

Poisson distribution: An Alternative Statistical Model to Predict Exact Scores of Football Matches

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Abstract—Poisson distribution is one of the widely used tools that can be used to predict the probability of the exact scores in sporting events, such as football matches. The purpose of this study is to suggest and analyze a new method to predict exact scores in football matches and to exhibit its results. While the common approach was based only on the goals by each team, in order to be given a better approach, we used not only the goals, but the final efforts' average by each team, too. The implementation of the new model on the results of the Greek football League revealed that the proposed model can be used to predict the exact score of football matches more accurately, than the previous one.

Keywords: Poisson distribution; Chi-square goodness of fit test; Exact scores football

1. Introduction

Poisson distribution and Chi-square goodness of fit tests are commonly used in the field of sports data. Sports attract lots of youngsters worldwide and especially football. Over the years, many studies have analyzed some statistical models using sports data-bases. Chalikias in [1] used the binomial distribution to predict the winner of the 49th International Shooting Sport Federation World Championship in double trap shooting held in 2006 in Zagreb, Croatia. Similarly, Tijms in [2] gave a real-life example of Bayesian thinking. In particular, he discussed how credible accusations are that the outcome of the draw for the quarter-finals in the 2013 European Champions League football was manipulated. Also a study that combines sports and statistical research compares Michael Jordan with LeBron James [3].

Concentrating on the combination of Poisson distribution and football, a lot of studies have given some impressive results so far. According to Karlis and Ntzoufras [4] replacing two independent Poisson distributions by a bivariate could give some beneficial results. Although the correlation between the two scores is discussed, they managed to improve the model fit and the prediction of the number of draws in football games using data sets from football and water polo. Moreover, Poisson distribution fits certain baseball data, and it is also applied to some football data by Keller [5]. Furthermore, Poisson distribution is used in football data sets from the World Cup tournament by Chu [6]. Specifically, the author used the Poisson distribution with λ the mean of the goals scored in the 90 minutes of regulation time of a soccer game to predict the probability of the exact scores. To assess the fit of the Poisson distribution, he used the chi-squared distribution.

Towards this direction, the purpose of the current study is to present and analyze a new method based on Poisson distribution in order to predict exact scores in football matches. In order to compare the initial Chu's model with the model we suggest, we will encounter the problem using chi square of goodness of fit test.

2. The prediction Model

Since the Poisson distribution can be used to predict the probability of the exact scores, our main thought was to estimate the parameter λ not only using the scores of previous matches. So we examined to add extra data, the final efforts of each team. As, the number of goals scored is usually a function of final efforts, as the team with the most final efforts tends to score more goals [7].

In order to estimate the parameter λ , considering both the goals and the final efforts, the rate parameter lambda could be calculated by in the following way: At first we estimate two parameters, c_1 and c_2 , which are considered as the effectiveness of the Home and the Visitor teams respectively. Especially, c_1 is the average of the division “goals / final efforts” of the Home Team in all matches of the first round of League and c_2 is the average of the division “goals / final efforts” of the Visitor Team in all matches of the first round of League. The utilization of those values is very important, as those values will be used as parameters which will be multiplied to final efforts of each team. The final step of our algorithm is a simple determination of our new-calculated rate parameters lambda, λ_1' for the home team and λ_2' for the visitor team, which are the average of the previous values. Particularly λ_1' is the average of the estimated goals of the home team and λ_2' is the average of the estimated goals of the visitor team in all matches of the first round of the League. Therefore, we are in place to estimate the new probabilities of certain goals in the second round of League using the Poisson distribution and in order to determine the goodness of fit of the new model we run the Chi-Square Goodness of Fit Test.

3. Implementation of the prediction models in the Greek football League

3.1. The Greek football League

The Greek Football League is the highest professional association football league in Greece. There were 17 clubs competing in the Greek Football League (2014-2015), playing a 32-game home-and-away series in two rounds (Table 1).

Table 1: Teams that participated in the Greek Football League.

