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A Novel Optimised Fractional Hausdorff Grey Model with Time Power Term (OFHGM$(1, 1, t^{\alpha})$) for Forecasting Greenhouse Gas Emissions

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Abstract-In this paper, we introduce a novel optimised Fourier-Markov discrete fractional Hausdorff grey model with a time power term (OFHGM$(1, 1, t^{\alpha})$) to model and forecast greenhouse gas (GHG) emissions. The Black Hole Optimization (BHO) algorithm is employed to obtain optimal parameters for the fractional order and the time-power coefficient. Numerical implementations already demonstrate promising results and we further improve the forecasting accuracy of the discrete fractional HGM $(1, 1, t^{\alpha})$ by employing the Fourier and Markov approaches. The performance of the OFHGM $(1, 1, t^{\alpha})$ developed is tested by modelling and forecasting GHG emissions. Further, a comparison with the modelling and forecasting performance against other grey forecasting models supports the superiority of the proposed OFHGM $(1, 1, t^{\alpha})$.

Keywords: Fractional Grey Model, Black Hole Optimization, Fractional Order Accumulating Operator, Greenhouse Gas Emission Forecasting.

1. Introduction

The production and consumption of energy is one of the major sources of anthropogenic greenhouse gas (GHG) emissions which accounts for around 78% of GHG emissions globally [1]. GHG are natural gases resulting from human-induced activities through production and consumption. They contribute directly or indirectly to global warming. Some main GHG are carbon dioxide (CO_2), methane (CH_4) and nitrous oxide (N_2O). According to Statistics Mauritius, the GHG emissions has increased by 3.1% from 2021 to 2022, with gross emissions increasing from 5,472 to 5,642 thousand tonnes of carbon dioxide (CO_2) equivalent and net emissions, after absorption by forest and land use practices from 5,136 to 5,308 thousand tonnes carbon dioxide equivalent [2]. Mauritius aims to reduce the overall GHG emissions by 40% in 2030, compared to the Business as Usual (BAU) scenario of around 6,900 kilotons (kt) CO_2 equivalent (including Land use, land-use change, and forestry (LULUCF)) in 2030 [3]. The rapid increase of atmospheric GHG concentrations remain the primary contributor to climate change [4], which has a great impact to the socio-economic plans and developments of a country [5]. Therefore, accurate forecasting of GHG emissions plays a significant role in the development of suitable energy policies and combating climate change. Different grey approaches for greenhouse gas emissions forecast have been proposed by scholars globally.

The GM $(1, 1)$ [6] is a time series model for predicting short-term problems and achieves satisfactory results with only a few data points [7], for exponentially increasing datasets. The GM $(1, 1)$ has been applied to forecast GHG emissions [8, 9], fuel production [10, 11] and energy consumption [12, 13]. Throughout the years, many scholars have put forward numerous ways and tools to improve the forecasting accuracy of the grey models. Wang and Li [14] proposed a non-equigap grey Verhulst model (NE grey Verhulst model) for forecasting CO_2 emissions and GDP per capita in China from 1990 to 2014. The model's coefficient was optimised using particle swarm optimization (PSO) algorithm. Xu et al. [15] proposed an adaptive grey model combined with a buffered rolling method (BR-AGM $(1, 1)$) to forecast China's greenhouse gas emissions from 2017 to 2025. Furthermore, Adarkwa et al. [16] proposed a Verhulst-GM $(1, N)$ model and emissions technical conversion to forecast the actual cement industry CO_2 emissions data from 2005 to 2018. Pao et al. [17] presented a nonlinear grey Bernoulli model (NGBM) to predict China's compound annual emissions, energy consumption and real GDP growth between 2011 and 2020, where a numerical iterative method was used to optimise the parameter of the proposed model. With the aim to avoid the over fitting problem, Xie et al. [18] proposed a robust reweighted multivariate grey model (RWGM $(1, N)$) to forecast the GHG emissions in European Union (EU) member countries from 2010 to 2016. The least absolute shrinkage and selection operator (LASSO) regression was considered to simplify the model through constraining the number of predictors. The result showed that the RWGM $(1, N)$ performed better than the GM $(1, N)$ and RGM $(1, N)$.

