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Analyzing Dependencies in the Electric Vehicle Sector Using the Canonical Vine Copula

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Abstract - In this paper, we analyze the dependence on the electric vehicle industry stocks using the canonical vine(C-vine) copula approach. We have used the most significant EV stock from the Indian stock market (NSE) based on their market capitalization. We have used the top seven companies. We analyze the daily close price of seven Indian EV stocks from December 30, 2015, to December 30, 2023. EV stocks in the analysis are TATAMOTORS, MARUTI, TVSMOTOR, Mahindra & Mahindra, BAJAJ-AUTO, OLECTRA, and EXIDEIND. The C-vine uses different copula families such as Gaussian, Student-t, Frank, Joe, BB1, Gumbel, Clayton and BB8 to model the dependence among EV stocks. Our empirical results show that Tata Motors has a strong positive dependence among other EV stocks, modelled by a C-vine copula with a joint maximum likelihood approach. These results shed light on the tail dependence structures among eclectic vehicle (EV) stocks, particularly, less-studied emerging markets like India that have shown rapid growth in the EV sector in last five-year. Such insights are pivotal for portfolio management strategies and formulating policies at the sector level for investors and policymakers.

***Keywords*:** Canonical Vine Copula (C-vine), Dependence, GARCH Model, Electric Vehicle (EV), Indian stock market (NSE)

1. Introduction

The electric vehicle sector has expanded significantly in the last five years in the Indian market. As of October 2023, the Indian government said 3 million Electric Vehicles (EVs) were registered in the country. But this number is projected to rise quickly, at a rate of 49% per year, until 2030. This will benefit the economy and the net-zero emissions goals over the next few decades. For this growth, many mini and micro-mobility vehicles must run on electricity or some other renewable energy source. The rising significance of the electric vehicle industry in the Indian economy is also seen in its growing influence on Indian stock market indices. Comprehending and measuring the relationship between financial factors is crucial in financial modelling due to their significant impact on the allocation of assets, management of risks, trading techniques, and policy creation [1]. The Pearson correlation test is commonly employed to quantify the relationship among random variables. However, it solely measures linear dependence and is most appropriate when dealing with the elliptical distribution of attribute returns. Nevertheless, the distribution of returns for financial variables does not follow an elliptical pattern. Instead, it displays distinct characteristics such as non-linearity, variation, and asymmetric effects. Consequently, relying on pearson correlations to assess dependence can be misleading [2]. If the return exhibits non-linear trends in asymmetric behaviour, it necessitates the utilization of advanced models. Studies on the correlation between financial asset returns typically concentrate on the interdependence of tails during times of strain and exceptional occurrences. According to Ayusuk et al. [3], the correlation between financial asset returns increases significantly during market crashes compared to periods of economic growth. Copula-based assessments address these limitations by enabling the modelling of both linear and non-linear dependence using any desired marginal distribution [4], even when the measurements exhibit skewed behaviour [5]. Copulas are used to analyze dependence and tail dependence of financial and non-financial markets, such as stocks, oil, and gold [6]. Vines are structures that can be used to model complex relationship patterns. They can do this by using an extensive collection of bivariate copulas. Vine tree allows for the flexible modelling of intricate dependence patterns by utilizing a diverse selection of bivariate copulas [7]. Nikoloulopoulos et al. [8] revealed that vine copulas derived from the bivariate t-copula family are optimal for multivariate financial asset return series. Lozza et al. [9] implemented R-vine copulas and found sufficient differences in appreciation and depreciation of the exchange rate. Hernandez et al. [10] employed the powerful R(regular)-, C(canonical)-, and D(drawable)-vine copulas to assess dependence during the global financial crisis comprehensively. Kumar et al. [11] used C- and R-vine copulas to analyze the conditional multivariate dependence structure in the Chinese context.

In this paper, we utilize a C-vine copula technique to understand the tail dependence in log returns of EV stocks. C-vine copulas represent a unique framework for modelling the unique characteristics of marginal and joint distributions, including kurtosis, skewness, non-linearity, and asymmetric dependency. The interdependence of the seven electric vehicle (EV) stocks is represented using the C-vines model. The significant findings of our study reveal that the correlation between the seven stocks is represented by a wide variety of copula families such as Gaussian, Student-t, Frank, Joe, BB1, Gumbel, Clayton and BB8 to model the dependencies. When comparing different C-vine copula models using the information criteria (AIC/BIC) method, our result shows that the C-vine copula with joint maximum likelihood estimation (MLE) is the most suitable model for analyzing the dependence of EV sector stocks in the Indian stock market.

