_{2.1n}\xi_{1t,nt(2)} + \cdots + \emptyset_{p.11}\xi_{1t,1t(p)} + \emptyset_{p.12}\xi_{1t,2t(p)} + \cdots + \emptyset_{p.1n}\xi_{1t,nt(p)} + \sigma^2_{\varepsilon_{1t}} \quad (9)$$

$E(Z_{1t}\varepsilon_{1t}) = \sigma^2_{\varepsilon_{1t}}$ (from correlated stationary processes)

$$\xi_{1t,1t} = \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.1j}\xi_{1t,jt(k)} + \sigma^2_{\varepsilon_{1t}} \tag{10}$$

Case 2: $for\ i = 2\ and\ \delta_2 = 0,$ multiply Equation (5) by Z_{2t} and take the expectations.
$E(Z_{2t}Z_{2t}) = E[Z_{2t}(\emptyset_{1.22}Z_{2t-1} + \emptyset_{1.23}Z_{3t-1} + \cdots + \emptyset_{1.2n}Z_{nt-1} + \emptyset_{2.22}Z_{2t-2} + \emptyset_{2.23}Z_{3t-2} + \cdots + \emptyset_{2.2n}Z_{nt-2} + \cdots + \emptyset_{p.22}Z_{2t-p} + \emptyset_{p.23}Z_{3t-p} + \cdots + \emptyset_{p.2n}Z_{nt-p} + \varepsilon_{2t})]$

$$\begin{aligned}\xi_{2t,2t} = {} & \emptyset_{1.22}\xi_{2t,2t(1)} + \emptyset_{1.23}\xi_{2t,3t(1)} + \cdots + \emptyset_{1.2n}\xi_{2t,nt(1)} + \emptyset_{2.22}\xi_{2t,2t(2)} + \emptyset_{2.23}\xi_{2t,3t(2)} + \cdots + \emptyset_{2.2n}\xi_{2t,nt(2)} + \cdots \\ & + \emptyset_{p.22}\xi_{2t,2t(p)} + \emptyset_{p.23}\xi_{2t,3t(p)} + \cdots + \emptyset_{p.2n}\xi_{2t,nt(p)} \\ & + \sigma^2_{\varepsilon_{2t}}\end{aligned} \tag{11}$$

where, $E(Z_{2t}\varepsilon_{2t}) = \sigma^2_{\varepsilon_{2t}}$ (from correlated stationary processes)

$$\xi_{2t,2t} = \sum_{k=1}^{p}\sum_{j=2}^{n} \emptyset_{k.2j}\xi_{2t,jt(k)} + \sigma^2_{\varepsilon_{2t}} \tag{12}$$

Case 3: $for\ i,j = m,n\ (m = n)\ and\ \delta_m = 0,$ multiply Equation (7) by Z_{mt} and take the expectations.
$E(Z_{mt}Z_{mt}) = E[Z_{mt}(\emptyset_{1.mn}Z_{nt-1} + \emptyset_{2.mn}Z_{nt-2} + \cdots + \emptyset_{p.mn}Z_{nt-p} + \varepsilon_{mt})]$

$$\xi_{mt,mt} = \emptyset_{1.mn}\xi_{mt,nt(1)} + \emptyset_{2.mn}\xi_{mt,nt(2)} + \cdots + \emptyset_{p.mn}\xi_{mt,nt(p)} + \sigma^2_{\varepsilon_{mt}} \tag{13}$$

where, $E(Z_{mt}\varepsilon_{mt}) = \sigma^2_{\varepsilon_{mt}}$ (from correlated stationary processes)

$$\xi_{mt,mt} = \sum_{k=1}^{p} \emptyset_{k.mn}\xi_{mt,nt(k)} + \sigma^2_{\varepsilon_{mt}} \tag{14}$$

Equations (10), (12) and (14) are upper diagonal variances of $Z_{1t}, Z_{2t}\ and\ Z_{mt}$ respectively.

ii. Lower Diagonal VAR Models and Their Variances

This section considers the conditions for identification of the upper diagonal VAR models from the general form. From Equation (1), the following set of models is obtained

$$Z_{it} = \begin{cases} \delta_1 + \emptyset_{k.1j}Z_{jt-k} + \varepsilon_{1t}, i = 1; j = 1; k = 1, \dots, p \\ \delta_2 + \emptyset_{k.2j}Z_{jt-k} + \varepsilon_{2t}, i = 2; j = 1,2; k = 1, \dots, p \\ \delta_3 + \emptyset_{k.3j}Z_{jt-k} + \varepsilon_{3t}, i = 3; j = 1,2,3; k = 1, \dots, p \\ \vdots \\ \delta_m + \emptyset_{k.mj}Z_{jt-k} + \varepsilon_{mt}, i = m; j = 1,2, \dots, n; k = 1, \dots, p \end{cases} \tag{15}$$

Equation (15) defines a set of Lower Diagonal Vector Autoregressive Models.

Proof:
Given Equation (1)

$$Z_{it} = \delta_i + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.ij} Z_{jt-k} + \varepsilon_{it}, i = 1, \dots, m$$

The parameters are as defined in Equation (1).