Among many others, is the grey model with the fractional order accumulation which was initially proposed by Wu et al. [19], used an r-order accumulated generating operation (r-AGO) called fractional order. Fractional grey models have been applied to many fields, especially in energy and environment forecasting [20, 21]. For instance, Wu et al. [22] proposed a fractional grey model FAGMO $(1, 1, k)$ to forecast China's nuclear energy consumption. On the other hand, Yuan et al. [23] presented a grey power model (GPM $(1, 1)$) based on a fractional order GPM $(1, 1)$ to forecast Wuhan's industry water consumption. Xin et al. [24] proposed a fractional time delayed grey model to forecast natural gas and coal consumption in China. Results of previous studies showed that optimising the parameter of fractional grey models can greatly enhance the prediction performance of grey models [25]. To further improve the applicability of the existing fractional grey models, Sahin [26] proposed an optimised fractional nonlinear grey Bernoulli model with rolling mechanism (ROFANGBM $(1, 1)$) under pre-pandemic and post pandemic scenarios. The author's aim was to predict the share of renewable in primary energy consumption and CO_2 emissions of the United States and China. Gao et al. [27] applied a fractional grey Riccati model (FGRM $(1, 1)$) for predicting CO_2 emissions, where the time response sequence of FGRM $(1, 1)$ was calculated using the Vieta's formulas and the bare bone fireworks algorithm was used to optimise the model. Moreover, Xin et al. [28] proposed a new information priority accumulation method into the grey forecasting model with the hyperbolic sinusoidal driving term NISinHGM $(1, 1)$ to predict the urban natural gas supply of China. The Whale Optimization Algorithm (WOA) was used to determine the optimal parameter of the new information accumulated priority.

This paper offers the following contributions:

1. Following the discretization technique [29], a novel discrete fractional Hausdorff grey model with time power term (DFHGM $(1, 1, t^{\alpha})$) is proposed, by adjusting the fractional order r and time power coefficient α. We also propose a novel FHGM $(1, 1, t^{\alpha})$ based on Fourier-Markov approaches to enhance the forecasting accuracy of the DFHGM $(1, 1, t^{\alpha})$.
2. The Black Hole Optimization (BHO) algorithm [30] is used to determine optimal parameter values for r and α.
3. The effectiveness of the proposed models is evaluated against other benchmark approaches and also in fitting Mauritius's GHG gas emissions from 2001 to 2020.

The rest of this paper is organized as follows: Section 2 presents the methodology of the FHGM $(1, 1)$, DFHGM $(1, 1, t^{\alpha})$, FDFHGM $(1, 1, t^{\alpha})$ and OFHGM $(1, 1, t^{\alpha})$. In section 3, the application and comparison of the proposed models, with numerical implementations are presented for the simulation and prediction of Mauritius's GHG emissions. Finally, section 4 concludes the paper.

2. Methodology

In this section, the grey models as well as the optimization technique used to determine the optimal parameters are discussed.

2.1. Fractional Hausdorff grey model (FHGM $(1, 1)$)

The purpose of the accumulated generating operator (AGO) in the GM $(1, 1)$ is to improve the smoothness of the fitting sequence. The AGO method decreases the noise of the original sequence and the randomness of the data [31]. Among the many improvements made to the GM $(1, 1)$, improving the grey accumulated generating operator (AGO) is one of them. Since the advent of fractional order, several researchers have proposed different types of fractional order accumulated generating operators. For instance, Chen et al. [32] proposed a fractional Hausdorff grey model (FHGM $(1, 1)$) with r-order accumulated generating operator (r-AGO). FHGM $(1, 1)$ is the GM $(1, 1)$ when $r = 1$. The modelling process of the FHGM $(1, 1)$ is defined as follows:

We define the original data sequence as $X^{(0)} = \{x^{(0)}(k)\}_{k=1}^{n}$ and construct the r-order accumulation sequence $X^{(r)} = \{x^{(r)}(k)\}_{k=1}^{n}$, where

$$x^{(r)}(k) = \sum_{i=1}^{k} x^{(0)}(i)\left[i^{r} - (1 - r)^{r}\right], \quad k = 1, 2, \dots, n, \tag{1}$$

where r is the fractional order. We define $Z^{(r)} = \{z^{(r)}(k)\}_{k=2}^{n}$, where

$$z^{(1)}(k) = \frac{1}{2}\left[x^{(1)}(k) + x^{(1)}(k-1)\right],\ k = 2, 3, ..., n. \tag{2}$$

The differential equation of the FHGM $(1, 1)$ is represented by

$$\frac{dx^{(r)}}{dt} + ax^{(1)} = b, \tag{3}$$

where a is the developing coefficient and b is the grey input. Setting $\hat{a} = [a, b]^T$ and

$$B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{pmatrix},\ Y = \begin{pmatrix} x^{(1)}(2) - x^{(1)}(1) \\ x^{(1)}(3) - x^{(1)}(2) \\ \vdots \\ x^{(1)}(n) - x^{(1)}(n-1) \end{pmatrix}, \tag{4}$$

the least squares the least squares estimate sequence of the FHGM $(1, 1)$ satisfies

$$\hat{a} = \left(B^T B\right)^{-1} B^T Y. \tag{5}$$

The solution to equation (3) is given by

$$\hat{x}^{(1)}(k) = \left(x^{(1)}(1) - \frac{b}{a}\right)e^{-a(k-1)} + \frac{b}{a},\ \ k = 2, 3, ..., n, \tag{6}$$

and the restored values of $\hat{x}^{(0)}(k)$ are given by

$$\hat{x}^{(0)}(k) = \begin{cases} x^{(1)}(k), & \text{for } k = 1. \\ \dfrac{\hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1)}{k^r - (k-1)^r}, & \text{for } k = 2, 3, ..., n. \end{cases} \tag{7}$$

2.2. The Discrete Fractional Hausdorff Grey Model with Time Power Term (DFHGM $(1, 1, t^{\alpha})$)

To enhance the adaptability of the grey model to diverse sequences, a time power term is incorporated into the grey model [29]. Moreover, given the advantages of the discrete fractional grey model [33], we propose a discrete fractional Hausdorff grey model with time power term, called DFHGM $(1, 1, t^{\alpha})$.The computational steps are summarized as follows:

The differential equation of the proposed model is given by

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = bt^{\alpha} + c, \tag{8}$$

where a is the development coefficient, $bt^{\alpha} + c$ is the grey action quantity and α is the time power term. By using the trapezoid formula, equation (8) can be rewritten as

$$x^{(1)}(k) - x^{(1)}(k-1) + az^{(1)}(k) = bk^{\alpha} + c. \tag{9}$$

Setting $\hat{a} = [a, b, c]^T$ and

$$B = \begin{pmatrix} -z^{(1)}(2) & 2^{\alpha} & 1 \\ -z^{(1)}(3) & 3^{\alpha} & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & n^{\alpha} & 1 \end{pmatrix},\ Y = \begin{pmatrix} x^{(1)}(2) - x^{(1)}(1) \\ x^{(1)}(3) - x^{(1)}(2) \\ \vdots \\ x^{(1)}(n) - x^{(1)}(n-1) \end{pmatrix}, \tag{10}$$

the least squares estimate sequence of the DFHGM $(1, 1, t^{\alpha})$ satisfies equation (5). The time discrete response function of equation (9) is given by

$$\hat{x}^{(1)}(k) = u_1^{k-1} x^{r}(1) + \sum_{i=0}^{k-2} u_1^i\left[(k-i)^{\alpha} u_2 + u_3\right],\ k = 2, 3, ..., n, \tag{11}$$

where $u_1 = \frac{1-0.5a}{1+0.5a}$, $u_2 = \frac{b}{1+0.5a}$, and $u_3 = \frac{c}{1+0.5a}$. The restored values $\hat{x}^{(0)}(k)$ are obtained using equation (7).