Team	Symbol	Team	Symbol	Team	Symbol	Team	Symbol	Team	Symbol
AsterasTripolis	T1	Kerkyra	T5	P.a.o.k.	T9	Panionios	T12	Platanias	T15
Atromitos	T2	Levadiakos	T6	Panathinaikos	T10	Panthrakikos	T13	ScodaXanthi	T16
Ergotelis	T3	Ofi	T7	Panaitolikos	T11	Pas Giannena	T14	Veria	T17
kalloni	T4	Olympiakos	T8						

In this section we present the collected results of all races of the Greek football League. First of all, we have collected the results of the first round, which are summarized in Table 2.

Table2 : Results of the first round of the Greek football League.

Race	Home Team	Visitor Team	Score	Race	Home Team	Visitor Team	Score	Race	Home Team	Visitor Team	Score
1st	T7	T11	1 0	7nth	T3	T8	2 3	13th	T17	T15	2 0
	T5	T13	2 1		T10	T7	1 2		T5	T2	1 1
	T9	T4	1 1		T9	T2	2 1		T8	T14	2 2
	T14	T1	3 1		T1	T11	1 1		T16	T9	4 2
	T2	T15	1 0		T12	T14	0 1		T11	T12	2 1
	T17	T16	3 2		T17	T6	2 1		T7	T6	0 2
	T6	T10	1 1		T15	T16	1 0		T1	T10	1 1
	T12	T3	2 1		T4	T5	1 0		T13	T3	0 0
2nd	T13	T14	1 1	8fth	T8	T10	1 0	14th	T12	T7	2 0
	T4	T3	2 0		T9	T17	4 1		T1	T17	2 0
	T11	T8	1 1		T7	T1	2 3		T3	T16	0 2
	T15	T9	0 4		T14	T4	0 0		T14	T2	1 0
	T16	T5	0 0		T2	T3	1 1		T9	T5	2 1
	T1	T6	2 1		T11	T13	3 1		T10	T13	4 0
	T10	T12	2 1		T6	T12	1 0		T4	T11	0 0
	T7	T17	0 1		T5	T15	1 2		T6	T8	1 2
3rd	T17	T5	2 1	9fth	T1	T8	0 0	15th	T5	T3	3 1

	T4	T10	1	0		T13	T7	2	0		T13	T1	1	1
	T12	T1	2	1		T3	T9	0	2		T11	T15	1	0
	T6	T10	1	1		T15	T14	0	0		T7	T4	1	1
	T14	T16	2	2		T17	T12	2	2		T2	T6	1	0
	T2	T11	0	0		T11	T16	5	2		T8	T12	2	0
	T8	T7	3	0		T4	T6	0	0		T14	T9	3	0
	T3	T15	0	3		T10	T2	2	0		T16	T10	4	2
4rth	T8	T17	3	0	10th	T3	T17	2	2	16th	T13	T17	1	1
	T15	T10	2	3		T5	T11	1	2		T15	T7	1	0
	T11	T9	0	1		T16	T7	2	1		T4	T8	0	5
	T1	T4	1	0		T8	T13	5	1		T12	T2	2	2
	T5	T14	2	0		T6	T15	0	0		T1	T16	2	1
	T16	T6	2	0		T12	T4	0	2		T3	T14	1	0
	T13	T12	1	0		T9	T10	1	2		T6	T9	1	2
5fth	T7	T2	1	0	11th	T2	T1	4	3	17th	T10	T5	2	0
	T2	T8	1	0		T17	T4	1	1		T17	T11	1	3
	T9	T7	4	0		T15	T12	3	0		T5	T1	1	0
	T15	T1	0	1		T1	T9	3	0		T9	T12	3	2
	T17	T14	2	0		T11	T14	0	0		T8	T15	2	1
	T12	T16	1	1		T13	T2	1	1		T2	T4	0	0
	T6	T5	2	3		T7	T5	3	2		T6	T3	1	1
6fth	T3	T11	1	1	12th	T10	T3	5	0		T14	T10	0	0
	T4	T13	2	0		T16	T8	1	3		T16	T13	0	0
	T8	T9	1	2		T4	T15	0	0					
	T11	T10	0	1		T6	T11	0	0					
	T14	T6	0	4		T2	T16	2	0					
	T16	T4	2	1		T5	T8	0	4					
	T5	T12	1	0		T3	T1	0	1					
	T13	T15	1	0		T14	T7	3	0					
	T7	T3	1	0		T9	T13	3	2					
	T2	T17	0	0		T10	T17	2	1					

In order to predict the exact scores of the first round of the Greek football League, we use the above data in two different ways: a) based on the Chu's prediction model [6] and b) based on the new prediction model we propose. A comparison of the two prediction models follows.