Case 1: $if\ i = 1; j = 1; k = 1, \dots, p$

$$Z_{1t} = \delta_1 + \emptyset_{1.11} Z_{1t-1} + \emptyset_{2.11} Z_{1t-2} + \cdots + \emptyset_{p.11} Z_{1t-p} + \varepsilon_{1t} \quad (16)$$

$$Z_{1t} = \delta_1 + \sum_{k=1}^{p} \emptyset_{k.11} Z_{1t-k} + \varepsilon_{1t} \quad (17)$$

Case 2: $if\ i = 2; j = 1, 2; k = 1, \dots, p$

$$Z_{2t} = \delta_2 + \emptyset_{1.21} Z_{1t-1} + \emptyset_{1.22} Z_{2t-1} + \emptyset_{2.21} Z_{1t-2} + \emptyset_{2.22} Z_{2t-2} + \cdots + \emptyset_{p.21} Z_{1t-p} + \emptyset_{p.22} Z_{2t-p} + \varepsilon_{2t} \quad (18)$$

$$Z_{2t} = \delta_2 + \sum_{k=1}^{p}\sum_{j=1}^{2} \emptyset_{k.2j} Z_{jt-k} + \varepsilon_{2t} \quad (19)$$

Case 3: $if\ i = m; j = 1, 2, \dots, n; k = 1, \dots, p$

$$Z_{mt} = \delta_m + \emptyset_{1.m1} Z_{1t-1} + \emptyset_{1.m2} Z_{2t-1} + \cdots + \emptyset_{1.mn} Z_{nt-1} + \emptyset_{2.m1} Z_{1t-2} + \emptyset_{2.m2} Z_{2t-2} + \cdots + \emptyset_{2.mn} Z_{nt-2} + \dots + \emptyset_{p.m1} Z_{1t-p} + \emptyset_{p.m2} Z_{2t-p} + \cdots + \emptyset_{p.mn} Z_{nt-p} + \varepsilon_{mt} \quad (20)$$

$$Z_{mt} = \delta_m + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.mj} Z_{jt-k} + \varepsilon_{mt} \quad (21)$$

Equations (17), (19) and (21) define a set Lower Diagonal VAR Models, and these complete the proof.

Variances of Lower Diagonal VAR Models

Variances of Z_{it}:

Let Z_{it} in Equation (3) be a stationary process that is distributed about the origin such that $E(Z_{it}) = 0, => \delta_1 = \delta_2 = \cdots = \delta_m = 0$.

Case1: $for\ i = 1,$ multiply Equation (16) by Z_{1t} and take the expectations.

$$E(Z_{1t} Z_{1t}) = E[Z_{1t}(\emptyset_{1.11} Z_{1t-1} + \emptyset_{2.11} Z_{1t-2} + \cdots + \emptyset_{p.11} Z_{1t-p} + \varepsilon_{1t})]$$

$$\xi_{1t,1t} = \emptyset_{1.11} \xi_{1t,1t(1)} + \emptyset_{2.11} \xi_{1t,1t(2)} + \cdots + \emptyset_{p.11} \xi_{1t,1t(p)} + \sigma^2_{\varepsilon_{1t}} \quad (22)$$

where, $E(Z_{1t}\varepsilon_{1t}) = \sigma^2_{\varepsilon_{1t}}$ (from correlated stationary process)

$$\xi_{1t,1t} = \sum_{k=1}^{p} \emptyset_{k.11}\xi_{1t,1t(k)} + \sigma^2_{\varepsilon_{1t}} \tag{23}$$

Case 2: $for\ i = 2\ and\ \delta_2 = 0$, multiply Equation (18) by Z_{2t} and take the expectations.

$$E(Z_{2t}Z_{2t}) = E[Z_{2t}(\emptyset_{1.21}Z_{1t-1} + \emptyset_{1.22}Z_{2t-1} + \emptyset_{2.21}Z_{1t-2} + \emptyset_{2.22}Z_{2t-2} + \cdots + \emptyset_{p.21}Z_{1t-p} + \emptyset_{p.22}Z_{2t-p} + \varepsilon_{2t})]$$

$$\xi_{2t,2t} = \emptyset_{1.21}\xi_{2t,1t(1)} + \emptyset_{1.22}\xi_{2t,2t(1)} + \emptyset_{2.21}\xi_{2t,1t(2)} + \emptyset_{2.22}\xi_{2t,2t(2)} + \cdots + \emptyset_{p.21}\xi_{2t,1t(p)} + \emptyset_{p.22}\xi_{2t,2t(p)} + \sigma^2_{\varepsilon_{2t}} \tag{24}$$

where, $E(Z_{2t}\varepsilon_{2t}) = \sigma^2_{\varepsilon_{2t}}$ (from correlated stationary processes)

$$\xi_{2t,2t} = \sum_{k=1}^{p}\sum_{j=1}^{2} \emptyset_{k.2j}\xi_{2t,jt(k)} + \sigma^2_{\varepsilon_{2t}} \tag{25}$$

Case 3: $if\ i = m; j = 1, 2, \ldots, n; k = 1, \ldots, p$ and $\delta_m = 0$, multiply Equation (20) by Z_{mt} and take the expectations.

