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A Phenomenological-Based Semiphysical Model for Hydrocyclones

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Abstract- In this paper, a Phenomenological-based Semiphysical Model (PBSM) is developed to predict the behavior of hydrocyclones. The developed model is based on physical principles for which the compromise between accuracy and computation effort is considered, allowing its use in real time operation. The model contains 92 nonlinear algebraic equations, which are solved in less that 1 second. Several experiments were taken in a pilot plant to identify some parameters of the model and to validate the results.

Keywords: hydrocyclone, phenomenological model, liquid-solid separation

1 Introduction

Nowadays, the effort to understand and quantify the separation mechanism in hydrocyclones can be classified from a point of view extremely theoretic or empiric (Venugopal & Chapperia, 2012). Although these separation equipment are widely used in the mineral processing industry to classify solids due to high separation efficiency and the relative easy operation, the design and modeling have been majority heuristic. Probably, the reason is due to the complexity of the involved phenomena. For empirical models, Murthy & Bhaskar (2012) mentioned that the most used models were developed by Lynch & Rao in 1975 and Plitt in 1976. However, these models can only be applied around the operating point where the experimental data were taken (Murthy & Bhaskar, 2012; Schubert, 2010). We want to point out that the empirical models cannot provide the phenomenological knowledge of the system because that kind of models only consider the system as a set of inputs/outputs without taking into account the involved physical principles.

On the other hand, the theoretical models based on first principles have been recently considered to understand the dynamics involved in this kind of separation process. The most relevant approaches correspond to the full and simplified Eulerian multiphase models (Manninen et al., 1996; Hirt & Nichols, 1981), which are solved using computational fluid dynamics (CFD). However, the high computational effort makes impossible its use in real time operation.

The task of this work is to provide a hydrocyclone model based on physical principles for which the compromise between accuracy and computation effort is considered, allowing its use in real time operation. To this end, the so-called Phenomenological-Based Semiphysical Model (PBSM) (Álvarez et al., 2009) is developed in Section 2. In Section 3 the prediction of efficiency of the separation process is shown using the simulated model. Conclusions are given in Section 4.

2 Mathematical Model

It is said that a model is phenomenologically based when its structure is developed through process matter, energy and momentum balances, and it can be also semiphysical when empirical formulations for various parameters are used as a part of the model (Álvarez et al., 2009). These families of models, specifically using concentrated parameters, have been commonly used in process analysis, design and control. In this sense, and looking for clarifying the model presentation, the ten steps of the procedure to obtain a Phenomenological Based Semi-physical Model (PBSM) is repeated here as follows:

1. Develop a verbal description and a process flow diagram that complement each other. These pieces of information must be clear and complete. Description and diagram are doing reference to the real process to be modeled.

2. Propose a modeling hypothesis and set a level of detail for the model according to model object or purpose. Two main options there exist: lumped parameters or distributed parameters.

3. Define as many process systems (PS) on the process to be modeled as required by the level of detail set A clue to PS determination is to look for physical walls into the process, distinguishable phases or any mass characteristics marking spatial differences.

4. Apply the principle of conservation of each determined PS. It is recommended to take almost next balances: total mass balance, n component mass balances, total energy balances. Momentum balances are indicated when significant pressure or density changes are presumed. This set of equation are the Dynamics Balance Equation (DBE), considering by default that all balances are originally dynamics but can be turned to static if the process has this behavior.

5. Select from DBE those equations with significant information for fulfill the model porpoise established in step 2. Ever some DBE are redundant or are merely a numerical equality.

6. Identify parameters, variables, and constants of the model. Fixed the values for all constant of the model. Remember that variables values will be found by the model after its solution.

7. Find constitutive equations for calculating the largest number of parameters in each processing system. Parameters without a constitutive equation must be identified from experimental data.

8. Verify the Degree of Freedom (DF) of the model (mathematical systems formed by all equations and constant values). DF=Number of equations Number of unknown variables-parameter. DF must be zero for a solvable model.

9. Build a computational model: a computer program able to solve the model.

10. Validate the model response using real operating conditions related to those used at step 2 for establish the model objective.

