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Model of Time Series with Asymmetric Persistence and Procedure of its Reconstruction from Data

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Extended Abstract

A number of econometric studies have demonstrated asymmetric dependences - there are often stronger correlations between financial objects when the market is going down than when it is going up. Some mechanisms that are sources of asymmetric correlations may also be present in a variety of geophysical, physical and biological processes. The existence of asymmetric correlations was demonstrated in the sunspot sequence and in the northern hemisphere temperature time series [1].

In order to describe the behavior of econometric data a number of models with asymmetric correlations were proposed [2]. In most cases these were appropriate generalizations of nonlinear models: TAR, Markov switching model and GARCH. In the first two, nonlinearities are introduced into the drift and in the third into the diffusion term. In this study, we propose a model based on the modified Langevin equation, because then both terms, drift and diffusion, can have nonlinear forms. Moreover, the Langevin equation is widely used in many fields of science and has a simple physical interpretation.

In order to introduce the asymmetric persistence into the discrete form of the standard Langevin model we assume that the next state $y(k+1)$ of the time series will depend not only on the current state $y(k)$ (as in Markov process) but also on the sign of previous increment, $d(k) = y(k) - y(k\text{-}1)$. Therefore, a new factor $c(d(k))$ that determines the sign of the diffusion term is introduced. If $d(k) > 0$ then with probability p the function $c = 1$ (or $c = -1$ with probability $1\text{-}p$) while if $d(k) < 0$ than $c = -1$ with probability q (or $c = 1$ with probability $1\text{-}q$). Parameters p and q determine the degree of persistence in upward and downward trends respectively.

For applications the model of time series should be completed by a procedure for reconstructing the model from the data. Following the method presented in Ref. [3] a system of two implicit nonlinear equations for the initial drift and diffusion functions has been derived. The system reduces to a fourth-degree algebraic equation and therefore may be solved analytically.

The procedure was tested on stationary time series generated by our model, assuming various linear and nonlinear forms of drift and diffusion functions and various values of the persistence parameters p and q. In all cases, the procedure correctly reconstructed the assumed forms of drift and diffusion functions. The procedure was also applied to a geophysical time series, obtaining a stochastic model of the studied process and asymmetric persistence parameters.

References

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