$$E(Z_{mt}Z_{mt}) = E[Z_{mt}(\emptyset_{1.m1}Z_{1t-1} + \emptyset_{1.m2}Z_{2t-1} + \cdots + \emptyset_{1.mn}Z_{nt-1} + \emptyset_{2.m1}Z_{1t-2} + \emptyset_{2.m2}Z_{2t-2} + \cdots + \emptyset_{2.mn}Z_{nt-2} + \ldots + \emptyset_{p.m1}Z_{1t-p} + \emptyset_{p.m2}Z_{2t-p} + \cdots + \emptyset_{p.mn}Z_{nt-p} + \varepsilon_{mt})]$$

$$\xi_{mt,mt} = \emptyset_{1.m1}\xi_{mt,1t(1)} + \emptyset_{1.m2}\xi_{mt,2t(1)} + \cdots + \emptyset_{1.mn}\xi_{mt,nt(1)} + \emptyset_{2.m1}\xi_{mt,1t(2)} + \emptyset_{2.m2}\xi_{mt,2t(2)} + \cdots + \emptyset_{2.mn}\xi_{mt,nt(2)} + \cdots + \emptyset_{p.m1}\xi_{mt,1t(p)} + \emptyset_{p.m2}\xi_{mt,2t(p)} + \cdots + \emptyset_{p.mn}\xi_{mt,nt(p)} + \sigma^2_{\varepsilon_{mt}} \tag{26}$$

where, $E(Z_{mt}\varepsilon_{mt}) = \sigma^2_{\varepsilon_{mt}}$ (correlated stationary process)

$$\xi_{mt,mt} = \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.mj}\xi_{mt,jt(k)} + \sigma^2_{\varepsilon_{mt}} \tag{27}$$

Equations (23), (25) and (27) are lower diagonal variances of $Z_{1t}, Z_{2t}\ and\ Z_{mt}\ respectively.$

b. MARDL Models and Their Variances

Definition

Let $\underline{Z_t} = (Z_{1t}, Z_{2t}, \ldots, Z_{mt})^I$ be the vector of response time variables, $\underline{\emptyset} = (\emptyset_{k.ij})$ is the coefficients vector, $\underline{Z}_{st}(s \neq i)$ is the non-lag predictor, $\underline{\emptyset}_{st(s\neq i)}$ is a vector of non-lag coefficients, $\underline{Z}_{t-k} = (Z_{1t-k}, Z_{2t-k}, \ldots, Z_{nt-k})^I$ is defined as the vector of the predictive lag time variables, $\underline{\delta} = (\delta_1, \delta_2, \ldots, \delta_m)^I$ is the vector of constants and $\underline{\varepsilon_t} = (\varepsilon_{t1}, \varepsilon_{2t}, \ldots, \varepsilon_{mt})^I$ is the vector of error terms associated with the vector of response time variables. The above definition is reduced to the following model,

$$Z_{it} = \delta_i + \sum_{s=1}^{m} \emptyset_{is}Z_{st} + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.ij}\, Z_{jt-k} + \varepsilon_{jt}, i = 1, \ldots, m(m = n), (i \neq s) \tag{28}$$

$\emptyset_{is}$ are non-lag coefficients of the predictor variables, $\emptyset_{k.ij}$ are lag contributions of j predictors to i responses at k lags, $\delta_{i(i=1,\ldots,m)}$ are constants

Equation (28) is the general Multivariate Autoregressive Distributed Lag Model (MARDL)

i. Upper Diagonal MARDL Models and Their Properties

Model Derivation

This section considers the conditions for identification of the upper diagonal MARDL models from the general form. From Equation (28), the following set of models is obtained

$$Z_{it} = \begin{cases} \delta_1 + \emptyset_{1s} Z_{st} \ + \ \emptyset_{k.1j} Z_{jt-k} + \varepsilon_{1t}, i = 1; j = 1, \dots, n; k = 1, \dots, p; (s \neq 1) \\ \delta_2 + \emptyset_{2s} Z_{st} \ + \ \emptyset_{k.2j} Z_{jt-k} + \varepsilon_{2t}, i = 2; j = 2, \dots, n; k = 1, \dots, p; (s \neq 2) \\ \delta_3 + \emptyset_{3s} Z_{st} \ + \ \emptyset_{k.3j} Z_{jt-k} + \varepsilon_{3t}, i = 3; j = 3, \dots, n; k = 1, \dots, p; (s \neq 3) \\ \vdots \\ \delta_m + \emptyset_{ms} Z_{st} \ + \ \emptyset_{k.mj} Z_{jt-k} + \varepsilon_{mt}, \ i = m; \ j = n; \quad k = 1, \dots, p; (s \neq m) \end{cases}$$
(29)

Equation (29) defines a set of Upper Diagonal Multivariate Autoregressive Distributed Lag Models (UDMARDL).

