Numerical Investigation of Droplet Dynamics in Recirculating Flows Using LBM

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Abstract –Droplet dynamics in a recirculating flow is investigated in this work. In this regard, Lattice Boltzmann simulation is performed to study the problem. To give a perspective on the physics of this problem, time evolution of the droplet behaviour and the global flow pattern is investigated. Finding show that droplets with different density ratios have different path lines and different shapes of deformations. Also, diameter effects as an important parameter that affects the dynamics of the droplet are studied. Another parameter which should be regarded is the first position of the droplet in the flow domain due to the forces that global flow pattern exert on the drop and change the shape of the drop. To create a recirculation flow a lid driven cavity with moving upper wall is considered here.

Keywords: Two-Phase Flow, Droplet, Recirculating Flow, Lattice Boltzmann Method.

1. Introduction

Investigation of droplet and bubble dynamics is one of the challenging phenomena in many science and engineering problems such as emulsification processes, food industry, polymer blending and oil recovery, and in deformation of biological cells (Bruijin, 1989). In these processes, two immiscible fluids are mixed to obtain a distribution of droplets of one of the liquids in the other. So, many investigations have done from experimental, numerical, and theoretical points of view.

Megias-Alguacil et al. (2005), A. Javadi et al. (2010), A. Javadi et al. (2012) and A. Javadi et al. (2014) have carried out experimental studies on droplet characteristics. Also, Chang Zhi et al. (2007) and Janssen et al. (2008) have done numerical studies on this area. Among many numerical studies, some researches applied lattice Boltzmann method (LBM) to simulate the dynamics, deformation and break up of droplet. Inamuro et al. (2003), Sman et al. (2008) and Farokhirad et al. (2013) are examples of those studied droplet dynamics by LBM.

The main aim of the present paper is to study the dynamics of a single droplet moving in a recirculating flow. According to the authors' investigations, little study is carried out on droplet dynamics in recirculating flows. The deformation and behaviour of the droplet is simulated by LBM. Compared with other two-phase LBMs based on (Shan and Chen, 1993, Swift et al., 1996), the present LBM (Lee, 2009, Lee and Liu, 2010) is capable of eliminating the parasitic currents and dealing with higher density and viscosity ratios, but it could be more computationally expensive. This paper is organized as follows: in section 2, the problem description and the numerical method applied in this study is briefly reviewed. Section 3, presents the validation and results. In section 5, the conclusion of the present study is explained in a few words.

2. Problem Description and Numerical Method

The schematic of the configuration analyzed in this study is illustrated in Fig. 1. A circular droplet is put in a cavity that the upper wall has the velocity U. The droplet will move and rotate in the cavity dependent on its density, its diameter, its first position in the cavity and the velocity of the moving wall.



Fig. 1. Geometrical configuration of the problem .

Two particle distribution functions, g_{α} and h_{α} , are applied in the present LBM for binary fluids (Lee and Liu, 2010). The function h_{α} is used as a phase-field function for the transport of the composition C of one component, and the function g_{α} is used for the calculation of pressure and momentum of the two-component mixture. The discrete Boltzmann equations for the phase-field advection equation and the pressure evolution and momentum equations are, respectively:

$$\frac{\partial h_{\alpha}}{\partial t} + \boldsymbol{e}_{\alpha} \cdot \nabla h_{\alpha} = -\frac{h_{\alpha} - h_{\alpha}^{eq}}{\lambda} + (\boldsymbol{e}_{\alpha} - \boldsymbol{u}) \cdot [\nabla C - \frac{C}{\rho c_s^2} (\nabla p - \mu \nabla C] \Gamma_{\alpha} + \nabla \cdot (M \nabla \mu) \Gamma_{\alpha}, \tag{1}$$

$$\frac{\partial g_{\alpha}}{\partial t} + \boldsymbol{e}_{\alpha} \cdot \nabla g_{\alpha} = -\frac{g_{\alpha} - g_{\alpha}^{eq}}{\lambda} + (\boldsymbol{e}_{\alpha} - \boldsymbol{u}) \cdot [\nabla \rho c_{s}^{2} (\Gamma_{\alpha} - \Gamma_{\alpha}(0)) + \mu \nabla C$$
⁽²⁾

