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An Modified EPQ Model with Deteriorating Production System and Deteriorating Product

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Abstract -The proposed study considers both product deterioration and production process deterioration in a traditional Economical Production Quantity model. In this model, backlog and rework process are also taken into consideration while the production process is assumed to be imperfect. Unserviceable products will be manufactured as a result. Unserviceable products are kept in storage until the end of the normal production run and then reworked to condition that can meet the standard of customer requirement. The production quality is also assumed to be deteriorative. The production process is assumed to be in control when the production run starts and the defective rate is relatively. At a certain point, the deterioration and switch of states occurs and the defective rate increases to a higher value. After the normal production cycle, the state of machine will be restored in control state. The overall objective of this study is to minimize the expected total cost by determining the combination of total production quantity and backlog quantity.

Keywords: EPQ; Deteriorating; Exponential; Perishable; Stochastic

1 Introduction

In the increasingly competitive global market, manufacturing companies need to reduce their operational cost as much as they can to be able to survive. While Inventory cost and production cost are two of the most important factors in their operation cost. Hence the inventory and production activities should be planned and scheduled to minimize the associated cost. Both academic world and industry have focused on this field for the last half-century. Among all the research models, Economic Production Quantity (EPQ) is a classical model developed to address the production-inventory control problem (Muller, 2003). But there are a lot of impractical assumptions made in original model. For example, demand is normally assumed to be constant and products are viewed as perfect and the quality does not change with time (Cardenas-Barron ,2001). In the recent years, different elements have been considered into the traditional model, For example, trade policy (Teng et al, 2009), stock and price dependent demand (Teng et al,2005). In this study, a model considering both product and process deterioration is proposed.

Rosenblatt and Lee (1986) first studied the Economic Production Quantity model with imperfect production process. In their model, a production process shifts from a good condition to a bad condition in a random time. The results showed that the production run time was shorter than the one in classical model. They also extended the model by taking setup up cost dependent deteriorate rate into consideration. Khouja and

Mehrez (1994) proposed a modified EPQ model where the production rate was taken as a control variable and the deterioration rate is dependent on the production rate.
For product deterioration, Goyal and Giri (2001) have summarized the causes for deterioration such as damage, dryness, vaporization etc. The earliest study on deteriorating products can be traced back to the 1960s, where Hadley and Whitin (1963) first developed an inventory model and the product having an obsolescence date. Later on, several production lot sizing inventory models considering deteriorating products were proposed. Mak (1982) looked at an exponentially decaying case and backlog was also allowed in his model.
The work in this paper extends the model proposed in (Tai, 2013) in which two EPQ models with deteriorating/imperfect products are developed. In Tai's study (2013), the inspection quality of imperfect products was also integrated and examined the impact of selling imperfect products on the corporate image. However, the deterioration of the production process was not considered in his model. Another similar research study can be found in the work done by Pal et al. Our main contribution to this research topic is to provide a modified EPQ model considering both product and production process deterioration at the same time. At the same time, rework and backordering are also modelled. In Section 2, mathematical modeling of the proposed model is provided and corresponding equations are listed as well. Section 3 details the numerical examples and analysis. In the end, the conclusions are made in Section 4.

2 Mathematical Modelling

The proposed model can be divided into three different scenarios according the occurrence of switch of machine states. In Scenario 1, machine switches states during the backlog state, while in Scenario 2, the switch of states occurs in inventory surplus period. Scenario 3 assumes that the switch happens after normal production time, so it won't affect the production process. Because of the page limit, only the equations for Scenario 1 is shown. Figure 1 illustrates the inventory behaviour of Scenario 1.
Figure 1 shows the inventory behavior of the perfect product. In this case, T_1 is divided into two separate periods $T_{1\alpha}$ and $T_{1\beta}$. During $T_{1\alpha}$, the defective rate remains as α, while in $T_{1\beta}$, the defective rate changes to β. During T_3, the imperfect products are reworked together, and in T_4 and T_5 the normal production process stops and the current inventory is consumed. The inventory in each time period can be described by the differential equations below:

$$I_{1\alpha}(t_{1\alpha}) = \Big((1-\alpha)p-\mu\Big)t_{1\alpha} - B, \quad 0 \le t_{1\alpha} \le T_{1\alpha} \tag{1}$$

$$I_{1\beta}(t_{1\beta}) = \Big((1-\beta)p-\mu\Big)t_{1\beta} + \Big((1-\alpha)p-\mu\Big)T_{1\alpha} - B, \quad 0 \le t_{1\beta} \le T_{1\beta} \tag{2}$$

$$I_2(t_2) = \Big(\frac{(1-\beta)p-\mu}{\delta}\Big)(1-\exp(-\delta t_2)), \quad 0 \le t_2 \le T_2 \tag{3}$$

$$I_3(t_3) = \Big(I_s - \frac{(1-\gamma)p-\mu}{\delta}\Big)\exp(-\delta t_3) + \frac{(1-\gamma)p_r-\mu}{\delta}, \quad 0 \le t_3 \le T_3 \tag{4}$$

$$I_4(t_4) = \Big(I_m + \frac{\mu}{\delta}\Big)\exp(-\delta t_4) - \frac{\mu}{\delta}, \quad 0 \le t_4 \le T_4 \tag{5}$$

$$I_5(t_5) = -\mu t_5, \quad 0 \le t_5 \le T_5 \tag{6}$$

The overall total cost function can be represented as

$$E\widetilde{TC} = \int_0^{B/a_1} TC_1(\theta)f(\theta)d\theta + \int_{B/a_1}^{Q/p} TC_2(\theta)f(\theta)d\theta + \int_{Q/p}^{y} TC_3(\theta)f(\theta)d\theta. \tag{7}$$