Proof:

Given Equation (28)

$$Z_{it} = \delta_i + \sum_{s=1}^{m} \emptyset_{is} Z_{st} + \sum_{k=1}^{p} \sum_{j=1}^{n} \emptyset_{k.ij} Z_{jt-k} + \ \varepsilon_{jt}, i = 1, \dots, m \ (i \neq s)$$

Case 1: $if \ i = 1; ; s = 1, \dots, m; j = 1, \dots, n; k = 1, \dots, p$

$$Z_{1t} = \ \ \delta_1 + \ \emptyset_{12} Z_{2t} + \emptyset_{13} Z_{3t} + \dots + \emptyset_{1n} Z_{nt} + \ \emptyset_{1.11} Z_{1t-1} + \emptyset_{1.12} Z_{2t-1} + \cdots + \ \emptyset_{1.1n} Y_{nt-1} \ + \emptyset_{2.11} Z_{1t-2} + \ \emptyset_{2.m2} Z_{2t-2} + \cdots + \ \emptyset_{2.1n} Z_{nt-2} + \cdots + \ \emptyset_{p.11} Z_{1t-p} + \ \emptyset_{p.12} \emptyset_{2t-p} + \cdots + \ \emptyset_{p.1n} Z_{nt-p} + \varepsilon_{1t} \qquad (30)$$

$$Z_{1t} = \delta_1 + \sum_{s=1}^{m} \emptyset_{1s} Z_{st} + \sum_{k=1}^{p} \sum_{j=1}^{n} \emptyset_{k.1j} Z_{jt-k} + \ \varepsilon_{1t} \qquad (31)$$

Case 2: $if \ i = 2; s = 1, \dots, m (s \neq 2); \ j = 2, \dots, n; i = 1, \dots, p$, we have,

$$Z_{2t} = \ \ \delta_2 + \ \emptyset_{21} Z_{1t} + \emptyset_{23} Z_{3t} + \dots + \emptyset_{2n} Z_{nt} + \ \emptyset_{1.21} Z_{1t-1} + \emptyset_{1.22} Z_{2t-1} + \cdots + \ \emptyset_{1.2n} Y_{nt-1} \ + \emptyset_{2.21} Z_{1t-2} + \ \emptyset_{2.22} Z_{2t-2} + \cdots + \ \emptyset_{2.2n} Z_{nt-2} + \cdots + \ \emptyset_{p.21} Z_{1t-p} + \ \emptyset_{p.22} \emptyset_{2t-p} + \cdots + \ \emptyset_{p.2n} Z_{nt-p} + \varepsilon_{2t} \qquad (32)$$

$$Z_{2t} = \delta_2 + \sum_{s=1}^{m} \emptyset_{2s} Z_{st} + \sum_{k=1}^{p} \sum_{j=1}^{n} \emptyset_{k.2j} Z_{jt-k} + \ \varepsilon_{2t} \qquad (33)$$

Case 3: $if \ i = m; s = 1, \dots, m - 1 (s \neq m); \ j = n; k = 1, \dots, p$ we have,

$$Z_{mt} = \ \ \delta_m + \ \emptyset_{m1} Z_{1t} + \emptyset_{m2} Z_{2t} + \dots + \emptyset_{m(n-1)} Z_{(n-1)t} + \ \emptyset_{1.m1} Z_{1t-1} + \emptyset_{1.m2} Z_{2t-1} + \cdots + \ \emptyset_{1.mn} Y_{nt-1} \ + \emptyset_{2.m1} Z_{1t-2} + \ \emptyset_{2.m2} Z_{2t-2} + \cdots + \ \emptyset_{2.mn} Z_{nt-2} + \cdots + \ \emptyset_{p.m1} Z_{1t-p} + \ \emptyset_{p.m2} \emptyset_{2t-p} + \cdots + \emptyset_{p.mn} Z_{nt-p} + \varepsilon_{mt} \qquad (34)$$

$$Z_{mt} = \delta_m + \sum_{s=1}^{m} \emptyset_{ms} Z_{st} + \sum_{k=1}^{p} \sum_{j=1}^{n} \emptyset_{k.mj} Z_{jt-k} + \ \varepsilon_{mt} \qquad (35)$$

Equations (31), (33) and (35) are a set of Upper Diagonal MARDL Models, and these complete the proof.

Variances of Upper Diagonal MARDL Models

Variances of Z_{it}:

Let Z_{it} in Equation (28) be a stationary process that is distributed about the origin such that $E(Z_{it}) = 0, => \delta_1 = \delta_2 = \cdots = \delta_m = 0$.

