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Sedimentary Waves in Laminar Flows of Pseudoplastic Fluids

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Abstract **–**In this paper, results of an experimental study aimed to determine the characteristics of bedforms generated by non-cohesive granular particles under the action of subcritical laminar flows of pseudoplastic fluids in open channels are reported. In terms of proper dimensionless parameters, conditions for existence of bedforms are determined. Relationships among dimensionless geometric characteristics and celerity with those parameters defining the flow conditions and sediment properties are also found.

Keywords**:** bedforms, non-Newtonian fluid, laminar flow, sedimentary waves

1. Introduction and Objective of the Paper

Sedimentary waves or bedforms are a characteristic feature of flows over deformable beds; particularly those formed by non-cohesive granular particles, and they can be found as much in nature as in industry. Most of the studies developed to characterize de properties of the bedforms (wavelength, height, celerity) have been carried out with Newtonian fluids, mainly water. However, in nature and industry, bedforms generated by non-Newtonian fluid flows are also found. The mixture of fine solid particles with water can be regarded as an equivalent fluid, behaving as pseudoplastic or plastic (i.e. yield stress) fluids. Common examples are mudflows in nature and slurry flows in industry. In nature, bedforms are a normal feature of river or stream morphologies, and geometric characterization of the bed deformation is useful for design purposes. In the other hand, most of the industrial applications involving solid-fluid mixtures require to transport the solids in the suspension mode. Thus, programed or accidental stop of the flow (as it occurs in the mining industry) results in sedimentation of the solids and their deposition on the bottom, generating bedforms that have to be taken into account in the dimensions of the canals when they are designed.

The objective of this paper is to present some experimental results obtained in open-channels regarding the geometric characteristics of bedforms generated by subcritical laminar flows of a pseudoplastic fluid on a bed constituted by uniform non-cohesive solid particles, the celerity at which these sedimentary waves move, and the flow conditions under which the forms are found.

2. Literature Review

As already mentioned, sedimentary waves, or bedforms, have been extensively studied in water flows, and many relations to predict their existence and characteristics have been proposed. Restricting our attention to flows of liquids, among the firsts (if not the first) description of the triangular shape of dunes in sand rivers is that by Du Buat (1816, p. 100). In his book, Du Buat describes the motion of the sand grains and the subsequently generation of bedforms depicted by two sloping planes (steeper the downstream one), and their periodic nature. A summarized description of the generation and evolution dynamics of bedforms in turbulent flows is presented in Leeder (1983). A classic reference is the work by Vanoni (1974), where the bedform that can be developed according to the flow, fluid and sediment characteristics are presented in a set of graphs. Also in turbulent flows, it is interesting to note the experimental work by Blasius (1910), who was the first one to show the importance of the Froude number in the type of bedforms generated. Regarding their dimensions, the study by Shinohara and Tsubaki published in 1959 and reported in Yalin (1977), and those by Yalin himself are worth to mention. Other relationships to estimate the bedform height and wavelength have been presented by Julien and Klaassen (1995), van Rijn (1984), Karim (1999), etc. Raudkivi (1997) and Baas (1999) have proposed relationships for the height and wavelength for ripples, but they are not dimensionally homogeneous. The studies by Coleman and Melville (1996) and Coleman and Elling (2000) were aimed to know the dimensions of bedforms in laminar flows. Gilbert (1914, p. 232) reports the results of "extensive observations of dunes in the river Loire" by Sainjon who in 1871 published an equation for the "rate of dune advance", or wave celerity. Yalin (1977) presents a couple of results obtained by Russian researchers regarding the bedform celerity. Coleman and Melville (1994) provide a relationship for the celerity of bedforms in their initial state of development ("sand wavelets").

The references above mentioned refer to sedimentary waves generated under the flow of Newtonian liquid, mostly water. Studies regarding bed forms in non-Newtonian fluid flows are limited, and most of them only describe presence of bedforms. Working with a clay-water mixture, behaving as a Bingham plastic fluid, Wan (1982) observed dunes and transition to flat bed in his experiments. He also determined the dune height and its migrating velocity. The sequence dune – flat bed – antidune was reported in Wan and Song (1987). Rickenmann (1990) in his Doctoral Thesis mention the study by Kikkawa and Fukukua from 1969 in which using very fine sand as washload, the authors detected the sequence dunes – flat bed – antidunes when the washload concentration increased. Bradley (1986), also reported by Rickenmann (1990), confirmed the presence of dunes and plane beds in flows of bentonite suspensions.