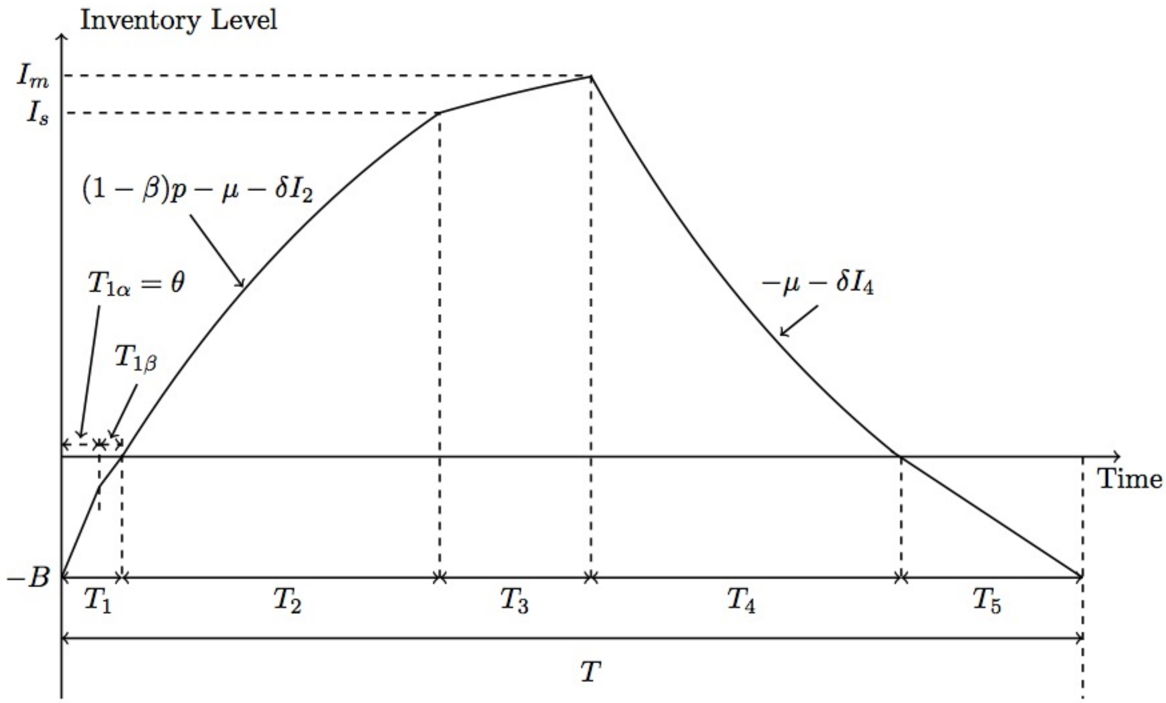

Fig. 1: The inventory behavior in Scenario 1

3 Results

In this section, numerical examples are first provided to prove the convexness of the cost function and to determine the optimal solutions for the proposed model. The graphs of expected total cost per product against both Q and B are shown in Figures 2 and 3. The two graphs also help prove the convexness of the proposed cost function.The optimum combination of Q and B can then be determined as $(Q^*,B^*) = (4223,456)$. And the corresponding optimal total cost per unit product is:

$$ETC^* = 29.3$$

4 Conclusions

On the topic of EPQ with an imperfect production system, a large amount of research has been done. Particularly for the deteriorating production systems, the extant literature has provided a comprehensive analysis. However, as discussed earlier, in some industries the phenomenon of product deterioration cannot be neglected. What is the inventory behavior under the condition of both product and processes deterioration needs to be examined and analyzed. This study has modelled such a problem and considered rework and backordering at the same time. Both the production and rework process are assumed to be imperfect, but only the normal production process is subject to deterioration. At the end of each production run, maintenance activities are applied to recover the production system back to a good condition. The optimal expected total cost per unit product is obtained by determining the total production quantity and backlog quantity at

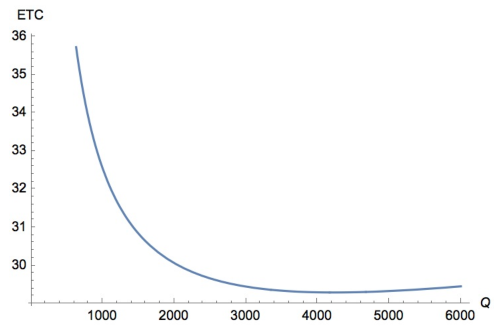

Fig. 2: The plot of expected total cost per unit product against *Q* and *B*

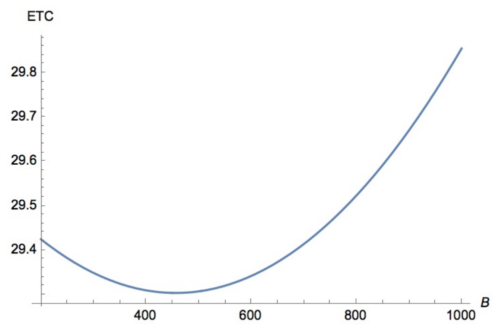

Fig. 3: The plot of expected total cost per unit product against *Q* and *B*

the same time. Numerical experiments are carried out and used to illustrate the performance of the proposed model.

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