where the equilibrium distribution functions are given as

$$h_{\alpha}^{eq} = t_{\alpha} C \left[1 + \frac{\boldsymbol{e}_{\alpha} \cdot \boldsymbol{u}}{c_s^2} + \frac{(\boldsymbol{e}_{\alpha} \cdot \boldsymbol{u})^2}{2c_s^4} - \frac{\boldsymbol{u} \cdot \boldsymbol{u}}{2c_s^2} \right],\tag{3}$$

$$g_{\alpha}^{eq} = t_{\alpha} \left[p + \rho c_s^2 \left(\frac{\boldsymbol{e}_{\alpha} \cdot \boldsymbol{u}}{c_s^2} + \frac{(\boldsymbol{e}_{\alpha} \cdot \boldsymbol{u})^2}{2c_s^4} - \frac{\boldsymbol{u} \cdot \boldsymbol{u}}{2c_s^2} \right) \right].$$
(4)

In these equations, e_{α} represents discrete lattice velocities in the direction of α , u is volume averaged velocity, c_s is the basic speed on the lattice, ρ is the mixture density, λ is the relaxation time, t_{α} is the weighting factor, μ is the chemical potential and M is the mobility in the Cahn-Hilliard diffusion. Also, Γ_{α} is defined as

$$\Gamma_{\alpha} = t_{\alpha} \left[1 + \frac{\boldsymbol{e}_{\alpha} \cdot \boldsymbol{u}}{c_s^2} + \frac{(\boldsymbol{e}_{\alpha} \cdot \boldsymbol{u})^2}{2c_s^4} - \frac{(\boldsymbol{u} \cdot \boldsymbol{u})}{2c_s^2} \right].$$
(5)

The composition, momentum and dynamic pressure can be obtained by taking the moments of h_{α} and g_{α} :

$$C = \sum_{\alpha} h_{\alpha}, \tag{6a}$$

$$\rho \boldsymbol{u} = \frac{1}{c_s^2} \sum_{\alpha} \boldsymbol{e}_{\alpha} \boldsymbol{g}_{\alpha}, \tag{6b}$$

(6c)

$$p = \sum_{\alpha} g_{\alpha}.$$

For detailed discretization of Eqs. (1) and (2), readers are referred to Lee and Liu (2010).

The mixture density ρ can be measured as $\rho = \rho_h C + \rho_l (1 - C)$, in which ρ_h and ρ_l are the bulk densities of two fluids. The mixing energy density for binary fluids can be calculated as $E_0 (C, \nabla C) = E_0 (C) + \kappa |\nabla C|^2/2$, where κ is the gradient parameter and $E_0 (C) = \beta C^2 (1 - C)^2$ is the bulk energy density with constant β (Lee and Liu, 2010). The equilibrium profile is obtained by minimizing the mixing energy. The equilibrium interface profile is then $C(z) = 1/2 + \tanh(2z/D)/2$, where z is the coordinate normal to the plane interface and D is the numerical interface thickness. Having D and interfacial tension σ , β and κ can be determined as $\beta = 12\sigma/D$ and $\kappa = \beta D^2/8$.

3. Results and Discussion

In this section, findings of the present study are illustrated. First, the results are verified and in continuance, physics of the flow is studied. In later sections, effect of parameters such as density ratio, droplet diameter and its first position on droplet deformation and its path line are studied.

3. 1. Verification of Results

In order to verify the results, a circular drop with an initial radius R = 23 in lattice unit is placed at the center of a cavity with 350×350 grid domain. The gas and liquid densities are $\rho_g = 0.01$, $\rho_l = 1$. The velocity vectors at the interface and global flow in the cavity are shown in Fig. 2 for different positions of the droplet in its path line when Re = 1000. This figure demonstrates the capability of the model for eliminating spurious currents, too. It should be mentioned that in all of our results, droplet radius unit is lattice unit and Reynolds number is measure as $Re = \frac{UL}{\nu}$, in which U is the velocity of the upper wall, L is the width of the cavity and ν is the kinematic viscosity. Also, σ is the surface tension of the fluid.