2. Methodology

2.1. Data

With the dependence on the electric vehicle (EV) sector, we mainly analyze the Indian stock market (NSE) based EV companies. We have used the top seven EV stocks based on market capitalization. We analyze the daily close price of seven Indian EV stocks from December 30, 2015, to December 30, 2023. The EV stocks used in the analysis are TATAMOTORS, MARUTI, TVSMOTOR, Mahindra & Mahindra, BAJAJ-AUTO, OLECTRA, and EXIDEIND. All the EV stock data is collected from Yahoo Finance using the ticker TATAMOTORS.NS, MARUTI.NS, TVSMOTOR.NS, M&M.NS, BAJAJ-AUTO.NS, OLECTRA.NS, EXIDEIND.NS. EV stock returns are determined by calculating the logarithmic difference between two consecutive close prices. This process resulted in 1976 daily log returns observations for seven EV stocks.

2.2. GARCH Model

The log-return data has high kurtosis, indicating the presence of heavy tails. Skewness is a prevalent occurrence in numerous scenarios. The fluctuation of returns is inconsistent and may demonstrate the tendency for large and minor changes to occur in clusters within the data. The initial stage involves accurately modelling normalized marginal distributions to incorporate ARCH effects [12]. We applied a GARCH (1,1) model [13] with standardized student-t to each of the seven EV stocks. This model appears to be the most suitable for the marginal density. Let the EV stocks log-returns series $Y_t, t = 1, \ldots, T$ with conditional volatility σ_t , at time t, is given as the GARCH model in Eqs. (1) - (2),

$$Y_t = \sigma_t Z_t \tag{1}$$

$$\sigma_t = \omega + \alpha Y_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

where coefficients $\omega > 0, \alpha \geq 0, \beta \geq 0$, $Z_t \sim iid(0,1)$ is student-t with mean 0 and variance 1.

2.3. Canonical vine copula(C-Vine)

Sklar theorem states that "any multivariate distribution function can be expressed as a copula function of its marginal distributions" [14]. In the last decade, copulas have been used very frequently in the financial industry [15]. In the subsequent phase of our approach, we proceed by employing the marginal data to choose copula functions and construct vine structures. These structures are called vine. Vine constructions are formed by pairing marginal densities, d, resulting in $\frac{d(d-1)}{2}$ pair copulas that are organized into $d-1$ trees [7]. The initial primary(root) node establishes the dependence by utilizing a single variable incorporating copulas for every pair. We utilize this node to represent the interdependence between the subsequent nodes and the second primary node in our model. For each tree, a root node is chosen based on the strongest dependencies among nodes. As a result, C-vine trees construct a star-like structure. Here, we use the expression [16] in Eq. (3) for C-vine density for root nodes $1 \cdots d$:

$$f(x) = \prod_{k=1}^{d} f_k(x_k) \times \prod_{i=1}^{d-1} \prod_{j=1}^{d-i} c_{i,i+j|1,\ldots,(i-1)} \left(F(x_i|x_1,\cdots,x_{i-1}), F\left(x_{i+j}\middle|x_1,\cdots,x_{i-1}\right)\right) \tag{3}$$

where the marginal densities f_k represent the individual marginal distributions of each variable, and the product symbolizes the joint density of all variables. The subsequent product terms involve bivariate copula densities $c_{i,i+j|1\cdots(i-1)}$ that model the dependence between pairs of variables in each tree. This methodology assures us that we may make well-informed decisions based on dependable facts, resulting in enhanced precision and more efficient results. The objective is to represent the most important interconnections in the early trees appropriately. Joe et al. [17] have provided valuable insights demonstrating how vine copulas can exhibit tail dependence in all bivariate margins. Their research highlights that such dependence can be achieved by ensuring that the bivariate copulas in the first tree exhibit tail dependence. By employing this method, one can successfully omit the difference in the following trees. The sequential and joint maximum likelihood (MLE) parameter estimation method is based on Kendall's τ for vine model selection [18,19]. To reduce the model complexity, we use different families of pair copulas. The copulas are meticulously picked to align with the selected tree. They can accurately represent positive and negative dependency, asymmetry in the tails, as well as lower and upper tail dependence [20]. Before creating the models, we always perform a bivariate independence test. This helps us obtain simpler and more easily manageable models.

3. Empirical Result

To estimate the dependence among EV stocks, we have used a C-vine copula that identifies the representative stocks in the EV sector. The first C-Vine tree structure and correlation among seven EV stocks with corresponding pair and normalized contour plots with the order of tree node as follows: 1 (TATAMOTORS), 4 (OLECTRA), 3 (TVSMOTOR), 5 (EXIDEIND), 6 (M.M), 2 (MARUTI), 7 (BAJAJ.AUTO) is shown in Fig. 1.