3.2. Estimating Greek football League's results based on Chu's model

Initially, we use the above results to estimate the parameters of the Poisson distribution in the way that was suggested by Chu [6]. Particularly, we estimate the λ_1 , which is the average goals scored by the home team and after that, the λ_2 for the visitor team which is the average goals scored by the visitor team in the 17 races of the first round of the Greek football League. After a very simple calculation we find that $\lambda_1 = 1,45588$ and $\lambda_2 = 0,95588$.

Then, we are in place to estimate the probabilities of certain goals in the second round of League, using the Poisson distribution. Also, we calculate the expected (theoretical) frequencies for each class, which are useful for running the procedure of Chi-Square Goodness of Fit Test, about the home and the visitor team in the 2nd round (Table 3).

Table 3: Chi square using the number of goals scored by home and visitor teams.

Number of goals	Home teams				Visitor teams			
	f_i	p_i	$\theta_i = n \cdot p_i$	$(f_i - \theta_i)^2 / \theta_i$	f_i	p_i	$\theta_i = n \cdot p_i$	$(f_i - \theta_i)^2 / \theta_i$
0	29	0,233	31,715	0,232	62	0,3845	52,2884	1,8037
1	41	0,340	46,173	0,579	42	0,3675	49,9815	1,2745
2	34	0,247	33,611	0,005	21	0,1756	23,8881	0,3492
3	19	0,120	16,311	0,443	11	0,0560	7,6114	1,5086
4	11	0,044	5,937	4,318	0	0,0134	1,8189	1,8189
5	1	0,013	1,729	0,307	0	0,0026	0,3477	0,3477
6	1	0,003	0,419	0,804	0	0,0004	0,0554	0,0554
	136	0,999	135,894	6,688	136	0,9999	135,9914	7,1581

The procedure of Chi-Square Goodness of Fit Test reveals a strong approximation to the real results for the home (1) and the visitor team (2). As we can see,

$$D^2 = \sum_{i=0}^6 \frac{(f_i - \theta_i)^2}{\theta_i} \cong 6,688 < 11.07 = \chi^2_5 \quad (1)$$

$$D^2 = \sum_{i=0}^6 \frac{(f_i - \theta_i)^2}{\theta_i} \cong 7,158 < 11.07 = \chi^2_5 \quad (2)$$

3.3. Estimating Greek football League's results based on the new prediction model

In order to present the new prediction model that we suggest, we also collected the final efforts of each team to score in the first round of the Greek Football League (Table 4).

Table 4: Final efforts of the teams in the Greek football League.

Race	Home Team	Visitor Team	Final Efforts		Race	Home Team	Visitor Team	Final Efforts		Race	Home Team	Visitor Team	Final Efforts	
1st	T7	T11	16	3	7nth	T3	T8	17	22	13th	T17	T15	13	14
	T5	T13	12	7		T10	T7	15	7		T5	T2	16	8
	T9	T4	16	8		T9	T2	12	2		T8	T14	24	7
	T14	T1	10	6		T1	T11	15	13		T16	T9	6	9
	T2	T15	14	10		T12	T14	12	7		T11	T12	17	13
	T17	T16	11	9		T17	T6	15	10		T7	T6	14	7
	T6	T10	5	16		T15	T16	10	12		T1	T10	17	9
	T12	T3	18	12		T4	T5	11	7		T13	T3	22	6
2nd	T13	T14	18	11	8fth	T8	T10	6	4	14th	T12	T7	8	2
	T4	T3	18	7		T9	T17	20	9		T1	T17	9	7
	T11	T8	6	11		T7	T1	6	12		T3	T16	6	12
	T15	T9	8	14		T14	T4	7	11		T14	T2	9	10
	T16	T5	10	8		T2	T3	7	10		T9	T5	13	14
	T1	T6	20	7		T11	T13	7	8		T10	T13	22	6
	T10	T12	18	7		T6	T12	17	8		T4	T11	11	6
	T7	T17	10	9		T5	T15	12	9		T6	T8	5	21
3rd	T17	T5	12	10	9fth	T1	T8	5	22	15th	T5	T3	12	14
	T4	T10	5	12		T13	T7	14	5		T13	T1	11	18
	T12	T1	13	9		T3	T9	7	10		T11	T15	21	7