3. 2. Physics of the Flow

To study different aspects of the flow pattern and droplet motion in the cavity, time evolution of flow pattern and droplet behavior in the cavity is illustrated in Fig. 3. In the first seconds, t = 0.15s, (Fig. 3a), the droplet moves in the directions of the flow in the cavity and a vortex is formed at the right top corner. By passing time at t = 0.3s, this droplet has slightly deformed and moved toward the upper wall, Fig. 3b. In later seconds, the primary vortex and the separation bubble has grown more and the droplet is in an upper position than before, t = 0.45s. Part d shows that by increasing time, the droplet has moved to near the upper wall and it has a great deformation, because of high shear rate in this region. Also, a small vortex is formed behind it and the primary vortex has moved toward the center of the cavity. Letting the flow in the cavity develop more, the droplet circulates by the global flow in the cavity and deforms according to the forces that would experience in different positions, such that it deforms much at the right corner and bottom of the cavity. Generally, it can be seen that the droplet experiences more shear in the right side of the cavity since the separation bubble is formed from this position and the developing of the global flow pattern in the cavity is from this region. Finally, in part i, it can be seen that two corner vortices of the bottom have formed and the droplet is moving upward again.

3. 3. Effect of Different Parameters

In order to have a better study of the considered problem, effect of density ratio on the droplet shape and its path line are studied in Fig. 4. By increasing the density ratio from 10 to 100, the deformation of the droplet and its path line changes considerably, such that the droplet has a dramatic deformation near the

wall where $\rho_h/\rho_l = 10$. Also, due to its lower inertia, it has moved longer than the heavier droplet. This phenomenon can be seen clearly in Fig. 4.



Fig. 2. Velocity vectors in the cavity for different positions of the droplet with C = 0.5 in its path line.

Another parameter which is studied is the effect of droplet diameter on its path line. As shown in Fig. 5, by expanding the droplet diameter, the droplet movement is slower and its path line has more curvature. This is because it has more inertia to resist against the global cavity flow. This point is more significant when the R = 35 lattice units.

Our findings show that first position of the droplet in the cavity has a great effect on its shape during circulating in it with global flow. Fig. 6 illustrates two droplets with two different first positions. In part a, droplet first position is from the center of the cavity while in part b, the droplet move from the center of left bottom quarter. In part b, the droplet begins to deform when it gets near to the upper wall and forms like an oval.

4. Conclusion

In this paper, a study of droplet behaviour in a recirculating flow is done numerically. Lattice Boltzmann method is applied to simulate the problem physics. To give a perspective of the flow field and droplet dynamics, time evolution of the fluid flow and droplet movement is investigated for Re = 1000, $\sigma = 0.001$ and R = 23 lattice units. Findings show that walls and the global flow pattern of the cavity have great effects on droplet motion such that the droplet experiences more shear near walls especially at the right hand side. In continuance, the problem is studied for different values of droplet diameter and density ratio. The results showed that because of experiencing different forces, the droplet moves different path with different shapes when the density ratio and the droplet size is changes. The higher density ratio and the higher droplet size, the shorter path line and the more curvature on the path. Also, first position of the droplet affects the shape and path line of the droplet.



Fig. 3. Time evolution of flow pattern and droplet behaviour in the cavity for C = 0.5, $\rho_h / \rho_l = 10$, Re = 1000R = 23 and $\sigma = 0.001$.



Fig. 4. Behavior and path lines of droplets with different density ratios, C = 0.5, Re = 1000, $\sigma = 0.001$ and R = 23.



Fig. 5. Effect of diameter on path lines of droplets, C = 0.5, Re = 1000, Density Ratio = 100 and $\sigma = 0.001$



Fig. 6. Effect of droplet first position on its, C = 0.5, Re = 1000, $\sigma = 0.001$ and R = 23.

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