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Effect of High-Prandtl Number on Microscale Flow and Heat Transfer

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Abstract **-**The effect of high-Prandtl number on steady fully developed microscale flow and heat transfer between two parallel plates exposed to a constant heat flux is investigated analytically in this study. The Prandtl number over several orders of magnitude ($5 < Pr < 10^3$) as well as different Knudsen number ($0 < Kn < 0.1$) and different modified Brinkman number ($-0.1 < Br < 0.1$) are considered. The momentum and energy equations are solved for first order boundaries of slip velocity and temperature jump. The influences of the Prandtl number, Knudsen number and Brinkman number on the temperature distribution and heat transfer characteristics are discussed.

Keywords**:** Slip flow, parallel plate, microchannel, viscous dissipation

1. Introduction

Fluid flow and heat transfer characteristics of microchannels have become the subject of intense research in recent years with the rapid developments in electronic industry. Many research has been conducted to understand the differences between micro and macro scale flows since microchannels are employed in many industrial applications such as micro-electric chip cooling, biochemical application, biomedical application, and microelectromechanical systems.

Fluid flow and heat transfer characteristics in microchannels depend on the mean free path of the fluid and the characteristic length of the flow field. This ratio is defined as Knudsen number which is a key parameter in microchannels. Knudsen number generally gives the idea for the suitable fluid flow model for the physical system. Hence, fluid flow in microchannels is classified into four groups depending on the Knudsen number (Beskok and Karniadakis, 1994). When $Kn < 0.001$, flow is modelled as continuum flow where Navier-Stokes equation can be used with no slip conditions. When $0.001 < Kn < 0.1$, flow is called slip flow regime and Navier Stokes approach is still used with modified slip boundary conditions. $0.1 < Kn < 10$ is defined as transition regime and $Kn > 10$ is defined as free molecular flow.

Aydin and Avcı (2007) studied analytically on microchannel heat transfer for the laminar flow of rarefied gas between two parallel plates with equal temperature or equal heat flux and investigated the effects of Knudsen and Brinkman number on Nusselt number. Jeong and Jeong (2006) studied micro flow between parallel plates for constant wall temperature and heat flux analytically and reported that the Nusselt number decreases when Knudsen number or Brinkman number increases and as Peclet number decreases. Tunc and Bayazitoglu (2002) investigated heat transfer in rectangular microchannels analytically without considering temperature jump and found that Nusselt number decreases when Knudsen number increases and Nusselt number increases with increasing Knudsen number. Zhang et al (2010) investigated the effects of viscous heating on heat transfer for parallel plates. They reported that viscous heating seriously distorts the temperature profiles. Their results also show that the viscous heating effect increases linearly and decreases non-linearly with an increase in Knudsen number.

Graetz (1983) solved first the convective heat transfer in a cylindircal tube for laminar flow and Barron et al. (1997) solved analytically for slip flow condition. Mecili (2013) investigated slug flow heat transfer between parallel plates for microchannels analytically by including slip flow effects. It is found that Nusselt number decreases with an increase of Knudsen number. Buonomo and Manca (2010) studied slip flow for natural convection in a vertical microchannel with constant heat flux numerically. Results show that the Nusselt number increases with an increase of Kn number for lower Ra numbers and decreases for higher Ra numbers. Bao and Lin (2008) studied heat transfer and fluid flow in micro Poisuille channel including compressibility effects by Burnett equations.

Prandtl number represents the ratio of viscous diffusion to thermal diffusion. When Pr is small, the thermal diffusivity dominates the heat transfer mechanisms. Pr number ranges between below 10^{-3} to above 10^{5}.

Yoo et al. (1994), Yoo (1998) and Custer and Shaughnessy (1997) investigated natural convection in very low Prandtl numbers. Yu et al. (2010) investigated laminar flow in horizontal cylindrical enclosure with an inner coaxial triangular cylinder numerically for Pr numbers between 10^{-2} and 10^{3}. They proposed correlations for Nusselt number with respect to Rayleigh number for several ranges of Pr number.