3. Experiments

3. 1. Experimental Setup and Materials

Experiments were carried out in a rectangular cross sectional tilting flume with dimensions 3.0×0.21×0.15 m^3 (length×width×height). Fluids were recirculated by means of a stainless steel centrifugal pump connected to a 0.20 m^3 tank at the suction side. A heat exchanger was installed in the recirculation loop to control the fluid temperature. Before the head reservoir of the flume, a small tank was installed in order to ensure a constant discharge. The channel slope could be adjusted between horizontal and 100%. Flow height could be controlled by means of a gate, located in the downstream end of the flume, which was followed by a trap to collect the particles transported by the flow. The solid particles that formed the bed were glass spheres of nearly uniform size, and they formed a layer 1 cm thick, which was flattened using a smoothing board at the beginning of the experiments. Three sizes were used in the experiments, characterized by the mean diameter of the glass beads, namely, 75, 106 and 202 μm. Density of particles was ρ_s = 2650 kg/m^3. A pseudoplastic fluid was generated by aqueous solutions of sodium carboxymethyl cellulose (CMC). These solutions are transparent, allowing visualization of the bed particles. All mixtures were carefully blended to avoid the formation of solid lumps in the mixture. To this purpose, they were first prepared in small, 800 ml beakers by slowly mixing the powder at the required concentrations with warm water. Density of the solutions was measured by a Gamma RTM Dr J

Ambrus densimeter, with a scale span from 1.00 to 1.05 relative to distilled water at 1 atm and 15 °C. Resulting densities were in the range 1003 – 1012.3 kg/m3. Rheology was determined using an Anton Paar Rheolab QC concentric cylinder equipment before and after each experiment. Both measurements were considered to fit the parameters of the Ostwald-de Waele rheological model, with a constitutive relationship that is greatly simplified for two dimension slender flows to:

$$\tau = K\dot{\gamma}^n \tag{1}$$

where τ is the shear stress, $\dot{\gamma}$ is the strain rate, K is the consistency coefficient and n is the flow index. Three different concentrations of CMC generated solutions with values (K, n) corresponding to (0.022, 0.86), (0.048, 0.82), and (0.032, 0.78). Dimensions of K are Pa s^n.

For each combination of fluid rheology and particle size, several flow depths and channel slopes were considered, totalling 108 experiments. Additionaly, 28 experiments with water were carried out.

3. 2. Experimental Flow Conditions

Experimental conditions were in the following ranges: channel slope: 1 – 3%; flow depth (h): 3.8 – 6.4 cm; mean flow velocity (U): 0.02 – 0.65 m/s. The Froude number, $Fr = U/\sqrt{gh}$, was in the range 0.06 – 0.7. A modified Reynolds number was used: $Re_K = \rho U^{2-n} h^n / K$ (here, ρ is the fluid density), which was in the range 68 – 1500.

In order to define the flow regime, Haldenwang et al. (2010) criterion was used. The criterion defines a modified Reynolds number given by $Re = 8\rho U^2/(K(8U/D_h)^n)$, where $D_h = 4A/P$ is the hydraulic diameter, with A the flow area and P the wetted perimeter. The flow regime is laminar when Re is less than a critical Reynolds number obtained from:

$$Re_C = 853.1\left(\frac{\mu|_{\dot{\gamma}100\,s^{-1}}}{\mu_w}\right)^{-0.21} Fr + 1.263\times 10^4\left(\frac{\mu|_{\dot{\gamma}100\,s^{-1}}}{\mu_w}\right)^{-0.75} \tag{2}$$

where $\mu|_{\dot{\gamma}100\,s^{-1}}$ is the equivalent dynamic viscosity at a shear rate of 100 s^{-1} and μ_w is the dynamic viscosity for water. In the experiments reported in this paper, the critical Reynolds number, Re_C, took values between 962 and 2654.

4. Experimental Results

When the experimental conditions were suitable for generation of bedforms, striations emerged few minutes after the start of the experiment, followed by a slight deformation of the bed in the central part of the channel that evolved into a well-defined wave as time went on. Later, this bedform began to expand across the channel width and to move in the flow direction. In most of the experiments, the initially separated waves began to join each other increasing their size. Finally, after a time lapse, the bedforms covered totally the bottom and moved downstream without growing their amplitude, moment that was considered that the bedforms had reached their equilibrium condition. The time needed by the bedforms to attain this condition depended on the flow discharge. For low discharges, equilibrium was reached in less than one hour, but larger discharges demanded times larger than four hours, after initiated the experiment. As an example, images of the bedforms generated, and later analyzed, are presented in Fig. 1. In some experiments, Barchan type dunes were observed and they were not included in the analysis in this paper.