Case1: $for\ i = 1,$ multiply Equation (30) by Z_{1t} and take the expectations.

$$E(Z_{1t}Z_{1t}) = E[Z_{1t}(\emptyset_{12}Z_{2t} + \emptyset_{13}Z_{3t} + \ldots + \emptyset_{1n}Z_{nt} + \emptyset_{1.11}Z_{1t-1} + \emptyset_{1.12}Z_{2t-1} + \cdots + \emptyset_{1.1n}Y_{nt-1} + \emptyset_{2.11}Z_{1t-2} + \emptyset_{2.m2}Z_{2t-2} + \cdots + \emptyset_{2.1n}Z_{nt-2} + \cdots + \emptyset_{p.11}Z_{1t-p} + \emptyset_{p.12}\emptyset_{2t-p} + \cdots + \emptyset_{p.1n}Z_{nt-p} + \varepsilon_{1t})]$$

$$\xi_{1t,1t} = \emptyset_{12}\xi_{1t,2t} + \emptyset_{13}\xi_{1t,3t} + \cdots + \emptyset_{1n}\xi_{1t,nt} + \emptyset_{1.11}\xi_{1t,1t(1)} + \emptyset_{1.12}\xi_{1t,2t(1)} + \cdots + \emptyset_{1.1n}\xi_{1t,nt(1)} + \emptyset_{2.11}\xi_{1t,1t(2)} + \emptyset_{2.12}\xi_{1t,2t(2)} + \cdots + \emptyset_{2.1n}\xi_{1t,nt(2)} + \cdots + \emptyset_{p.11}\xi_{1t,1t(p)} + \emptyset_{p.12}\xi_{1t,2t(p)} + \cdots + \emptyset_{p.1n}\xi_{1t,nt(p)} + \sigma^2_{\varepsilon_{1t}} \quad (36)$$

where, $E(Z_{1t}\varepsilon_{1t}) = \sigma^2_{\varepsilon_{1t}}$ (from correlated stationary processes)

$$\xi_{1t,1t} = \sum_{s=1}^{m} \emptyset_{1s}\, \xi_{1s} + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.1j}\, \xi_{1t,jt(k)} + \sigma^2_{\epsilon_{1t}}, \qquad s \neq 1 \quad (37)$$

Case 2: $for\ i = 2, \delta_2 = 0,$ multiply Equation (32) by Z_{2t} and take the expectations.

$$E(Z_{2t}Z_{2t}) = E[Z_{2t}(\emptyset_{21}Z_{1t} + \emptyset_{23}Z_{3t} + \ldots + \emptyset_{2n}Z_{nt} + \emptyset_{1.21}Z_{1t-1} + \emptyset_{1.22}Z_{2t-1} + \cdots + \emptyset_{1.2n}Y_{nt-1} + \emptyset_{2.21}Z_{1t-2} + \emptyset_{2.22}Z_{2t-2} + \cdots + \emptyset_{2.2n}Z_{nt-2} + \cdots + \emptyset_{p.21}Z_{1t-p} + \emptyset_{p.22}\emptyset_{2t-p} + \cdots + \emptyset_{p.2n}Z_{nt-p} + \varepsilon_{2t})]$$

$E(Z_{2t}Z_{2t}) = E(Z^2_{2t}) = \xi_{2t,2t}$ (Variance of Z_{2t})

$$\xi_{2t,2t} = \emptyset_{21}\xi_{2t,1t} + \emptyset_{23}\xi_{2t,3t} + \cdots + \emptyset_{2n}\xi_{2t,nt} + \emptyset_{1.21}\xi_{2t,1t(1)} + \emptyset_{1.22}\xi_{2t,2t(1)} + \cdots + \emptyset_{1.2n}\xi_{2t,nt(1)} + \emptyset_{2.21}\xi_{2t,1t(2)} + \emptyset_{2.22}\xi_{2t,2t(2)} + \cdots + \emptyset_{2.2n}\xi_{2t,nt(2)} + \cdots + \emptyset_{p.21}\xi_{2t,1t(p)} + \emptyset_{p.22}\xi_{2t,2t(p)} + \cdots + \emptyset_{p.2n}\xi_{2t,nt(p)} + \sigma^2_{\varepsilon_{2t}} \quad (38)$$

where, $E(Z_{2t}\varepsilon_{2t}) = \sigma^2_{\varepsilon_{2t}}$ (from correlated stationary processes)

$$\xi_{2t,2t} = \sum_{s=1}^{m} \emptyset_{2s}\, \xi_{2s} + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.2j}\, \xi_{2t,jt(k)} + \sigma^2_{\epsilon_{2t}}, \qquad s \neq 2 \quad (39)$$

Case 3: $for\ i, j = m, n\ (m = n)\ and\ \delta_m = 0,$

multiply Equation (34) by Z_{mt} and take the expectations.

$$E(Z_{mt}Z_{mt}) = E[Z_{mt}(\emptyset_{m1}Z_{1t} + \emptyset_{m2}Z_{2t} + \ldots + \emptyset_{mn}Z_{nt} + \emptyset_{1.m1}Z_{1t-1} + \emptyset_{1.m2}Z_{2t-1} + \cdots + \emptyset_{1.mn}Y_{nt-1} + \emptyset_{2.m1}Z_{1t-2} + \emptyset_{2.m2}Z_{2t-2} + \cdots + \emptyset_{2.mn}Z_{nt-2} + \cdots + \emptyset_{p.m1}Z_{1t-p} + \emptyset_{p.m2}\emptyset_{2t-p} + \cdots + \emptyset_{p.mn}Z_{nt-p} + \varepsilon_{mt})]$$