One of the key elements during process model construction is to establish an appropriate modeling hypothesis. When a Phenomenological Based Semi-physical Model (PBSM) is being constructed, some assumptions about the phenomena taken place must be formulated. Those assumptions are normally dedicated to declare as constant some variables of the process. There are a group of considerations sometimes called assumptions too, but very different of fixing variables to given values. To these considerations is better to call as modeling hypothesis. Such a hypothesis is based on one or more abstraction of the current phenomena into pre-stated phenomena, easily linked to but simpler than current process phenomena. This abstraction suggest to create a mental image conformed by enough pre-stated phenomena in order to cover interesting characteristics of the process and to write a description of real process behavior using the abstraction. That description is the modeling hypothesis. In this way, the final representation seems like the real phenomena and give the opportunity of simulate real behaviors using supposed pre-stated behaviors.

The power of this approach is evident because consolidated knowledge is used for constructing new knowledge, which ends validated for the new phenomena model. Note that abstraction does not try to offer an explanation about the real mechanism of the modeled process. Instead of that, abstraction has the intention of facilitating to the user a fast way to model the process without loss the rigor and formalism. In addition, modular construction will help to model complex processes ever the process can be broken into single parts and each one of those parts can be modeled by pre-stated phenomena. In the next section we state the modeling hypothesis to develop the hydrocyclone model.

2.1 Modeling hypothesis

In order to develop the PBSM for the hydrocyclone, the following assumptions, which are inspired by Schubert (2010); Wang et al. (2008), are considered:

- The lost of pressure at the feed nozzle of the hydrocyclone is computed using a modified lost of pressure for a venturi, which represents suitably the recovered pressure after the contraction.
- After the nozzle, the feed flow splits in two flows, namely underflow (UF) and overflow (OF). Both flows create hypothetical spiral paths in pipe form, which travel together until a point where the overflow changes its direction. Thus, the overflow is characterized according to its direction, namely down overflow and up overflow.
- The up overflow is bounded by the air core and the vortex finder, while the underflow is bounded by the hydroclyclone wall and the down overflow pipe.
- The cross-area of each pipe is constant through each path.
- The particles moving trough the underflow pipe describe a unique trajectory, i.e., the particles do not have independent movements as considered in other models (Wang et al., 2008).

According to Pana-Suppamassadu & Amnuaypanich (2007) there exists a critical inner pressure for which the number of turns for each spiral and the average velocity of the fluid are maximum. Therefore, we assume that the number of turns for each pipe is not fixed but it depends of the inner pressure, the solid concentration of each stream and the liquid-solid properties.

Due to our model is based on transport of a pulp (solids and water) through spirals with pipe form, the cross-area of each pipe must be computed. To this end, the air core plays an important role because it determines the suitable operation of the hydrocyclone and bounds the diameter of the cross-area of the up overflow. The air-core is consider invariant, i.e., the diameter does not change once a stable operating point is reached. The air core formation is due to the ventilator effect of the inner vortexes (de Brito Dias et al., 2008; Elsayed & Lacor, 2010).

2.2 Mass and energy balances

In order to establish the mass and energy balances, we consider the five subsystems sketched in Fig. 1:

- S-I: From point 1, inlet of the hydrocyclone, to point 2, the hydrocyclone nozzle input.
- S-II: From point 2, to points 3 and 5. Here we consider the hypothetical split.
- S-III: The down overflow spiral from point 5 to 5low.
- S-IV: The up overflow spiral from point 5low to 6atm.
- S-V: The underflow spiral from point to 4atm.

We assume that the pulp is incompressible and for each subsystem no heat transfer occurs. The drop pressure between points 1 and 2 corresponds to a sudden contraction, which is modeled as a Venturi flowmeter. As assumed above, at point 2 the feed stream splits in two hypothetical pipes, namely, the underflow and overflow. At this split point, the pressures P_2, P_3 and P_5 are assumed to be equal. The particles moving in the overflow pipe descend until a point where a hypothetical pressure P_{5low} is reached. At this point, the particles change the direction and ascend until the point 6, located at output of the vortex finder. On the other hand, the particles moving in the underflow pipe reach the point 4 which is located in the output of the apex. Between the points 4-4atm and 6-6atm, the losses correspond to the transport in the pipe and the output pressures correspond to the atmospheric pressure.