Ali et al. (2007) investigated the effect of Prandtl number on the hydrodynamic and thermal behaviour of a laminar buoyant free jet. Yoo (1998) studied the effect of Prandtl number on branching and dual solutions for natural convection in a horizontal annulus. Koca et al. (2007) conducted a study on Prandtl number effect on natural convection in triangular enclosures with heating from below. Results showed that heat transfer enhances with the increase of Prandtl number and Prandtl number effects the temperature and flow fields. Sahu et al. (2009) studied the effects of Reynolds and Prandtl numbers on heated 2D square cylinder exposed to an unsteady cross-flow numerically. Results show that local Nusselt number increases with increase of the Prandtl number. Nourgaliyev et al. (1997) made an experimentally and computational study on the effect of Prandtl number on heat transfer for volumetrically heated liquid pools and they found that the effect of the Prandtl number on Nu number is significant particularly on the bottom of the enclosures. They also found that increasing Rayleigh number magnifies the effect of Prandtl number on Nu number. Benkhelifa et al. (2000) conducted a study on the effect of Prandtl number on natural convection heat transfer in cylindrical enclosures and concluded that heat transfer enhances with increasing Prandtl number. Kishore and Gu (2011) investigated the flow and heat transfer characteristics of spheroid particles numerically for moderate Prandtl and Reynolds numbers.

2. Analysis

This study is based on analytical examination of heat transfer and slip flow characteristics in enclosures between two parallel plates exposed to constant heat flux. The coordinate system and geometry of the problem are given in Fig.1. The flow is assumed to be laminar, steady, incompressible and fully developed thermally and hydrodynamically. The thermophysical properties of the fluid are taken to be constant.

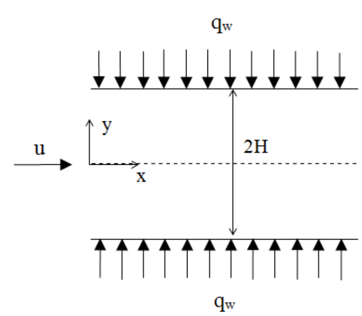

Fig. 1. The geometry and the coordinate system.

The momentum equation and the energy equation including viscous dissipation are:

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx} \tag{1}$$

$$u\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{du}{dy}\right)^2 \tag{2}$$

where μ, α, ν, c_p and T are dynamic viscosity, thermal diffusivity, kinematic viscosity, specific heat and temperature respectively. Navier Stokes approach assumes that the fluid adjacent to the wall obeys no slip condition. Hence, the temperature of the fluid reaches the temperature of wall. On the other hand, in slip flow regime in microchannels, fluid no longer reaches the velocity of the wall or the wall temperature. This means that fluid on the surface slips along the wall having a tangential velocity. The tangential velocity is called slip velocity. Thus, the temperature of the fluid on the surface is also different from the surface temperature. This temperature difference is called temperature jump.

When the wall is stagnant, first order boundaries of slip velocity and temperature jump are defined as follows:

$$u_s = \frac{F_v - 2}{F_v}\lambda\left.\frac{du}{dy}\right|_w \tag{3}$$

$$T_s - T_w = \frac{F_t - 2}{F_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{Pr}\left.\frac{\partial T}{\partial y}\right|_w \tag{4}$$

where u_s is the slip velocity, λ is molecular mean free path, F_v is the tangential momentum accommodation coefficient, T_s is the temperature of the fluid at the surface, T_w is the wall temperature, F_t is the thermal accommodation coefficient, γ is the ratio of specific heats and Pr is Prandtl number. F_v and F_t are considered equal to unity since they take values near unity for most of the engineering applications (Zhang, 2007).

2. 1. Velocity Profile

Dimensionless parameters are defined as:

$$X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{u_m}, \quad U_s = \frac{u_s}{u_m}, \quad P = \frac{pH}{\mu u_m} \tag{5}$$

where u_m is the mean velocity along the x axis and defined as:

$$u_m = \frac{1}{2H}\int_{-H}^{H} u\,dy \tag{6}$$

Dimensionless momentum equation in x-direction for hydrodynamically fully developed Poiseuille flow and boundary conditions can be defined as:

$$\frac{d^2U}{dY^2} = \frac{dP}{dX} \tag{7}$$

$$\left.\frac{dU}{dY}\right|_{Y=0} = 0 \quad \text{at} \quad Y = 0 \tag{8}$$

$$U_s = -2Kn\left.\frac{dU}{dY}\right|_{Y=1} \quad \text{at} \quad Y = 1 \quad \text{and} \quad Kn = \frac{\lambda}{H} \tag{9}$$

Subjected to the boundary conditions defined in Eqs. (8) and (9), analytical solution of the Eq. (7) is obtained as

$$U = \frac{3}{2}\left[\frac{1 - Y^2 + 4Kn}{1 + 6Kn}\right] \tag{10}$$

2. 2. Temperature Profile

Constant heat flux on the wall is defined as:

$$q_w = k\frac{\partial T}{\partial y}\bigg|_{y=H} \tag{11}$$

The variation of fluid temperature along x axis equals to the variation of the wall temperature. Thus