$E(Z_{mt}Z_{mt}) = E(Z^2_{mt}) = \xi_{mt,mt}$ (Variance of Z_{mt})

$$\xi_{mt,mt} = \emptyset_{m1}\xi_{mt,1t} + \emptyset_{m2}\xi_{mt,2t} + \cdots + \emptyset_{mn}\xi_{mt,nt} + \emptyset_{1.m1}\xi_{mt,1t(1)} + \emptyset_{1.m2}\xi_{mt,2t(1)} + \cdots + \emptyset_{1.mn}\xi_{mt,nt(1)} + \emptyset_{2.m1}\xi_{mt,1t(2)} + \emptyset_{2.m2}\xi_{mt,2t(2)} + \cdots + \emptyset_{2.mn}\xi_{mt,nt(2)} + \cdots + \emptyset_{p.m1}\xi_{mt,1t(p)} + \emptyset_{p.m2}\xi_{mt,2t(p)} + \cdots + \emptyset_{p.mn}\xi_{mt,nt(p)} + \sigma^2_{\varepsilon_{mt}} \quad (40)$$

where, $E(Z_{mt}\varepsilon_{mt}) = \sigma^2_{\varepsilon_{mt}}$ (from correlated stationary processes)

$$\xi_{mt,mt} = \sum_{s=1}^{m-1} \varphi_{ms}\, \xi_{ms} + \sum_{k=1}^{p}\sum_{j=1}^{n} \varphi_{k.mj}\, \xi_{mt,jt(k)} + \sigma^2_{\epsilon_{mt}}, \qquad s \neq m \tag{41}$$

Equations (37), (39) and (41) are upper diagonal variances of $Z_{1t}, Z_{2t}\ and\ Z_{mt}\ respectively$.

ii. Lower Diagonal of MARDL Models and Their Variances

This section considers the conditions for identification of the lower diagonal VAR models from the general form.

From Equation (28), the following set of models is obtained

$$Z_{it} = \begin{cases} \delta_1 + \emptyset_{1s} Z_{st} + \varphi_{k.1j} Z_{jt-k} + \varepsilon_{1t}, i = 1; j = 1; k = 1, \dots, p; (s \neq 1) \\ \delta_2 + \emptyset_{2s} Z_{st} + \varphi_{k.2j} Z_{jt-k} + \varepsilon_{2t}, i = 2; j = 1, 2; k = 1, \dots, p; (s \neq 2) \\ \delta_3 + \emptyset_{3s} Z_{st} + \varphi_{k.3j} Z_{jt-k} + \varepsilon_{3t}, i = 3; j = 1, 2, 3; k = 1, \dots, p; (s \neq 3) \\ \vdots \\ \delta_m + \emptyset_{ms} Z_{st} + \varphi_{k.mj} Z_{jt-k} + \varepsilon_{mt}, \ i = m;\ j = 1, 2, 3, \dots, n; \quad k = 1, \dots, p; (s \neq m) \end{cases} \tag{42}$$

Equation (42) defines a set of Lower Diagonal Multivariate Autoregressive Distributed Lag Models (LDMARDL).

Proof:

Given Equation (28) as

$$Z_{it} = \delta_i + \sum_{s=1}^{m} \emptyset_{is} Z_{st} + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.ij}\, Z_{jt-k} + \varepsilon_{jt}, i = 1, \dots, m\ (i \neq s)$$

Case 1: $if\ i = 1; s = 1, \dots, m\ (s \neq 1); j = 1; k = 1, \dots, p$

$$Z_{1t} = \delta_1 + \emptyset_{12} Z_{2t} + \emptyset_{13} Z_{3t} + \dots + \emptyset_{1n} Z_{nt} + \emptyset_{1.11} Z_{1t-1} + \emptyset_{2.11} Z_{1t-2} + \dots + \emptyset_{p.11} Z_{1t-p} + \varepsilon_{1t} \tag{43}$$

$$Z_{1t} = \delta_1 + \sum_{s=1}^{m} \emptyset_{1s}\, Z_{st} + \sum_{k=1}^{p} \emptyset_{k.11}\, Z_{1t-k} + \varepsilon_{1t} \tag{44}$$

Case 2: $if\ i = 2; s = 1, \dots, m\ (s \neq 2);\ j = 1, 2; k = 1, \dots, p$

$$Z_{2t} = \delta_2 + \emptyset_{21} Z_{1t} + \emptyset_{23} Z_{3t} + \dots + \emptyset_{2n} Z_{nt} + \emptyset_{1.21} Z_{1t-1} + \emptyset_{1.22} Z_{2t-1} + \emptyset_{2.21} Z_{1t-2} + \emptyset_{2.22} Z_{2t-2} + \cdots + \emptyset_{p.21} Z_{1t-p} + \emptyset_{p.22} Z_{2t-p} + \varepsilon_{2t} \tag{45}$$

$$Z_{2t} = \delta_2 + \sum_{s=1}^{m} \emptyset_{2s} Z_{st} + \sum_{k=1}^{p}\sum_{j=1}^{2} \emptyset_{k.2j}\, Z_{jt-k} + \varepsilon_{2t} \tag{46}$$