we estimate the parameters and which the average of the divisions “goal final effort of home the visit team respective all match of first round the Greek football

	T6	T10	14	12		T15	T14	12	6		T7	T4	12	8
	T14	T16	15	7		T17	T12	13	8		T2	T6	13	8
	T2	T11	8	6		T11	T16	18	11		T8	T12	14	7
	T8	T7	23	5		T4	T6	11	10		T14	T9	13	6
	T3	T15	15	15		T10	T2	14	5		T16	T10	13	9
4rth	T8	T17	13	5	10th	T3	T17	11	11	16th	T13	T17	9	8
	T15	T10	10	9		T5	T11	5	11		T15	T7	16	6
	T11	T9	8	9		T16	T7	9	6		T4	T8	5	19
	T1	T4	19	4		T8	T13	28	10		T12	T2	15	10
	T5	T14	6	10		T6	T15	13	6		T1	T16	8	7
	T16	T6	7	7		T12	T4	17	5		T3	T14	16	9
	T13	T12	10	5		T9	T10	8	9		T6	T9	20	7
5fth	T7	T2	8	9	11th	T2	T1	13	10	17th	T10	T5	17	9
	T2	T8	6	8		T17	T4	15	8		T17	T11	12	5
	T9	T7	18	7		T15	T12	22	8		T5	T1	15	14
	T15	T1	11	8		T1	T9	10	6		T9	T12	7	11
	T17	T14	19	3		T11	T14	14	7		T8	T15	11	11
	T12	T16	23	10		T13	T2	3	12		T2	T4	14	9
	T6	T5	19	5		T7	T5	18	13		T6	T3	16	8
	T3	T11	16	7		T10	T3	14	4		T14	T10	10	6
6fth	T4	T13	16	5	12th	T16	T8	5	8		T16	T13	10	1
	T8	T9	14	5		T4	T15	12	9					
	T11	T10	11	14		T6	T11	12	3					
	T14	T6	8	11		T2	T16	7	10					
	T16	T4	20	5		T5	T8	9	25					
	T5	T12	6	10		T3	T1	8	13					
	T13	T15	14	13		T14	T7	15	4					
	T7	T3	15	11		T9	T13	14	2					
	T2	T17	9	3		T10	T17	13	5					

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League (Table 5).

Table 5: The quotient of the divisions “goals / final efforts”.

Race	Home Team	Visitor Team	Race	Home Team	Visitor Team	Race	Home Team	Visitor Team	Race	Home Team	Visitor Team
1st	0,063	0	6fth	0,071	0,4	10th	0,182	0,182	14th	0,25	0
	0,167	0,143		0	0,071		0,2	0,182		0,222	0
	0,063	0,125		0	0,364		0,222	0,167		0	0,167
	0,3	0,167		0,1	0,2		0,179	0,1		0,111	0
	0,071	0		0,167	0		0	0		0,154	0,071
	0,273	0,222		0,071	0		0	0,4		0,182	0
	0,2	0,063		0,067	0		0,125	0,222		0	0
	0,111	0,083		0	0		0,308	0,3		0,2	0,095
2nd	0,056	0,091	7nth	0,118	0,136	11th	0,067	0,125	15th	0,25	0,071
	0,111	0		0,067	0,286		0,136	0		0,091	0,056
	0,167	0,091		0,167	0,5		0,3	0		0,048	0
	0	0,286		0,067	0,077		0	0		0,083	0,125
	0	0		0	0,143		0,333	0,083		0,077	0
	0,1	0,143		0,133	0,1		0,167	0,154		0,143	0