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} \tag{12}$$

Dimensionless temperature is defined as

$$\theta(Y) = \frac{T - T_s}{\frac{q_w H}{k}} \tag{13}$$

Therefore, the energy equation becomes

$$\frac{\partial^2\theta}{\partial Y^2} = \left[aU - Br\left(\frac{dU}{dY}\right)^2\right] \tag{14}$$

where $a = \frac{Hu_m}{q_w}\frac{dT_s}{dx}$ and Br is the modified Brinkman number for constant heat flux condition and it is defined as:

$$Br = \frac{\mu u_m^2}{q_w H} \tag{15}$$

Dimensionless boundary conditions are:

$$\frac{d\theta}{dY}\bigg|_{Y=0} = 0 \quad \text{at} \quad Y = 0\,, \tag{16}$$

$$\theta = 0\,, \quad \frac{d\theta}{dY}\bigg|_{Y=1} = 1 \;\text{ at }\; Y = 1 \tag{17}$$

Under the thermal boundary conditions in Eqs. (16) and (17), the solution of energy equation can be obtained as:

$$\theta(Y) = \left[\frac{3Br}{(1+6Kn)^3} + \frac{1}{1+6Kn}\right]\left[-\frac{1}{8}Y^4 + \frac{3}{4}Y^2 + 3Kn(Y^2 - 1) - \frac{5}{8}\right] - \frac{3Br}{4(1+6Kn)^2}\left[Y^4 - 1\right] \tag{18}$$

a is defined as:

$$a = \left[\frac{3Br}{(1+6Kn)^2} + 1\right] \tag{19}$$

The dimensionless temperature can be transformed into a more appropriate form by the following relation:

$$\frac{T_s - T_w}{\frac{q_w H}{k}} = -\frac{4\gamma}{\gamma+1}\frac{Kn}{Pr} \tag{20}$$

Thus, the dimensionless temperature in terms of wall temperature can be defined as:

$$\tilde{\theta} = \frac{T - T_w}{\frac{q_w H}{k}} = \left[\frac{3Br}{(1+6Kn)^3} + \frac{1}{1+6Kn}\right]\left[-\frac{1}{8}Y^4 + \frac{3}{4}Y^2 + 3Kn(Y^2-1) - \frac{5}{8}\right] - \frac{3Br}{4(1+6Kn)^2}[Y^4 - 1] - \frac{4\gamma}{\gamma+1}\frac{Kn}{Pr} \tag{21}$$

The mean temperature of the fluid can be defined as:

$$T_m = \frac{\int \rho u T dA}{\int \rho u dA} \tag{22}$$

Dimensionless mean temperature in terms of Br number is

$$\tilde{\theta}_m = \frac{T_m - T_w}{\frac{q_w H}{k}} = -\frac{1}{3}\left[1 + \frac{12\gamma}{\gamma+1}\frac{Kn}{Pr}\right] - \frac{2Br}{35(1+6Kn)^4} - \frac{11Br}{35(1+6Kn)^3} - \frac{2(1+21Br)}{105(1+6Kn)^2} - \frac{2}{15(1+6Kn)} \tag{23}$$

Convective heat transfer coefficient for forced convection is:

$$h = \frac{q_w}{T_w - T_m} \tag{24}$$

Thus, Nusselt number can be defined as:

$$Nu = -\frac{2}{\tilde{\theta}_m} \tag{25}$$

Therefore, Nusselt number can be obtained in terms of Br, Kn and Pr numbers as follows:

$$Nu = 2\left\{\frac{1}{3}\left[1 + \frac{12\gamma}{\gamma+1}\frac{Kn}{Pr}\right] + \frac{2Br}{35(1+6Kn)^4} + \frac{11Br}{35(1+6Kn)^3} + \frac{2(1+21Br)}{105(1+6Kn)^2} + \frac{2}{15(1+6Kn)}\right\}^{-1} \tag{26}$$

3. Results and Discussion

Slip flow and heat transfer between two parallel microplates exposed to a constant heat flux has been investigated in this study. The variation of Nusselt number has been investigated for different Kn values in the range of $0 \leq Kn \leq 1.0$ and different Br values in the range of $-0.1 \leq Br \leq 0.1$ and high Prandtl numbers.