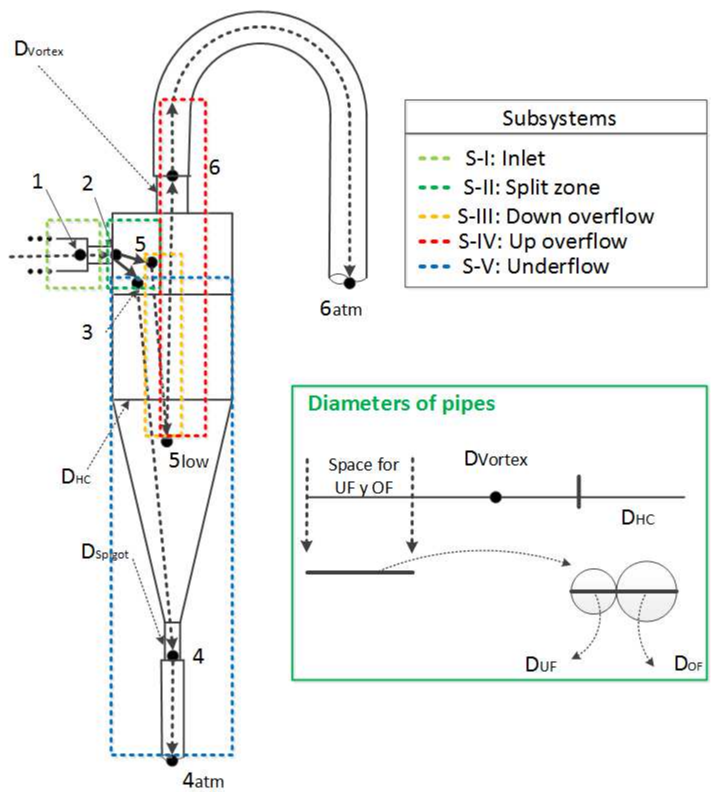

Fig. 1: Subsystems of the hydrocyclone to apply the mass and energy balances. The space available for the hypothetical underflow and overflow spirals.

The energy balance for each subsystem where the pipe section is considered with constant cross-area, gives the following:

$$P_{i+1} = P_i + g\,\rho_i\,(z_i - z_{i+1}) - \rho_i\,h_{f\,i\to i+1} \tag{1}$$

where P_i is the pressure, g the gravity, ρ_i the density, z_i the height with respect to the hydrocyclone input, and $h_{f\,i\to i+1}$ is the friction loss between points i to $i+1$. The indexes i and $i+1$ represent the input and output points in the pipe section. The friction losses $h_{f\,i\to i+1}$ are computed according to the 2-K method (Darby, 2001),

$$h_{f\,i\to i+1} = K_{i\to i+1}\frac{v_i^2}{2}, \tag{2}$$

where, v_i is the fluid flow linear velocity, and $K_{i\to i+1}$ is a factor computed as a contribution of pipe section and accessories.

For S-I, the hydrocyclone nozzle at the entry is modeled as a venturi flow-meter. Thus, a sudden contraction term, $\triangle P_{SC\,j}$ is consider in Eq. (1),

$$P_2 = P_1 + g\,\rho\,(z_1 - z_2) - \rho\,h_{f\,1\to 2} - \triangle P_{SC\,j}. \tag{3}$$

$\triangle P_{SC_1}$ is computed from the expression to compute the velocity at venturi throat (Darby, 2001), i.e.,

$$\triangle\,P_{SC_1} = \frac{\rho_1}{2}\left(\frac{v_2}{C_F\,C_d}\right)^2(1-\beta^4), \tag{4}$$

where v_2 is the fluid flow linear velocity, $C_d = 0.96$ for Reynolds number between 10.000 and 100.000, C_F is a correction factor for v_2, and β is the relation between the diameter of throat and the diameter of the inlet line.

In S-II we only consider the split into two streams, namely underflow and overflow. Energy losses caused at that point are not considered. In order to characterize the particle size distribution (PSD) for each stream (feed, underflow and overflow), three range of particle sizes (µm) are considered, namely, $J = \{\text{fine}, \text{medium}, \text{gross}\}$. The material balance results,

$$\dot{m}_2 = \dot{m}_3 + \dot{m}_5, \tag{5}$$

where $\dot{m}_i$ represents the mass flow in each stream.