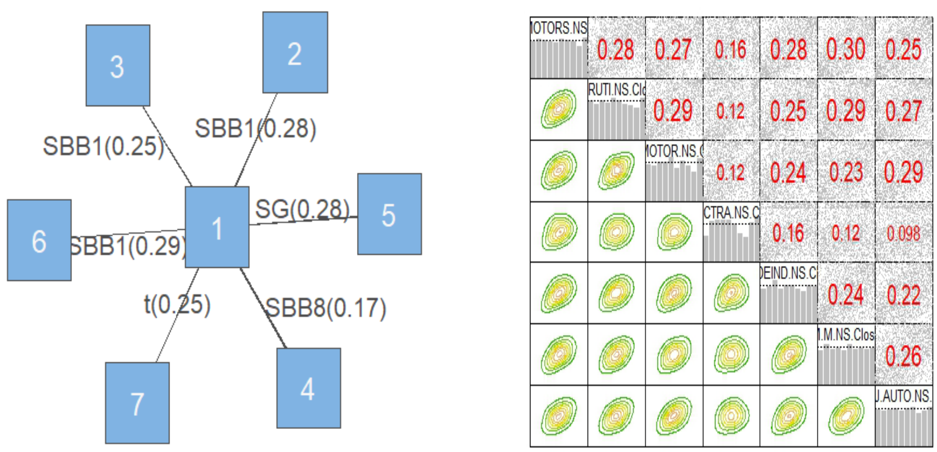

Fig 1: C-vine tree-1 structure (left) and Copula data pair plots, histograms of copula margins, and normalized contour plots for EV sector companies (right).

In the first C-vine tree, we see that Tata Motors shows strong positive correlations among EV stocks, with Kendall's τ value from 0.098 to 0.30. The normalized contour forms clearly show positive dependence, and the shapes imply that the student-t copula is present for bivariate pairs. The primary node 1 (TATAMOTORS) in the first C-vine tree is selected to maximize Kendall's tau value on this particular node. We have used copula families for selection purposes, such as BB1, BB8, Gaussian, Frank, Student-t, Gumbel, etc. Fig. 2 shows all C-vine tree structures with their corresponding copula families with Kendall's τ. Table 1 contains a complete collection of estimated parameters for C-vine specification. We compare C-vine models using sequential and maximum likelihood methods. We analyze C-vine models based on information criteria using AIC and BIC values [21], along with the estimated loglikelihood and number of parameters, in Table 2. Furthermore, we examine the loglikelihood values of sequential (seq) and maximum likelihood (mle) methods. It is observed that the loglikelihood values of sequential and maximum likelihood methods are almost identical.