The data obtained from the experiments were processed and they are presented in the following paragraphs. It was found that presence of bedforms can be determined in terms of the Froude and modified Reynolds numbers, as shown in Fig. 2. Black filled symbols correspond to experiments with bedforms, open symbols correspond to those experiments without movement of the particles, and × symbols correspond to the case where particle motion existed, but not noticeable bedform could be distinguished. Circles, triangles and squares correspond to fluid with rheology defined by n equal to 0.77, 0.82 and 0.86, respectively. Rhomboids define experiments with water. From the experimental data presented in Fig. 2, a region where presence of bedforms is observed can be defined. In the figure, it is delimited by two straight lines, and the region satisfies the following inequalities:

$$Re_K \geq 10^{\frac{Fr+1.84}{1.15}} \text{ and } Fr \geq 0.2 \tag{3}$$

Because there is no dependency on the properties of the solid particles, application of the above condition (or Fig. 2) should be limited to particles with densities and sizes similar to those used in the experiments reported in this paper.

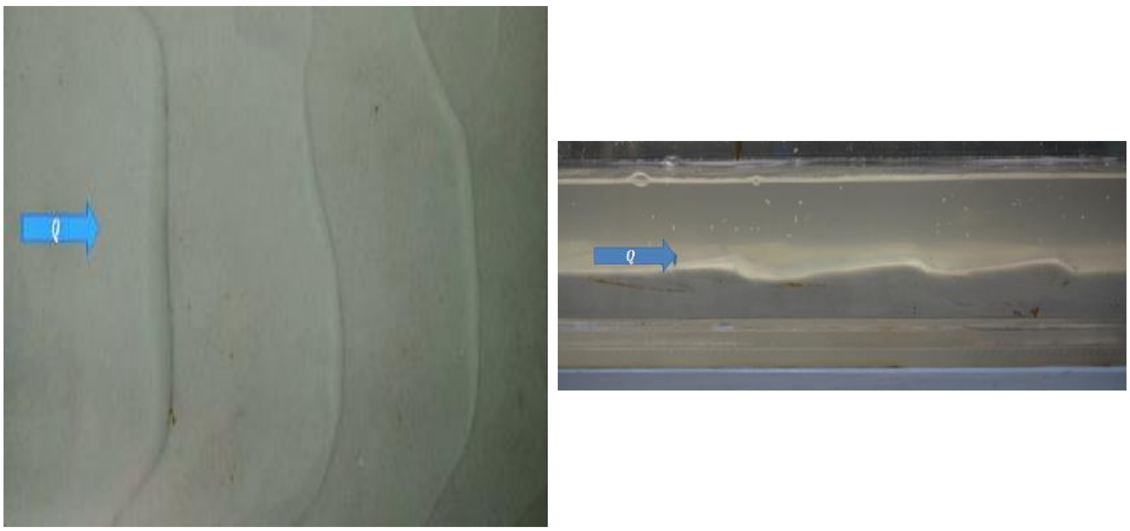

Fig. 1. Images of bedforms generated in the experiments and analysed in this paper.
The left panel is a top view of the sedimentary waves, expanding across the flume.
The right panel is a lateral view of the bedforms. Both images are not at the same scale.

It has been found that the wavelength of bedforms in their initial stages depends on the sediment diameter (e.g., Coleman and Melville, 1996). Thus, a modified Reynolds number based on the particle diameter, d, is defined as $Re_{Kd} = \rho U^{2-n} d^n / K$. The length of the bedform, λ, is made dimensionless with then flow depth. The experimental data characterizing the length of the sedimentary form has been plotted in Fig. 3, and a fitting to data is given by

$$\frac{\lambda}{h} = 0.75 Re_{Kd}^{0.38} \tag{4}$$

Considering that the friction coefficient is related to the amplitude Δ of the wave, it is valid to assume that, in some way Δ can be related to the total shear acting on the bottom. The bottom shear stress for a gradually varied flow (τ_0) of a pseudoplastic fluid can be related to that corresponding to uniform flow (τ_{0U}) according to the following relationship (Tamburrino et al., 2014):

$$\frac{\tau_0}{\tau_{0U}} = \left(\frac{2n+1}{n}\right)^n \frac{Fr^2}{Re_K \mathrm{sen}\theta} \tag{5}$$