Case 3: $if\ i = m; s = 1, \dots, m-1 (s \neq m);\ j = 1, 2, \dots, n; k = 1, \dots, p$

$$Z_{mt} = \delta_m + \emptyset_{m1} Z_{1t} + \emptyset_{m2} Z_{2t} + \emptyset_{m3} Z_{3t} + \dots + \emptyset_{m\,(n-1)} Z_{(n-1)t} + \emptyset_{1.m1} Z_{1t-1} + \emptyset_{1.m2} Z_{2t-1} + \cdots + \emptyset_{1.mn} Z_{nt-1} + \emptyset_{2.m1} Z_{1t-2} + \emptyset_{2.m2} Z_{2t-2} + \cdots + \emptyset_{2.mn} Z_{nt-2} + \cdots + \emptyset_{p.m1} Z_{1t-p} + \emptyset_{p.m2} Y_{2t-p} + \cdots + \emptyset_{p.mn} Z_{nt-p} + \varepsilon_{mt} \tag{47}$$

$$Z_{mt} = \delta_m + \sum_{s=1}^{m} \emptyset_{ms} Z_{st} + \sum_{k=1}^{p}\sum_{j=1}^{n} \emptyset_{k.mj}\, Z_{jt-k} + \varepsilon_{mt} \tag{48}$$

Equations (44), (46) and (48) are a set of Lower Diagonal MARDL Models, and these complete the proof.

Variances of Lower Diagonal Multivariate Autoregressive Distributed Lag (MARDL) Models
Variances of Z_{it}:
Let Z_{it} in Equation (32) be a stationary process that is distributed about the origin such that $E(Z_{it}) = 0, => \delta_1 = \delta_2 = \cdots = \delta_m = 0$.
Case1: $for\ i = 1$, multiply Equation (43) by Z_{1t} and take the expectations.

$$E(Z_{1t}Z_{1t}) = E[Z_{1t}(\emptyset_{12}Z_{2t} + \emptyset_{13}Z_{3t} + \ldots + \emptyset_{1n}Z_{nt} + \emptyset_{1.11}Z_{1t-1} + \emptyset_{2.11}Z_{1t-2} + \ldots + \emptyset_{p.11}Z_{1t-p} + \varepsilon_{1t})]$$

Let $\xi_{1t,1t}$ be Variance of Y_{1t}.

$$\xi_{1t,1t} = \emptyset_{12}\xi_{1t,2t} + \emptyset_{13}\xi_{1t,3t} + \ldots + \emptyset_{1n}\xi_{1t,nt} + \emptyset_{1.11}\xi_{1t,1t(1)} + \emptyset_{2.11}\xi_{1t,1t(2)} + \cdots + \emptyset_{p.11}\xi_{1t,1t(p)} + \sigma^2_{\varepsilon_{1t}} \quad (49)$$

where, $E(Z_{1t}\varepsilon_{1t}) = \sigma^2_{\varepsilon_{1t}}$ (from correlated stationary process)

$$\xi_{1t,1t} = \sum_{s=1}^{m} \varphi_{1s}\,\xi_{1s} + \sum_{k=1}^{p} \varphi_{k.11}\xi_{1t,1t(k)} + \sigma^2_{\varepsilon_{1t}} \quad (50)$$

Case 2: $for\ i = 2; j = 1, 2; k = 1, \ldots, p\ and\ \delta_2 = 0$, multiply Equation (45) by Z_{2t} and take the expectations.

$$E(Z_{2t}Z_{2t}) = E[Z_{2t}\big(\emptyset_{21}Z_{1t} + \emptyset_{23}Z_{3t} + \ldots + \emptyset_{2n}Z_{nt} + \emptyset_{1.21}Z_{1t-1} + \emptyset_{1.22}Z_{2t-1} + \emptyset_{2.21}Z_{1t-2} + \emptyset_{2.22}Z_{2t-2} + \cdots + \emptyset_{p.21}Z_{1t-p} + \emptyset_{p.22}Z_{2t-p} + \varepsilon_{2t}\big)]$$

Let $\xi_{2t,2t}$ *be* Variance of Y_{2t}.

$$\xi_{2t,2t} = \varphi_{21}\xi_{2t,1t} + \varphi_{23}\xi_{2t,3t} + \cdots + \boldsymbol{\varphi_{2n}}\xi_{2t,nt} + \boldsymbol{\varphi_{1.21}}\xi_{2t,1t(1)} + \varphi_{1.22}\xi_{2t,2t(1)} + \varphi_{2.21}\xi_{2t,1t(2)} + \varphi_{2.22}\xi_{2t,2t(2)} + \cdots + \varphi_{p.21}\xi_{2t,1t(p)} + \varphi_{p.22}\xi_{2t,2t(p)} + \sigma^2_{\varepsilon_{2t}} \quad (51)$$

where, $E(Z_{2t}\varepsilon_{2t}) = \sigma^2_{\varepsilon_{2t}}$ (from correlated stationary processes)

$$\xi_{2t,2t} = \sum_{s=1}^{m} \varphi_{2s}\,\xi_{2s} + \sum_{k=1}^{p}\sum_{j=1}^{2} \emptyset_{k.2j}\xi_{2t,jt(i)} + \sigma^2_{\varepsilon_{2t}} \quad (52)$$

