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Numerical Investigation on the Dissipation Property of the Rotated Roe Scheme

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Abstract -Rotated Riemann solvers have been investigated for the purpose of eliminate the abnormality phenomena in the vicinity of strong shock. It was proved that the rotation procedure will introducing intrinsic dissipation on Riemann solver. However, the dissipation property of rotated schemes haven't been thoroughly investigated. In this work, numerical investigation is made for the rotated Roe scheme. The results show that the presented rotated Roe scheme, which has a predefined rotation angle, is more dissipative than original Roe scheme. Thereinto, carbuncle is eliminated by rotated scheme, even without entropy fixing. Besides, laminar boundary layer simulations show that the boundary-layer-resolving capability of rotated Roe scheme is deteriorated. The results prove that the intrinsic dissipation is introduced by the rotation procedure, and the dissipative level is directly related with the rotation angle.

Keywords: rotated Roe scheme; strong shock; carbuncle; boundary-layer-resolving capability; dissipation

1 Introduction

In recent years, in order to eliminate the shock instabilities, such as carbuncle phenomenon and odd-even decoupling, rotated Riemann solvers have been investigated. (Ren 2003) presented a robust finite volume shock capturing scheme based on the rotated approximate Riemann solver (Roe 1981). The rotated Roe scheme demonstrates a robust shock-capturing capability, which the carbuncle phenomenon can be eliminated completely. Based on such rotated framework, (Nishikawa & Kitamura 2008) combined the Roe scheme with the Rusanov (Rusanov 1961) or HLL (Harten et al. 1983) schemes to eliminate the abnormity in the vicinity of strong shock. Thereinto, a fewer-wave solver (Rusanov or HLL) is automatically applied in the direction normal to the shocks to suppress carbuncles and a full-wave solver (Roe) is applied across shear layers to avoid an excessive amount of dissipation. Similarly, (Zha et al. 2011) used ECUSP scheme (Jameson 1995) instead of the Roe scheme for the fluxes in the direction parallel to the shock. (Huang et al. 2011) try to solve the problem of the numerical shock instability in HLLC (Toro et al. 1994) solver, one of their methods is rotated HLLC scheme.

Despite the ample works on rotated Riemann solvers, the details of dissipation property have not been quantitatively revealed. (Ren 2003) provided a theoretical analysis of rotated Roe scheme, which its dissipations corresponding to the contact and shear waves was examined. (Nishikawa & Kitamura 2008) shown that the RHLL scheme provide well resolution in boundary layer problem, that means the scheme can well resolve the contact discontinuity. Both of these researches are inspiring, yet further works are still necessary.

In this paper, rotated Roe scheme with several given rotation angles is numerical investigated. Because the rotation angle is given, the performance of rotated scheme is isolated with the direction of flow features. Firstly, simulations of hypersonic flow around blunt-body are conducted, the robustness of rotated scheme is proved by the results. And then the laminar boundary layer flow is simulated, therefore the boundary-layer-resolving capability of rotated scheme could be examined.

2 Rotated Riemann solver

The rotated Riemann solvers are based on the decomposition of normal direction $\mathbf{n}$, which is the outward unit vector normal to interface, into two orthogonal directions,

$$\mathbf{n} = \alpha_1 \mathbf{n}_1 + \alpha_2 \mathbf{n}_2, \mathbf{n}_1 \cdot \mathbf{n}_2 = \mathbf{0} \tag{1}$$

where $\alpha_1 = \mathbf{n}_1 \cdot \mathbf{n}$ and $\alpha_2 = \mathbf{n}_2 \cdot \mathbf{n}$. In order to keep the same left and right states in both directions, the vector $\mathbf{n}_1$ and $\mathbf{n}_2$ are chosen to make $\alpha_1 \geqslant 0$ and $\alpha_2 \geqslant 0$. At the mesh interface, the numerical flux is then decomposed correspondingly into the following form:

$$\Phi(\mathbf{n}) = \alpha_1 \Phi(\mathbf{n_1}) + \alpha_2 \Phi(\mathbf{n_2}) \tag{2}$$

The physical feature of the flow problem can be taken into account to determine the choice of $\mathbf{n_1}$ and $\mathbf{n_2}$. (Ren 2003) pointed out that it is effective for Roe solver to align $\mathbf{n_1}$with the velocity-difference vector between two adjacent cells:

$$\mathbf{n_1} = \begin{cases} \mathbf{n}, & \text{if} \|\Delta \mathbf{V}\| \leqslant \varepsilon \\ \dfrac{\Delta \mathbf{V}}{\|\Delta \mathbf{V}\|}, & \text{otherwise} \end{cases} \tag{3}$$

Here, different from Ren's work, the rotation angles are predefined, thus the decomposition of $\mathbf{n}$ is defined as:

$$\mathbf{n}_1 = \mathbf{T}_1 \mathbf{n}, \mathbf{n}_2 = \mathbf{T}_2 \mathbf{n} \tag{4}$$

$$\mathbf{T}_1 = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix}, \mathbf{T}_2 = \begin{bmatrix} \cos(\alpha - \pi/2) & \sin(\alpha - \pi/2) \\ -\sin(\alpha - \pi/2) & \cos(\alpha - \pi/2) \end{bmatrix} \tag{5}$$

3 Numerical results

3.1 Case 1: inviscid hypersonic flow around a blunt-body

The free stream Mach number is 20, and these test problems are computed on a quadrilateral grid with 80 cells in the radial direction and 120 cells in the circumferential direction, and a unstructured triangular grid with 12156 cells. Simulations are conducted in first order spatial accuracy, and entropy fix of (Harten 1983) is implemented on original (grid aligned) Roe scheme.