2.3. Parameter Optimization

Many studies have shown that the model parameters have a major impact on the forecasting accuracy of the grey model. We construct a simple optimization problem for searching the optimal parameters for the proposed models, given as

$$\min avg(error(i)) = \frac{1}{n-1}\sum_{i=2}^{n}\frac{\left|\hat{x}^{(0)}(i) - x^{(0)}(i)\right|}{x^{(0)}(i)} \times 100\,\%,$$

$$s.t\begin{cases} \hat{x}^{(r)}(k) = u_1^{k-1}x^r(1) + \sum_{i=0}^{k-2} u_1^i\left[(k-i)^{\alpha}u_2 + u_3\right] \\ \hat{a} = \left[a, b, c\right]^T = \left(B^TB\right)^{-1}B^TY, \end{cases} \tag{12}$$

where $x^{(0)}(i)$ are the actual values and $\hat{x}^{(0)}(i)$ are the predicted values. Due to its nonlinear characteristics and complexity, we propose to solve equation (12) using a metaheuristic algorithm, namely Black Hole Optimization Algorithm (BHO) [30] to determine the optimal parameters of the models.

2.4. Fourier Discrete Fractional Hausdorff Grey Model with Time Power Term (FDFHGM $(1, 1, t^{\alpha})$)

The Fourier series [34] is used to filter out the high-frequency terms (noise) and consider the low-frequency error values of a model. Therefore, in order to improve the forecasting accuracy of the DFHGM $(1, 1, t^{\alpha})$, this study uses the Fourier series to modify the residual of the model. We define the residual series as $E^{(0)} = \{\varepsilon^{(0)}(k)\}_{k=2}^{n}$, where

$$\varepsilon^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k), \ k = 2, 3, \ldots, n.$$

We use the Fourier series for modifying the error values of the DFHGM $(1, 1, t^{\alpha})$ using the following equation

$$\hat{\varepsilon}^{(0)}(k) = \frac{1}{2}a_0 + \sum_{i=1}^{Z}\left[a_i cos\left(\frac{2\pi i}{n-1}\right)k + b_i sin\left(\frac{2\pi i}{n-1}\right)k\right], \ k = 1, 2, \ldots, n,$$

where $Z = \frac{n-1}{2} - 1$ which is the minimum deployment frequency of the Fourier series and can only take integer values. The forecasting series of the DFHGM $(1, 1, t^{\alpha})$ can be modified as

$$\hat{x}_{\varepsilon}^{(0)}(k) = \begin{cases} x^{(0)}(k), & \text{for } k = 1. \\ \hat{x}^{(0)}(k) + \hat{\varepsilon}^{(0)}(k), & \text{for } k = 2, 3, \ldots, \end{cases}$$

where $\hat{x}_{\varepsilon}^{(0)}(k)$ represents the restored value of the FDFHGM $(1, 1, t^{\alpha})$ and $\hat{x}^{(0)}(k)$ is the restored value from the DFHGM $(1, 1, t^{\alpha})$.

2.5. The Optimised Fractional Hausdorff Grey Model with Time Power Term (OFHGM $(1, 1, t^{\alpha})$)

To further improve the forecasting accuracy of the FDFHGM $(1, 1, t^{\alpha})$, we introduce the Fourier-Markov DFHGM $(1, 1, t^{\alpha})$. The original data are first modelled by the FDFHGM $(1, 1, t^{\alpha})$ and then the residual errors between the predicted values and the actual values for all previous time steps are obtained. The model establishes the transition behaviour of those residual errors by Markov transition matrices and then the possible correction for the predicted value can be made from those Markov matrices [35]. According to the relative error between the predicted value and the actual value of the grey model, the relative error is divided in S states. We define S states for each time step where each state is an interval whose width is equal to a fixed portion of the range between the maximum and minimum of the residual errors. Each state S_{ij} has a boundary represented by $S_{ij} = \left[L_{ij}, U_{ij}\right]$, where L_{ij} and U_{ij} are the lower bound and upper bound of the j^{th} state for the i^{th} time step of the residual error series. After determining the state that has the greatest probability, then the boundary of the state will be used to predict the future data using