The shear stress for a gradually flow is related to the energy gradient, J, by means of the relation $\tau_0 = \rho g h J$ where g is the acceleration due to gravity. For uniform flow, $\tau_{0U} = \rho g h \,\mathrm{sen}\,\theta$, where θ is the angle formed by the bottom of the channel with the horizontal. Thus, Eq. 5 becomes

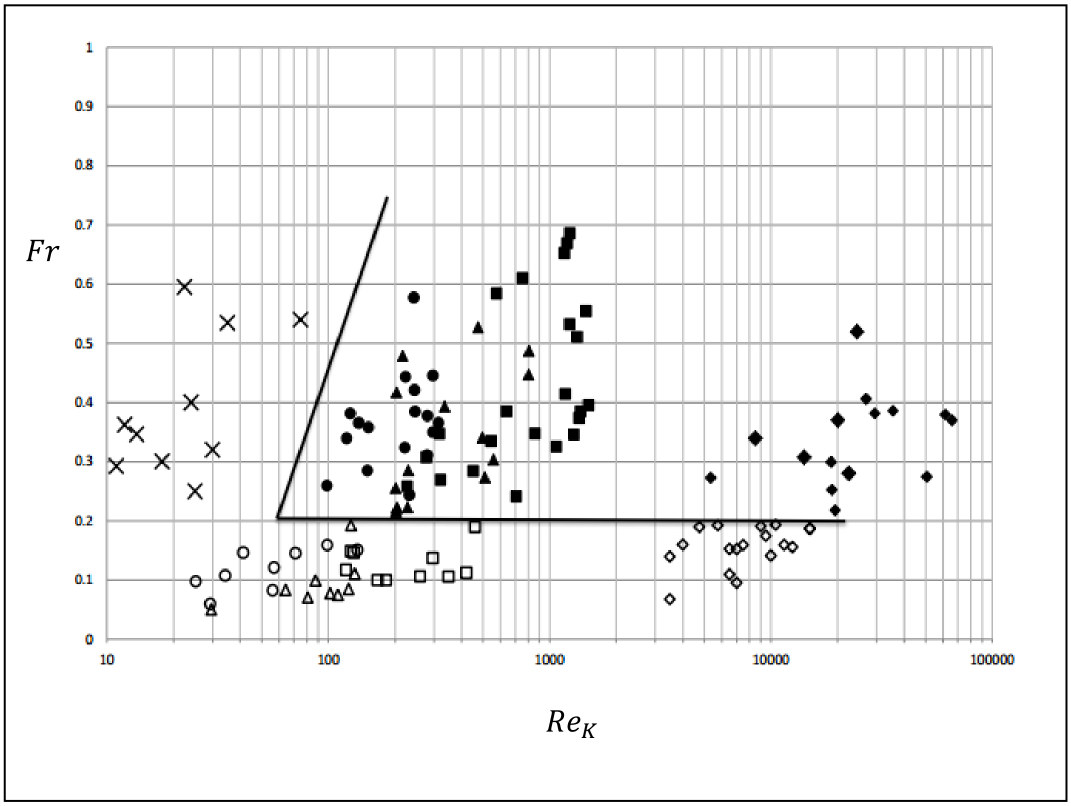

Fig. 2. Flow conditions showing presence of bedforms. Black filled symbols correspond to those flows with sedimentary waves.