To analyze the particle size distribution in each stream, we first compute the volume fraction using the following expression

$$\phi_{J,i} = \frac{\int_{PS_J}^{PS_{J+1}} R_i(x)dx}{\int_{PS_{min}}^{PS_{max}} R_i(x)dx} \tag{6}$$

where PS_J, PS_{J+1} are the minimum and maximum particle size of fraction J, respectively. PS_{min}, PS_{max} are the minimum and maximum particle size in the pulp, respectively. $R_i(x)$ is the cumulative distribution Rosin-Rammler function,

$$R_i(x) = 100 - 100\exp\left(-0.693\left(\frac{x}{d_{50,i}}\right)^{n_i}\right), \tag{7}$$

where $d_{50,i}$ is the cut point of solid separation in the hydrocyclone, x is the particle size, and n_i is a parameter that must be identified using experiments. $d_{50,i}$ is employed to predict the separation efficiency. Once the volume fractions $\phi_{J,i}$ are computed, the mass fraction $w_{J,i}$ can be determined using the dry base mass fraction $w_{J,i,db}$ and the solid concentration $C_{s,J}$,

$$w_{J,i} = w_{J,i,db} \times C_{s,J} = \frac{\phi_{J,i}\rho_J}{\rho_s} \times C_{s,J}, \tag{8}$$

where ρ_J is the material density and ρ_s is the solid total density. Using the mass fractions $w_{J,i}$, the density ρ_i and viscosity υ_i of the pulp can be computed by the following expressions (Neesse & Dueck, 2007),

$$\frac{1}{\rho_i} = \frac{1-\Sigma_J(w_{J,i})}{\rho_w} + \Sigma_J\left(\frac{w_{J,i}}{\rho_J}\right), \quad J = \{\text{fine}, \text{medium}, \text{gross}\}, \tag{9}$$

$$\upsilon_i = \upsilon_w\left(1 + a_i\Sigma_J\left(w_{J,i}\frac{\rho_i}{\rho_J}\right) + b_i\Sigma_J\left(w_{J,i}\frac{\rho_i}{\rho_J}\right)^2 \exp\left(c_i\Sigma_J\left(w_{J,i}\frac{\rho_i}{\rho_J}\right)\right)\right), \tag{10}$$

where ρ_w is the water density, υ_w the water viscosity, and a_i, b_i and c_i are empiric parameters identified using experiments.

For S-III, IV and V, we apply the Bernoulli's equation (1) taking into account that these hypothetical trajectories are spirals with a constant cross-section area. The friction losses for each spiral are computed using Eq. (2), for which the term $K_{i\to i+1}$ is computed as follows

$$K_{i\to i+1} = NS_i \times K_{S,i}, \tag{11}$$

where NS_i is the number of turns that the particles travel in each trajectory and $K_{S,i}$ is the energy loss due to the travel in a spiral. The number of spirals is computed as a function of the inlet pressure P_i in each pipe section, while $K_{S,i}$ is computed using the following equation (Jayanti, 2013),

$$K_{S,i} = 2\left(\frac{\Pi f_{D,i} R_{b,i}}{D_i} + K_{b,i}\right), \tag{12}$$

where $f_{D,i}$ is the Darcy factor, $R_{b,i}$ is the curvature radio of the spiral, D_i the diameter of the hypothetical pipe and $K_{b,i}$ is a compensation term for a turn in the hypothetical pipe. We want to point out that $R_{b,i}$ is bounded according to the considered subsystem, i.e., for the down overflow (S-III) $R_{b,i}$ is a function of the vortex finder diameter, D_{vortex}, and the pipe diameter, while for the up overflow (S-IV) the air-core diameter, $D_{aircore}$, reduces the space where this spiral can travel. Neesse & Dueck (2007) proposed the following expression to compute the air-core diameter,

$$D_{aircore} = D_{HC}\left(1 + \frac{2\alpha}{\beta^2}\left(\frac{D_{HC}}{D_{2i}}\right)^2\left(\frac{D_{2i}}{D_{vortex}}\right)^{2\gamma}\right)^{-2\alpha}, \tag{13}$$

where D_{HC} is the hidrocyclone diameter, D_{2i} the nozzle diameter, and the parameters α, β, γ are identified using experiments.