Case 3: $if\ i = m; j = 1, 2, \ldots, n; k = 1, \ldots, p$ and $\delta_m = 0$, multiply Equation (47) by Z_{mt} and take the expectations.

$$E(Z_{mt}Z_{mt}) = [Z_{mt}\big(\emptyset_{m1}Z_{1t} + \emptyset_{m2}Z_{2t} + \emptyset_{m3}Z_{3t} + \ldots + \emptyset_{m\,(n-1)}Z_{(n-1)t} + \emptyset_{1.m1}Z_{1t-1} + \emptyset_{1.m2}Z_{2t-1} + \cdots + \emptyset_{1.mn}Z_{nt-1} + \emptyset_{2.m1}Z_{1t-2} + \emptyset_{2.m2}Z_{2t-2} + \cdots + \emptyset_{2.mn}Z_{nt-2} + \cdots + \emptyset_{p.m1}Z_{1t-p} + \emptyset_{p.m2}Y_{2t-p} + \cdots + \emptyset_{p.mn}Z_{nt-p} + \varepsilon_{mt}\big)]$$

$E(Z_{mt}Z_{mt}) = E(Z^2_{mt}) = \xi_{mt,mt}$ (Variance of Z_{mt})

$$\xi_{mt,mt} = \emptyset_{m1}\xi_{mt,1t} + \emptyset_{m2}\xi_{mt,2t} + \emptyset_{m3}\xi_{mt,3t} + \cdots + \emptyset_{m(n-1)\xi_{mt,nt(1)}} + \emptyset_{1.m1}\xi_{mt,1t(1)} + \emptyset_{1.m2}\xi_{mt,2t(1)} + \cdots + \emptyset_{1.mn}\xi_{mt,nt(1)} + \emptyset_{2.m1}\xi_{mt,1t(2)} + \emptyset_{2.m2}\xi_{mt,2t(2)} + \cdots + \emptyset_{2.mn}\xi_{mt,nt(2)} + \cdots + \emptyset_{p.m1}\xi_{mt,1t(p)} + \emptyset_{p.m2}\xi_{mt,2t(p)} + \cdots + \emptyset_{p.mn}\xi_{mt,nt(p)} + \sigma^2_{\varepsilon_{mt}} \quad (53)$$

where, $E(Z_{mt}\varepsilon_{mt}) = \sigma^2_{\varepsilon_{mt}}$ (correlated stationary process)

$$\xi_{mt,mt} = \sum_{s=1}^{m} \varphi_{ms}\,\xi_{ms} + \sum_{j=1}^{p}\sum_{k=1}^{n} \varphi_{i.mk}\xi_{mt,kt(i)} + \sigma^2_{\varepsilon_{mt}} \quad (54)$$

Equations (50), (52) and (54) are lower diagonal variances of Z_{1t}, Z_{2t} *and* Z_{mt} *respectively*.

3. Empirical Results

This section considers results obtained from the diagonal VAR and MARDL models. These include variances of the processes and error variances. To validate the models and their variance properties, data on Nigeria Gross Domestic Product, Crude Oil Petroleum, Agricultural production and Telecommunication are obtained for the analysis and estimation of model parameters. The source of data is the CBN Statistical Bulletin with the range from 1988- 2020. The results are presented in Tables below.

Table1: Variances of the Processes from the two Upper and Lower Diagonal Models

VARIABLES	UDVAR	UDMARDL	LDVAR	LDMARDL
GDP	0.0002	0.0003	0.0003	0.0034
C/PETROLEUM	0.0003	0.0027	0.0003	0.0011
AGRIC	0.0009	0.0050	0.0009	0.0009
TELECOM	0.0003	0.0033	0.0014	0.0035

Table2: Variances of the Errors from the two Upper and Lower Diagonal Models

VARIABLES	UDVAR	UDMARDL	LDVAR	LDMARDL
GDP	0.0002	0.0003	0.0003	0.0001
C/PETROLEUM	0.0001	0.0006	0.0002	0.0004
AGRIC	0.0006	0.0003	0.0008	0.0003
TELECOM	0.0023	0.0031	0.0020	0.0015

Discussion and Conclusion

The goal of this work was to identify diagonal vector autoregressive and multivariate autoregressive distributed lag models from generalised models with parameter constraints to the upper and lower diagonal elements of the coefficient matrices. The parameter restriction is required for parameter reduction in accordance with the principle of parsimony, as well as to avoid the appearance of multicollinearity in a multiple relationship between the response and the predictor lag variables. The prerequisites for identification of the diagonal VAR and MARDL models have been identified in this work, and the models' validity has been demonstrated. To compare the performances of the new classes of multivariate lag models, data from certain macroeconomic variables such as Nigeria Gross Domestic Product (GDP), Crude Oil Petroleum (C/PET), Agriculture (AGRIC), and Telecommunication (TELECOM) are used after the first order difference of the logarithm of the series to achieve stationarity. Using the model parameters, the models were estimated, and the variances of the processes and errors were derived. According to the findings as shown in Tables “1” and “2”, the two models have about the same comparative advantage. As a result, the two models complement each other in the modelling of multivariate lag variables.

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