Variation of the Nusselt number with the Brinkman number for constant Knudsen and Prandtl numbers is shown in Fig. 2. The intersection seen at negative Br near zero changes the Kn-Nu interaction in opposite direction. Positive values of the Brinkman number means that the fluid is being heated by a hot wall. Therefore, increase in Kn number increases heat transfer rate as a result of higher temperature jump. Similar trend for the variation of Nusselt number with Brinkman number was also observed in the study conducted by Zhang et al. (2010). It is also clearly seen from Fig. 2 that the effect of Pr number on heat transfer is negligible for low values of Kn number. On the other hand, the effect of Pr number on heat transfer is more pronounced for high values of Kn number.

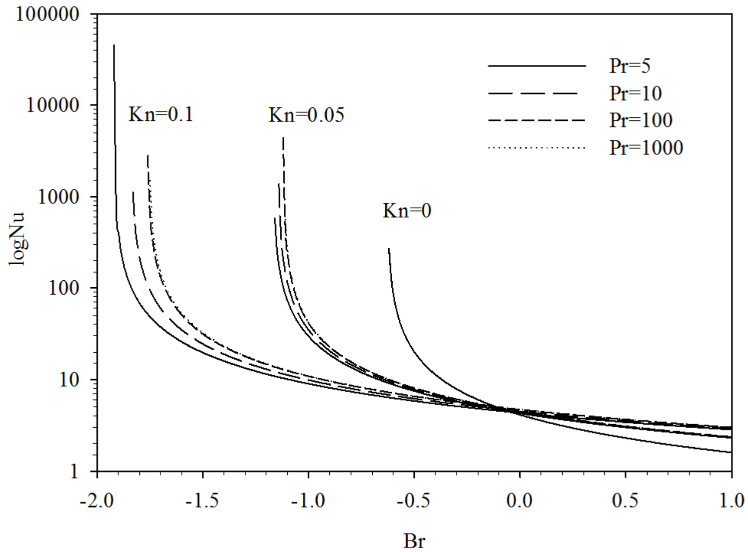

Fig. 2. Variation of the Nusselt number with the Brinkman number for various values of Kn and Pr numbers.

Variation of the Nusselt number with Prandtl number is shown in Fig. 3 for various values of Knudsen and Brinkman numbers. The effect of Prandtl number on Nusselt number becomes more significant when the Prandtl number is below 40. For values above 40, Nu number is approximately constant. As the Prandtl number gets higher values, the thermal boundary layer gets thinner. Therefore, Nu number converges to a constant value. For high values of Knudsen number, Nu number converges to a constant value for higher values of Prandtl number. This is as a result of higher temperature jump for high values of Knudsen number. It can therefore be concluded that Pr number becomes more effective for smaller microchannels.

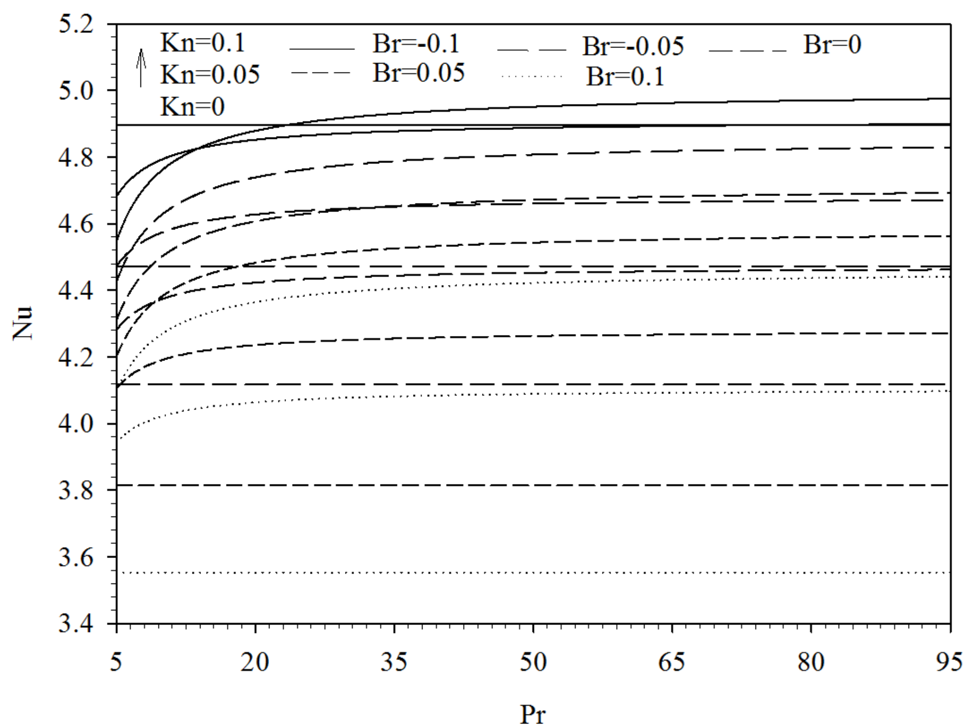

Fig. 3. Variation of the Nusselt number with the Prandtl number for various values of Kn and Br numbers.