	0,111	0,143		0,1	0		0,357	0		0,231	0
	0	0,111		0,091	0		0,2	0,375		0,308	0,222
3rd	0,167	0,1	8fth	0,167	0	12th	0	0	16th	0,111	0,125
	0,2	0		0,2	0,111		0	0		0,063	0
	0,154	0,111		0,333	0,25		0,286	0		0	0,263
	0,071	0,083		0	0		0	0,16		0,133	0,2
	0,133	0,286		0,143	0,1		0	0,077		0,25	0,143
	0	0		0,429	0,125		0,2	0		0,063	0
	0,13	0		0,059	0		0,214	1		0,05	0,286
	0	0,2		0,083	0,222		0,154	0,2		0,118	0
	4rth	0,231		0	9fth		0	0		13th	0,154
0,2		0,333	0,143	0		0,063	0,125	0,067	0		
0		0,111	0	0,2		0,083	0,286	0,429	0,182		
0,053		0	0	0		0,667	0,222	0,182	0,091		
0,333		0	0,154	0,25		0,118	0,077	0	0		
0,286		0	0,278	0,182		0	0,286	0,063	0,125		
0,1		0	0	0		0,059	0,111	0	0		
0,125		0	0,143	0		0	0	0	0		
5fth	0,167	0									
	0,222	0									
	0	0,125									
	0,105	0									
	0,043	0,1									
	0,105	0,6									
	0,063	0,143									
	0,125	0									

The next step in lambda calculation is to calculate the average of the quotients which are given before. For the home team we find that $c_1=0,124156$ and for the visitor team we find that $c_2=0,111998$. These values are the parameters that will be multiplied by the final efforts of each team, which are summarized in Table 6.

Table 6: Estimated scores using the final efforts and goals.

Race	Home Team	Visitor Team	Race	Home Team	Visitor Team	Race	Home Team	Visitor Team	Race	Home Team	Visitor Team
1st	1,987	0,336	6fth	1,738	0,56	10th	1,366	1,232	14th	0,993	0,224
	1,49	0,784		1,366	1,568		0,621	1,232		1,117	0,784
	1,987	0,896		0,993	1,232		1,117	0,672		0,745	1,344
	1,242	0,672		2,483	0,56		3,476	1,12		1,117	1,12
	1,738	1,12		0,745	1,12		1,614	0,672		1,614	1,568
	1,366	1,008		1,738	1,456		2,111	0,56		2,732	0,672
	0,621	1,792		1,862	1,232		0,993	1,008		1,366	0,672
	2,235	1,344		1,117	0,336		1,614	1,12		0,621	2,352
2nd	2,235	1,232	7nth	2,111	2,464	11th	1,862	0,896	15th	1,49	1,568
	2,235	0,784		1,862	0,784		2,732	0,896		1,366	2,016
	0,745	1,232		1,49	0,224		1,242	0,672		2,607	0,784

	0,993	1,568		1,862	1,456		1,738	0,784		1,49	0,896
	1,242	0,896		1,49	0,784		0,372	1,344		1,614	0,896
	2,483	0,784		1,862	1,12		2,235	1,456		1,738	0,784
	2,235	0,784		1,242	1,344		1,738	0,448		1,614	0,672
	1,242	1,008		1,366	0,784		0,621	0,896		1,614	1,008
3rd	1,49	1,12	8fth	0,745	0,448	12th	1,49	1,008	16th	1,117	0,896
	0,621	1,344		2,483	1,008		1,49	0,336		1,987	0,672
	1,614	1,008		0,745	1,344		0,869	1,12		0,621	2,128
	1,738	1,344		0,869	1,232		1,117	2,8		1,862	1,12
	1,862	0,784		0,869	1,12		0,993	1,456		0,993	0,784
	0,993	0,672		0,869	0,896		1,862	0,448		1,987	1,008
	2,856	0,56		2,111	0,896		1,738	0,224		2,483	0,784
	1,862	1,68		1,49	1,008		1,614	0,56		2,111	1,008
4rth	1,614	0,56	9fth	0,621	2,464	13th	1,614	1,568	17th	1,49	0,56
	1,242	1,008		1,738	0,56		1,987	0,896		1,862	1,568
	0,993	1,008		0,869	1,12		2,98	0,784		0,869	1,232
	2,359	0,448		1,49	0,672		0,745	1,008		1,366	1,232
	0,745	1,12		1,614	0,896		2,111	1,456		1,738	1,008
	0,869	0,784		2,235	1,232		1,738	0,784		1,987	0,896
	1,242	0,56		1,366	1,12		2,111	1,008		1,242	0,672
	0,993	1,008		1,738	0,56		2,732	0,672		1,242	0,112
5fth	0,745	0,896									
	2,235	0,784									
	1,366	0,896									
	2,359	0,336									
	2,856	1,12									
	2,359	0,56									
	1,987	0,784									
	1,987	0,56									