$$\hat{x}_m^{(0)}(k) = \hat{x}_{\varepsilon}^{(0)}(k)\left[1 + 0.5\left(L_{ij} + U_{ij}\right)\right].$$

2.6. Model-Performance Metrics

To comprehensively evaluate the performance of the models discussed in this paper, we examine the error associated with model fitting. The metrics considered are the mean absolute percentage error (MAPE) and the root mean square error (RMSE) given by:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{X_i - Y_i}{X_i}\right| \times 100 \text{ and } RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_i - Y_i)^2}{n}},$$

where n is the data sample size, X_i are the actual values and Y_i are the predicted values.

3. Results

3.1. Test Case Scenario 1: Total greenhouse gas emissions (GHG)

Forecasting total greenhouse gas (GHG) emissions in Mauritius is vital, especially, with the island being a tourist destination. As such, effective climate strategies can be set and future emission trends modelled. The government can set realistic reduction targets and prioritize actions in key sectors like energy, transportation, and agriculture. Accurate forecasting also helps to allocate the island's limited resources efficiently for cleaner technologies and renewable energy initiatives. Early identification of potential emission gaps is allowed, ensuring timely interventions to stay on track. Moreover, GHG forecasts aid Mauritius in meeting its international reporting obligations, enhancing climate resilience, and preparing for the impacts of climate change on vulnerable sectors such as tourism and agriculture.

As a first test case, we consider the total greenhouse gas emissions (GHG) [36] for the period 2001 to 2020 in Mauritius to train the models, as was discussed in Section 2. A decrease in the MAPE from 3.272% for the GM $(1, 1)$ to 0.051% for the OFHGM $(1, 1, t^{\alpha})$ can be observed in Table 1. According to Figure 1, the OFHGM $(1, 1, t^{\alpha})$ closely follows the actual data throughout the entire period. The OFHGM $(1, 1, t^{\alpha})$ appears to be the most accurate model in predicting the actual gas emissions. We also observe that the GM $(1, 1)$ fails to conform to the actual data.

Table 1: The performance of the models based on MAPE and RMSE for total greenhouse gas emissions (GHG).

	GM $(1, 1)$	DFHGM $(1, 1, t^{\alpha})$	FDFHGM $(1, 1, t^{\alpha})$	OFHGM $(1, 1, t^{\alpha})$
MAPE	3.272	1.534	0.393	0.051
RMSE	0.1420	0.0737	0.0153	0.0021