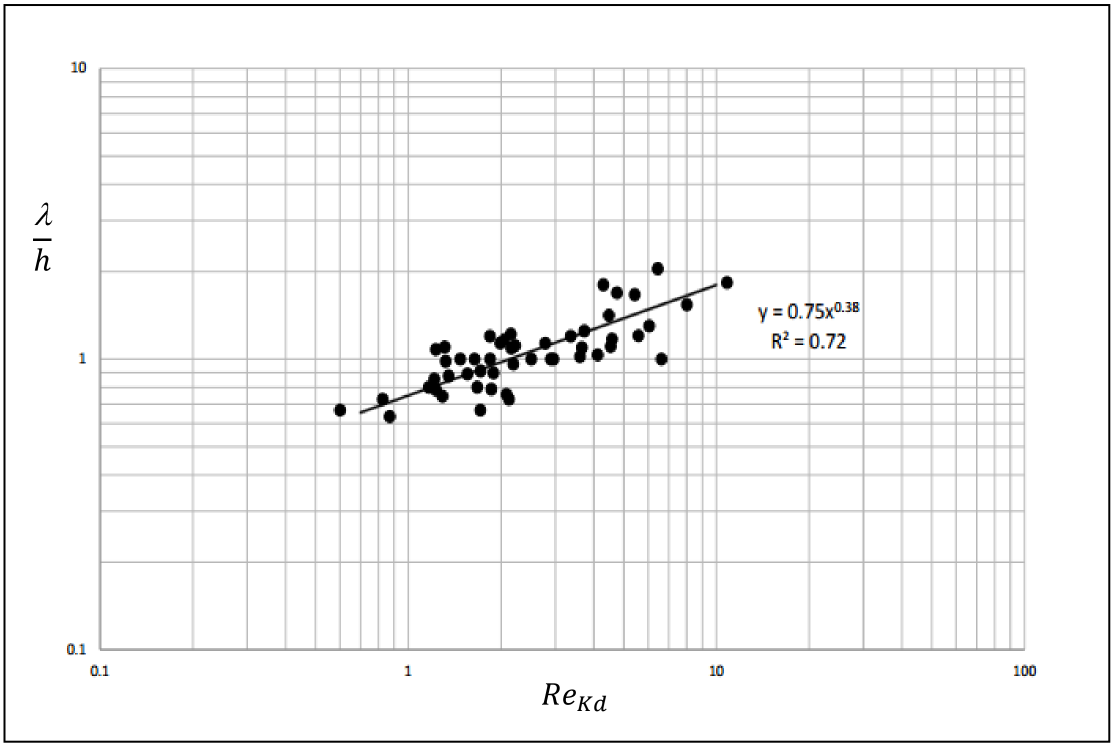

Fig. 3. Dimensionless wavelength of bedforms

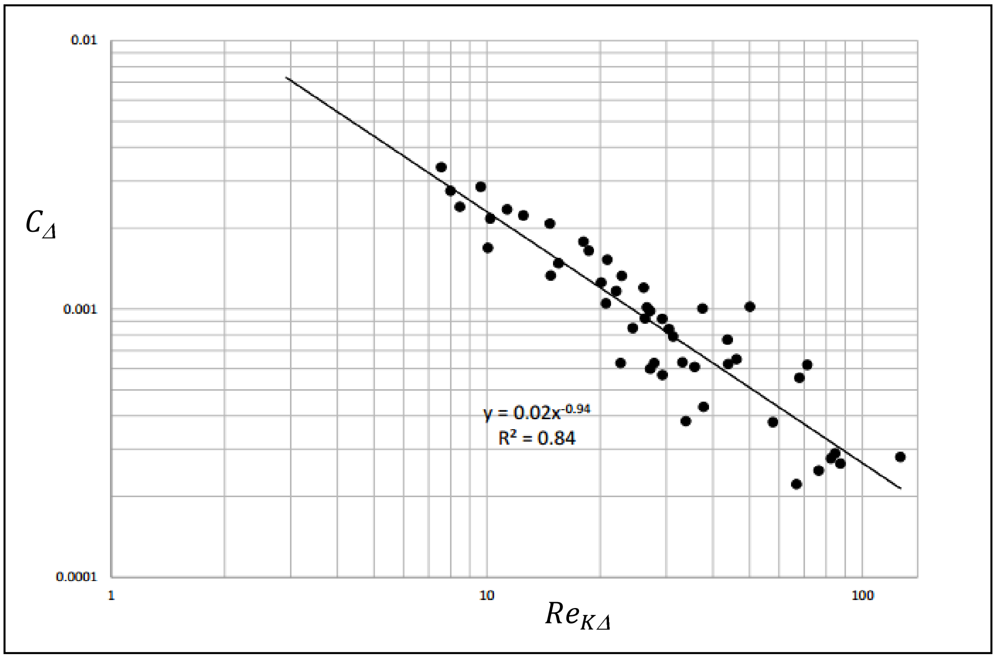

Fig. 4. Dimensionless height of bedforms

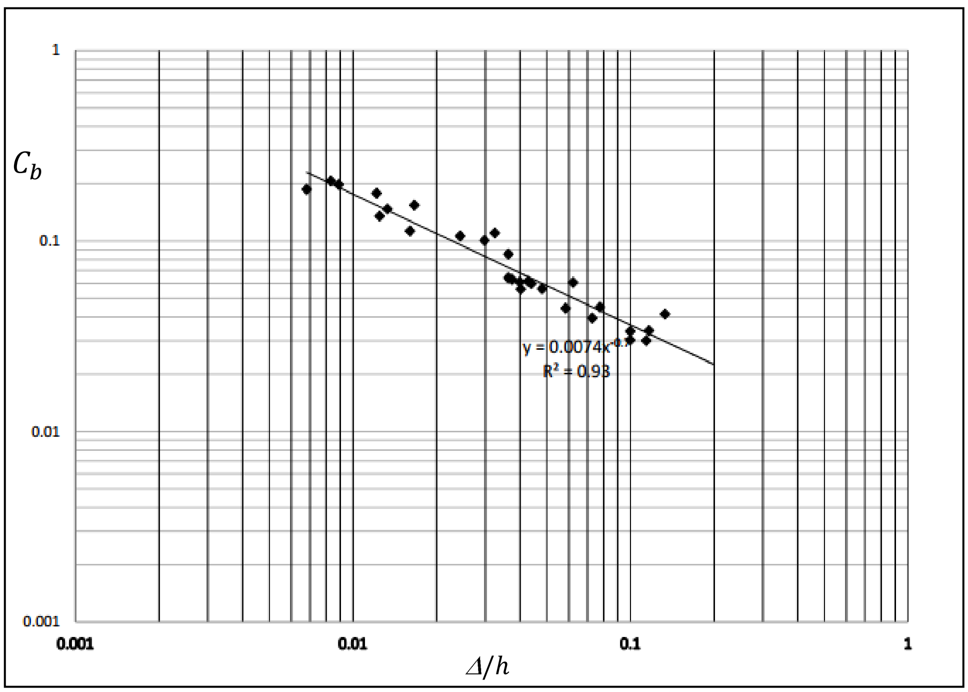

Fig. 5. Dimensionless celerity of the bedforms

$$J = \left(\frac{2n+1}{n}\right)^n \frac{Fr^2}{Re_K} \tag{6}$$

Thus, it is licit to expect a relationship between J and a dimensionless form of the bedform height, Δ. Because the flow is in the laminar regime, the following bedform modified Reynolds number is proposed: $Re_{K\Delta} = \rho U^{2-n} \Delta^n / K$. Defining the right hand side of Eq. 6 as C_Δ , the dimensionless bedform height is presented in Fig. 4. The best fit to the data gives the following relationship:

$$C_{\Delta} = 0.02\mathrm{Re}_{K\Delta}^{-0.94} \tag{7}$$

It is interesting to note the exponent of the Reynolds number very close to -1. Thus, considering that $C_{\Delta} \sim Re_{K\Delta}^{-1}$, then $(\Delta/h) \sim Fr^{-2}$, resulting that the wave amplitude depends on the flow depth (like dunes in turbulent Newtonian flows) and it is independent of the consistency (or viscosity) coefficient of the fluid. The proportionality coefficient is a function of the flow index, n.