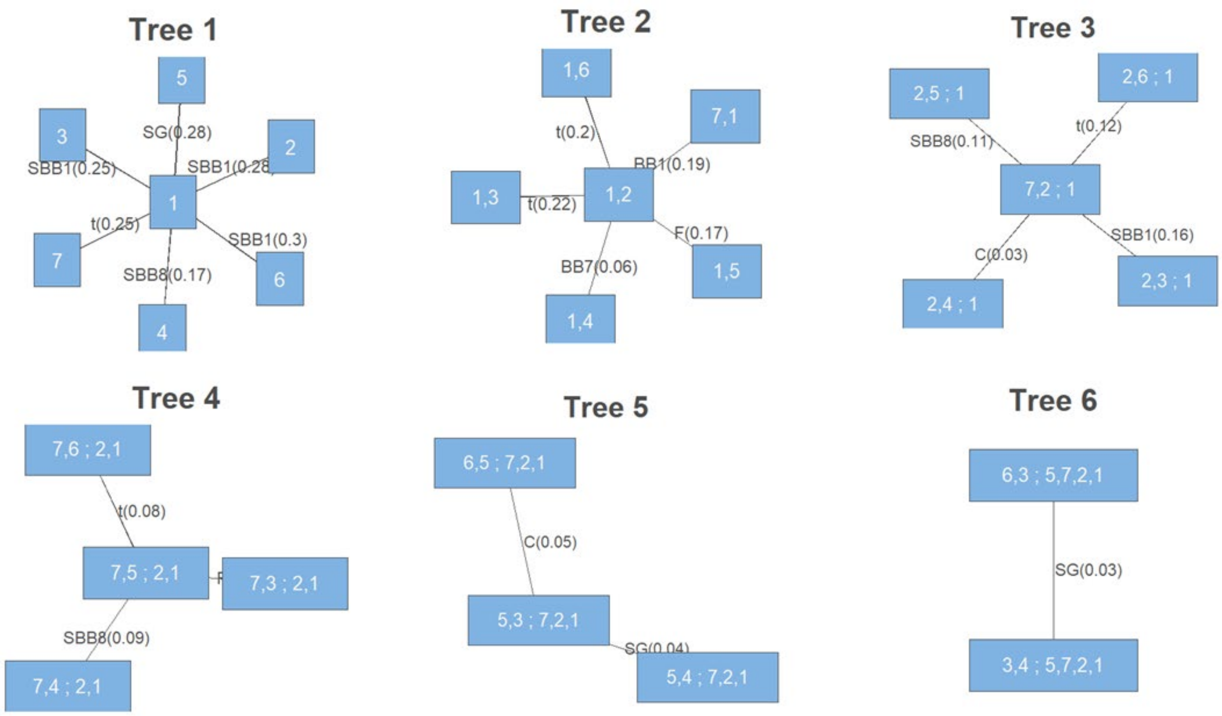

Fig. 2: C-vine tree structure (node symbols: 1(TATAMOTORS), 2(MARUTI), 3 (TVSMOTOR), 4 (OLECTRA), 5(EXIDEIND), 6(M.M), 7(BAJAJ.AUTO)).

Table 1: C-vine copula parameter estimates, Kendall's τ , and upper/lower tail dependence(utd/ltd) with copula families and node symbols: 1(TATAMOTORS), 2(MARUTI), 3 (TVSMOTOR), 4 (OLECTRA), 5(EXIDEIND), 6(M.M), 7(BAJAJ.AUTO).

Tree	Edge	Family	Copula	Par 1	Par 2	tau	utd	ltd
1	1,4	20	Survival BB8	1.43	0.97	0.17	-	-
	1,3	17	Survival BB1	0.1	1.28	0.25	0.00	0.29
	1,5	14	Survival G	1.39	0.00	0.28	-	0.35
	1,6	17	Survival BB1	0.16	1.32	0.3	0.03	0.31
	1,2	17	Survival BB1	0.08	1.33	0.28	0.00	0.32
	1,7	2	Student-t	0.38	11.19	0.25	0.05	0.05
2	2,4;1	9	BB7	1.03	0.1	0.06	0.04	0.00
	2,3;1	2	Student-t	0.34	11.81	0.22	0.04	0.04
	2,5;1	5	Frank	1.56	0.00	0.17	-	-
	2,6;1	2	Student-t	0.31	17.76	0.2	0.01	0.01
	2,7;1	7	BB1	0.12	1.16	0.19	0.18	0.01
3	7,4;2,1	3	Clayton	0.06	0.00	0.03	-	0.00
	7,3;2,1	17	Survival BB1	0.1	1.13	0.16	0.00	0.15
	7,5;2,1	20	Survival BB8	1.36	0.91	0.11	-	-
	7,6;2,1	2	Student-t	0.19	29.07	0.12	0.00	0.00
4	5,4;7,2,1	20	Survival BB8	1.48	0.76	0.09	-	-
	5,3;7,2,1	5	Frank	0.75	0.00	0.08	-	-
	6,5;7,2,1	2	Student-t	0.12	14.1	0.08	0.00	0.00
5	3,4;5,7,2,1	14	Survival G	1.04	0.00	0.04	-	0.05
	6,3;5,7,2,1	3	Clayton	0.11	0.00	0.05	-	0.00
6	6,4;3,5,7,2,1	14	Survival G	1.03	0.00	0.03	-	0.04

However, we also conducted an asymptotic independence (ind) test, slightly decreasing the number of parameters. The results of our study indicate that C-vine models with maximum likelihood estimation (mle) based on all copula families, which one has the minimum AIC and BIC values, is the best C-vine model for dependence. We also perform the model test with the student-t copula.

Table 2: C-vine model comparison.

C-Vine copula model	Loglikelihood	Parameter	AIC	BIC
C-vine-seq-all	1720	35	-3371	-3175
C-vine-mle-all	1721	35	-3373	-3177
C-vine-ind-seq-all	1711	32	-3358	-3179
C-vine-ind-mle-all	1712	32	-3360	-3181
C-vine-seq-t	1720	35	-3371	-3175
C-vine-mle-t	1721	35	-3372	-3177
C-vine-ind-seq-t	1711	32	-3358	-3179
C-vine-ind-seq-t	1712	32	-3360	-3181

4. Conclusion

The electric vehicle sector has distinct peculiarities compared to other industry sectors. In the past decade, the Indian market has experienced significant expansion in EV startups and companies. This has recently garnered considerable attention from both academic researchers and industry professionals. This article offers a distinct viewpoint by analyzing the interdependence within the electric vehicle (EV) sector industry, specifically focusing on more than five years. Our findings suggest a significant positive correlation within the electric vehicle (EV) industry. The C-Vine specification is chosen as the optimal dependence model using a joint maximum likelihood estimate technique. TATAMOTORS is represented in the central node, showing strong positive dependence on the other six EV stocks. The conclusions are significant for money managers and investors seeking to optimize their portfolios. Nevertheless, it is essential to acknowledge that examining specific firms reveals the substantial diversity within the electric vehicle industry.

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