In the final step of the prediction model that we suggest, we have to determinate the lambda parameters for the home and the visitor team, respectively. In particular λ_1' is the average of the estimated goals of the home team and λ_2' is the average of the estimated goals of the visitor team, in all matches of the first round of the League. After a very simple calculation we find that $\lambda_1' = 1,571172$ and $\lambda_2' = 0,992353$.

Next, we are able to estimate the probabilities of certain goals in the second round of League. We also calculate the expected (theoretical) frequencies for each class, which are useful for running the procedure of Chi-Square Goodness of Fit Test, about the home team in the 2nd round (Table 7).

Table 7: Chi square using the number of final efforts and goals for home and visitor teams.

Number of goals	Home teams				Visitor teams			
	f_i	p_i	$\theta_i = n \cdot p_i$	$(f_i - \theta_i)^2 / \theta_i$	f_i	p_i	$\theta_i = n \cdot p_i$	$(f_i - \theta_i)^2 / \theta_i$
0	29	0,208	28,2610	0,0193	62	0,371	50,416	2,662
1	41	0,326	44,403	0,261	42	0,368	50,030	1,289
2	34	0,256	34,882	0,022	21	0,183	24,824	0,589
3	19	0,134	18,269	0,029	11	0,060	8,211	0,947
4	11	0,053	7,176	2,038	0	0,015	2,037	2,037
5	1	0,017	2,255	0,698	0	0,003	0,404	0,404
6	1	0,004	0,590	0,284	0	0,000	0,067	0,067

	136	0,999	135,836	3,352	136	1,000	135,989	7,995
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The results of the Chi-Square Goodness of Fit Test show a strong approximation to the real results for the home (3) and the visitor team (4). As we can see,

$$D^2 = \sum_{i=0}^6 \frac{(f_i - \theta_i)^2}{\theta_i} \cong 3,352 < 11.07 = \chi^2_5 \tag{3}$$

$$D^2 = \sum_{i=0}^6 \frac{(f_i - \theta_i)^2}{\theta_i} \cong 7,995 < 11.07 = \chi^2_5 \tag{4}$$

3.4. Comparing the two prediction models

In order to compare Chu’s model [6] with the prediction model that we suggest, we will encounter the problem using chi square of goodness test. For the home team the results give a strong and crucial difference between the two models. As we can see, for the home team, the value of D^2 of the new model is much lower than the value of D^2 in Chu's model. For the visitor team, however, the value of D^2 of the new model is higher than the value of D^2 of Chu’s model. But surely, this cannot be a real problem because the difference between the two D^2 is worthless. Moreover, if we face the problem cumulatively, the new calculated D^2 is smaller than the previous way D^2 (Table 8).

Table 8: D^2 Comparison.

	Home Team	Visitor Team	Sum
Chu’s model	6,69	7,16	13,85
New model	3,35	7,99	11,34

4. Discussion and Conclusion

In this study we present and analyze a new method based on the Poisson distribution for the accurate prediction of football match results. The results of this study show that using the number of final attempts and the number of goals scored by each team improves the model fit and the prediction of the number of goals scored in football matches. It is therefore recommended to investigate whether this new prediction model can be used to accurately predict the results of other sports.

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