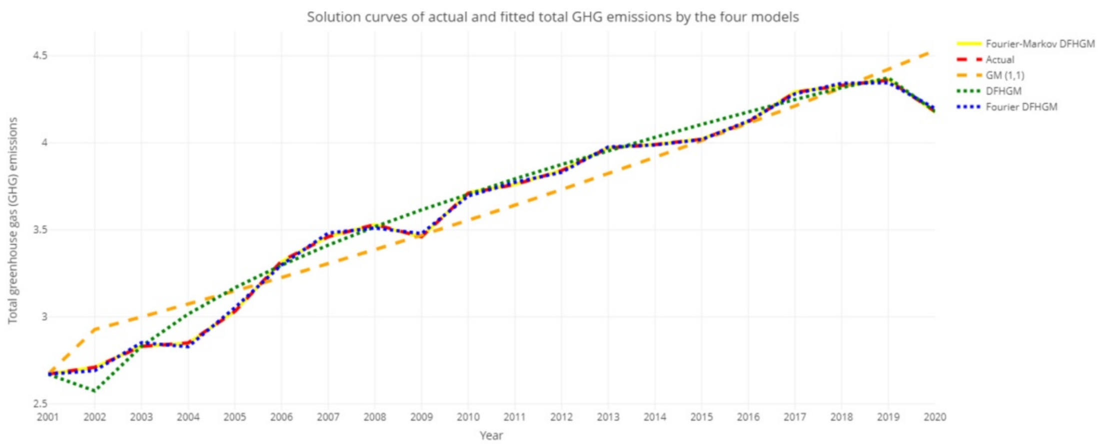

Figure 1: Comparison of plots for total greenhouse gas emissions (GHG).

3.2. Test Case Scenario 2: National inventory of greenhouse gas emissions (carbon dioxide)

Forecasting the national inventory of greenhouse gas emissions, particularly carbon dioxide, is crucial for helping the government set achievable emissions reduction targets and monitor progress across key sectors, including energy, transportation, and industry. This information guides the allocation of resources for renewable energy investments and energy-efficient technologies. Thus, as a second test case scenario, we take national inventory of greenhouse gas emissions (carbon dioxide) [37] from 2001 to 2020.

Table 2 shows the performance of the models based on MAPE and RMSE for CO_2 gas emissions, which again shows that OFHGM $(1, 1, t^{\alpha})$ is the most accurate model in predicting the actual gas emissions. We can observe in Figure 2 that the plot of OFHGM $(1, 1, t^{\alpha})$ is a better fit compared to the other grey models. The plot shows that the structure of the OFHGM $(1, 1, t^{\alpha})$ self-adapts to conform to the actual data.

Table 2: The performance of the models based on MAPE and RMSE for CO_2 gas emissions.

	GM $(1, 1)$	DFHGM $(1, 1, t^{\alpha})$	FDFHGM $(1, 1, t^{\alpha})$	OFHGM $(1, 1, t^{\alpha})$
MAPE	4.356	1.873	0.532	0.073

RMSE	180.69	83.07	20.15	2.882

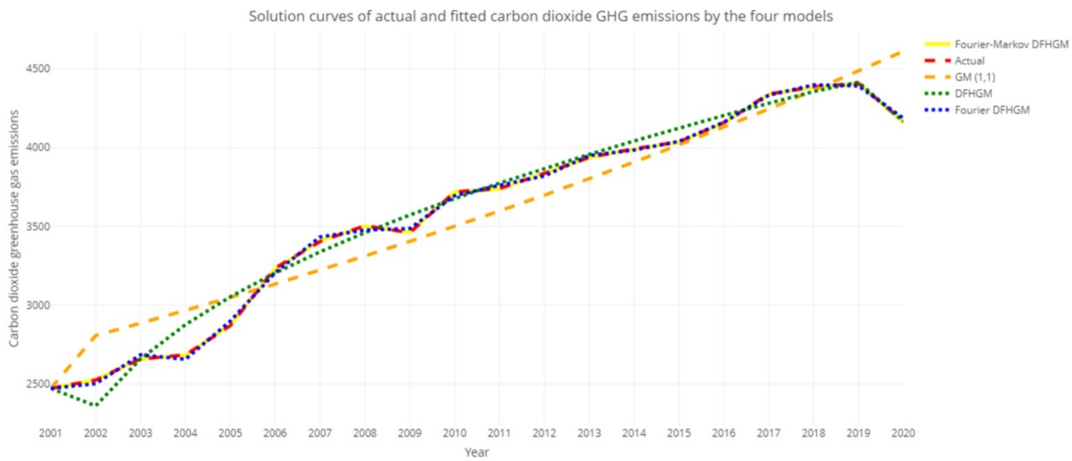

Figure 2: Comparison of plots for carbon dioxide gas emissions.