Fig.1 shows that Roe scheme with entropy fixing shows carbuncle phenomenon in quadrilateral grid, but the rotated Roe (without entropy fixing and $\alpha = 45°$) presents clean bow shock and neat density isolines in same grid. Even in the triangular grid, the rotated Roe scheme shows well result. As a reference, the result of van Leer scheme (van Leer 1982), which is a carbuncle free scheme, is shown in fig.1 as well.

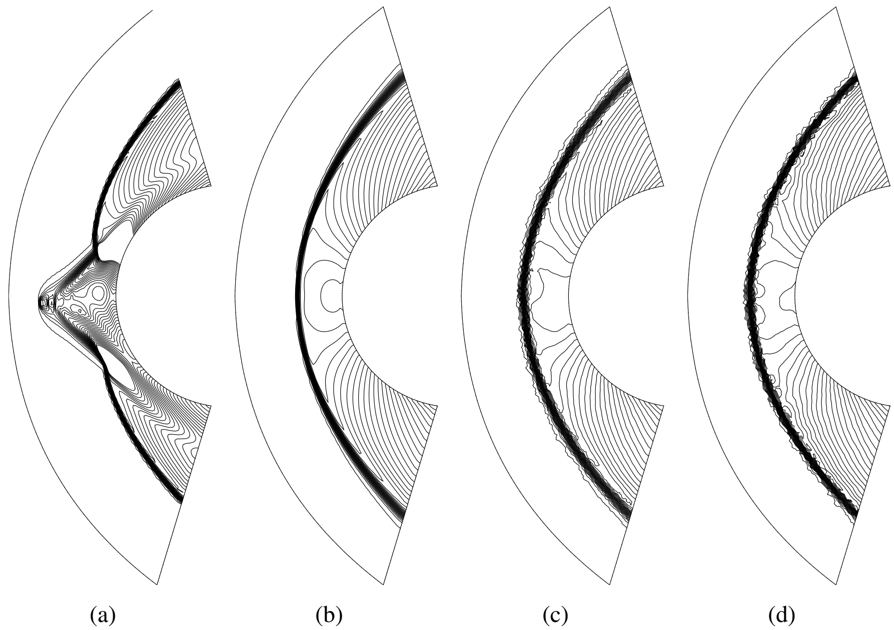

Fig. 1: M = 20 blunt-body test. (a) Roe with entropy fix in quadrilateral grid. (b) Rotated Roe in quadrilateral grid. (c) Rotated Roe in triangular grid. (d) van Leer in triangular grid.

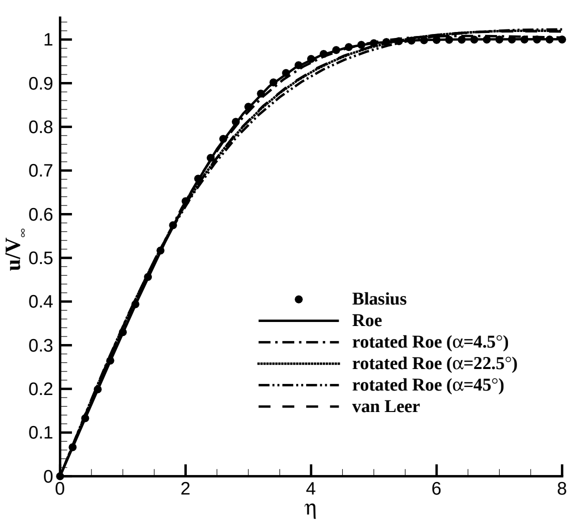

Fig. 2: Velocity profile of laminar boundary layer flow simulations

3.2 Case 2: laminar boundary layer flow

In the former case, rotated Roe scheme presents stabler result compare with entropy fixed Roe scheme. In order to feature out the dissipation more quantitatively, the laminar boundary layer flow simulations are conducted. Fig.2 shows that the rotated Roe with $\alpha = 45^\circ$ is more dissipative compare with the van Leer scheme. Furthermore, the dissipation of rotated Roe scheme is related with the rotation angle α.

4 Conclusion

In this paper, the dissipation property of rotated Roe scheme is investigated through two type of numerical cases. Different from the original rotated Roe scheme, which is presented by Ren (Ren 2003), rotation angles of rotated Roe scheme are fixed and predefined in this study, therefore the dissipation property is isolated with flow features. Simulations of hypersonic inviscid flow around blunt body are conducted in both quadrilateral and triangular grid, the results prove that the rotated Roe scheme is more robust than original grid aligned Roe scheme. The results of laminar boundary layer flow simulations prove that the boundary-layer-resolving capability is deteriorated by rotation procedure and the dissipation level is directly related with rotation angle.

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