Thus, the complete model corresponds to the set of equations developed for each subsystems and some constitutive equations related with fluid mechanics which have been avoided for simplicity of the presentation. The resulting model contains 92 nonlinear algebraic equations. As we mention above, some of the model parameters were identified using experiments taken form a pilot plant. The other ones correspond to hydrocyclone geometry and operational parameters. The model was solved in less than 1 second.

3 Results

During parameters identification of the model several assumptions were verified directly on experimental assembly. One of the major facts was the existence of spirals inside the hydrocyclone body. It was evident from direct contact with inside part of hydrocyclone that there are deep channels caused by solids flowing as a part of the pulp. Those channels have a perfect spiral form, indicating that main part of our modeling hypothesis is right. Other verified assumption during experimental section was that of air-core as a physical limit for up overflow inside the hydrocyclone. It was confirmed that when air-core disappears the hydrocyclone operation is totally abnormal.

In order to validate the proposed model, several tests were conducted under variation of feed pressure but maintaining feed pulp concentration. At each test three samples were taken during hydrocyclone operation: one of feed pulp, a second from overflow conduction and the third from underflow stream. The model was fed with next feed data: pressure, volumetric flow, density and viscosity. The model predicts: discharge pressure and $d_{50,i}$ for overflow and underflow. In Table 1 the results for one of validation points is presented.

Table 1: Experimental vs. Predicted values obtained using the model.

	OF pressure	UF pressure	$d_{50,OF}$	$d_{50,UF}$
Measured value	85326 Pa abs	85326 Pa abs	28 μm	43 μm
Predicted value	87730 Pa abs	90697 Pa abs	32.2 μm	45.2 μm
% Model error	2.82%	6.30%	15.10%	5.11%

Experimental values of $d_{50,i}$ were found using the particle size distribution from an automatic particle analyzer (MALVERN 3000). Modeled values for $d_{50,i}$ were obtained with only three representative particles sizes: fine, medium and gross particle ranges. Both discharge pressures were taken as the atmospheric pressure value at experimental assembly location. As can be seen from results, a good agreement between experimental and predicted values was obtained. Maximum error was for the predicted value of $d_{50,OF}$. However, due to the low size of particles contained into overflow, an error of 4 μm is acceptable for operative purpose. The other model predictions are all under 7% of error, which is a good agreement. We want to point out that after parameters identification, three critical values were analyzed in their final values: *i*) the point of direction change of overflow stream inside the hydrocyclone, *ii*) the flow area for overflow stream as area difference between air-core flow area and vortex-finder free area, and *iii*) friction factors for overflow and underflow streams. The obtained value for the first parameter was congruent with expected values: $\frac{3}{4}$ of cylindrical height of hydrocyclone body, because a point inside conical section were abnormal and just at vortex finder height was impossible. The second parameter, overflow stream flow area, was found ever as a positive number, directly related to air-core characteristics. Finally, friction factors, taken as Darcy factors, for overflow and under flow streams, exhibited values bigger than known values for solid-free liquids. In contrast, when values for Darcy factors regarding pulps were found in the literature, a similar order was found when comparing with Darcy factor identified for the present model.

4 Conclusion

A proposed model for hydrocyclone operation was presented and validated with data from a real assembly used for mineral processing. The modeling hypothesis is simpler compared with other approaches to the same modeling task, but results indicated usefulness of proposed model due to its low computational cost at good precision. A common mechanical energy balance (Bernoullis equation) was applied in order to cover the behavior of pulp inside the hydrocyclone. Through that approach, physical characteristics of overflow and underflow streams were predicted reaching a good agreement between experiments and model predictions. Several future works must be executed in order to conform a final model: to verify using Computational Fluid Dynamics (CFD) the point of direction change from down overflow to up overflow, to propose a best split equation and to propose a better sub-model for air-core dimensions using CFD and video-supported images.

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