3.3. Test Case Scenario 3: National inventory of greenhouse gas emissions (nitrous oxide)

As a final test case, we also examine the national inventory of greenhouse gas emissions, specifically nitrous oxide, using data from Statistics Mauritius [38]. According to the results presented in Table 3, the OFHGM $(1, 1, t^{\alpha})$ significantly outperforms the GM $(1, 1)$ across both key performance metrics. The OFHGM $(1, 1, t^{\alpha})$ achieves the lowest MAPE just 0.095% and a RMSE of 0.00064. These results indicate that the OFHGM $(1, 1, t^{\alpha})$ delivers far more accurate and precise predictions of nitrous oxide emissions than the other grey models, making it a more reliable tool for forecasting future emission trends in Mauritius. Furthermore, Figure 3 shows a comparison of the plots of the grey models.

Table 3: The performance of the models based on MAPE and RMSE for nitrous oxide gas emissions.

	GM $(1, 1)$	DFHGM $(1, 1, t^{\alpha})$	FDFHGM $(1, 1, t^{\alpha})$	OFHGM $(1, 1, t^{\alpha})$
MAPE	3.296	2.526	0.662	0.095
RMSE	0.0242	0.0204	0.0044	0.00064

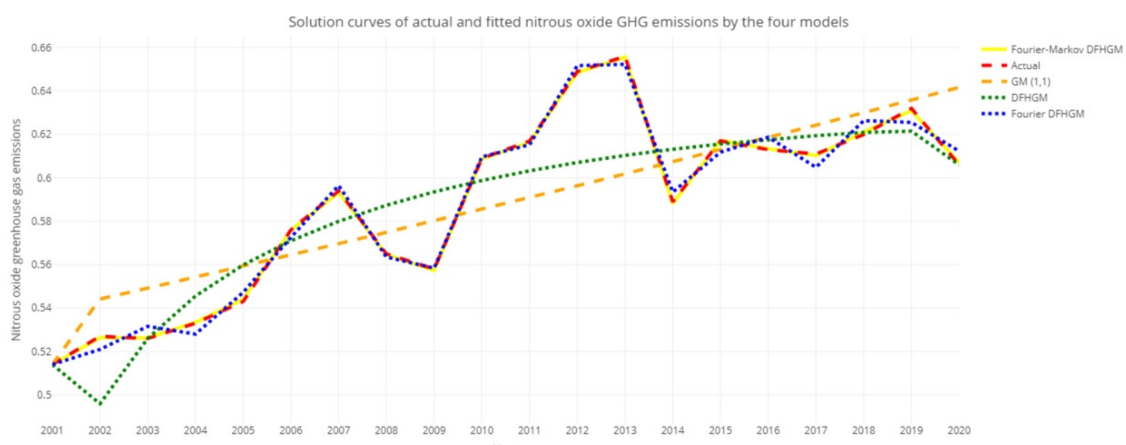

Figure 3: Comparison of plots for nitrous oxide gas emissions.

4. Conclusion

Forecasting greenhouse gas (GHG) emissions is crucial for understanding the future impact of climate change and developing effective mitigation strategies. Accurate emission forecasts help policymakers, researchers, and industries plan for emissions reduction targets and assess the effectiveness of environmental policies. Various forecasting methods, including statistical, machine learning, and grey models, are used to predict GHG emissions based on historical data and trends. These models enable better decision-making in environmental planning, climate change research, and sustainability efforts. As the global focus on reducing carbon footprints intensifies, advanced forecasting techniques, such as the one proposed in this study, provide critical insights into managing and mitigating the effects of GHG emissions. In conclusion, the proposed Fourier-Markov discrete fractional Hausdorff grey model with a time power term (OFHGM $(1, 1, t^{\alpha})$) demonstrates a highly effective approach for forecasting greenhouse gas emissions. By integrating the discrete fractional Hausdorff grey model with Fourier and Markov models, and optimizing the parameters using the Black Hole Optimization algorithm, the OFHGM $(1, 1, t^{\alpha}$) achieves remarkable forecasting accuracy. The model outperforms other grey forecasting models, as evidenced by the MAPE and RMSE, making it a promising tool for predicting GHG emissions, as illustrated in its application to Mauritius.

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