During the experiments, it was observed that the celerity c of the bedform decreases when its height increases, as it has been reported by other authors in Newtonian fluid flows. Having this in mind, an inverse relationship between the dimensionless celerity and the dimensionless bedform height was expected, as it is observed in Fig. 5, where C_b is defined as $C_b = c\ g\left(\frac{\rho_s}{\rho} - 1\right) d/u_*^3$, with u_* the shear velocity. The best fit to the data is given by

$$C_b = 0.0074\left(\frac{\Delta}{h}\right)^{-0.7} \tag{8}$$

5. Conclusion

The paper is a contribution to the study of the characteristics of bedforms generated in non-cohesive granular beds under the action of laminar flows of pseudoplastic fluids. It is found that, roughly, bedforms develop for Froude number, Fr, greater than 0.2, and modified Reynolds number, Re_K, greater than approximately 80. It has to be noted that, because the conditions for developing do not involve the particle characteristics (diameter and density), this condition is valid only for the solids particles used in the study. Relationships for the height, wavelength and celerity of the bedforms were also determined. Qualitatively, they behave as their counterpart generated in Newtonian fluid flows, i.e., wavelength grows with the particle diameter and celerity decreases with the height. Given the laminar regime of the flow, an unexpected result was obtained regarding the independence of the height with the consistency coefficient, K, of the fluid, aspect that demands further studies.

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