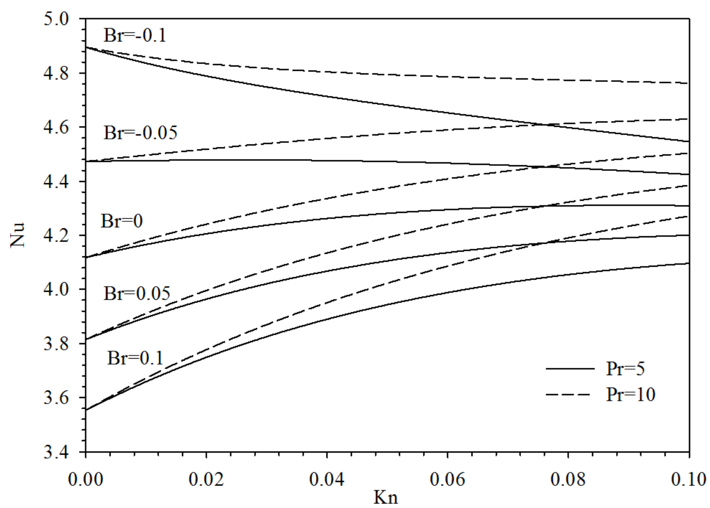

Fig. 4. Variation of the Nusselt number with the Knudsen number for various values of Br and Pr numbers.

The variation of Nu number with Kn number is shown in Fig. 4 for various values of Br number and Pr number. For Knudsen numbers below 0.01, the effect of Prandtl number on Nusselt number is negligible. Negative values of Br number corresponds to the case that the fluid is cooled by a cold wall. Therefore, Nu number shows a decrease with increasing Kn number depending on higher temperature jump. Positive values of Br number corresponds to the case that the fluid is heated by a hot wall. Therefore, Nu number shows an increase with increasing Kn number depending on higher temperature jump. As the Pr number is increased, the Nu number gets higher values depending on a thinner thermal boundary layer. The effect of Pr number on Nu number is more pronounced for high values of Kn number.

Variation of Nu number with the Kn number is given in Fig. 5 for various values of Br and Pr numbers. The Prandtl effect becomes less significant for high values of Pr number. The effect of viscous heating on heat transfer rate is more significant for low values of Kn number. For high values of Pr number, the effect of Br number on heat transfer rate becomes less effective with a decrease in Br number when Br number is negative.

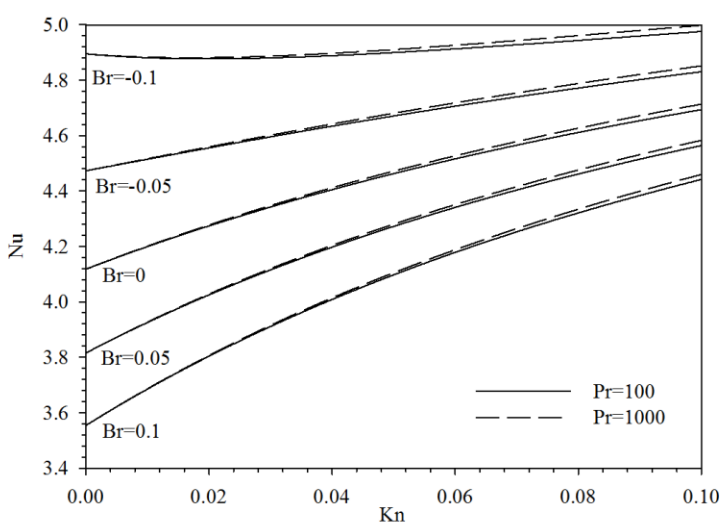

Fig. 5. The effect of Knudsen number on Nusselt Number for various values of Brinkman and Prandtl number.

4. Conclusion

The effect of high-Prandtl number on heat transfer and slip flow between two parallel plates exposed to a constant heat flux is investigated analytically in this study. The results show that the effect of Pr number on heat transfer is negligible for low values of Kn number. On the other hand, the effect of Pr number on heat transfer is significant for high values of Kn number. For high values of the Prandtl number, the effect of Br number on heat transfer rate becomes less effective for low values of Br number when Br number is negative.

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