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Concentration Distribution of Solid Particles Transported by a Pseudoplastic Fluid

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Abstract - Concentration distributions of solid particles transported by pseudoplastic fluids in laminar regime were determined experimentally. Practically all the distributions show minimum concentration of solids near the centre of the pipe and higher toward the walls and even, for some conditions, the highest values near the top of the pipe. This behaviour of the concentration distribution was explained when diffusive fluxes of the solid particles due to concentration gradient and viscosity gradient are considered. An existing diffusive model was modified in order to take into account negatively buoyant particles and a pseudoplastic fluid as a carrier. It was found that the head loss plays an important role in the fluxes of solid particles due to gradients of concentration and viscosity, increasing them enough to overcome the effect of gravity.

***Keywords*:** pseudoplastic fluids, concentration distribution, laminar flow, diffusive fluxes

1. Introduction and Objective

The hydraulic transport of solids in pipes is used in many industrial processes, particularly in the mining industry. Usually, it is considered that the flow has to be turbulent to keep the solid particles in suspension, and the upward velocity of the turbulent eddies must be greater than the sedimentation velocity of the particles [1]. Practically all of the studies regarding transport of solids in pipes have been developed using water as a carrier fluid. However, the rheological properties of the solids and water mixture can be modified depending on the concentration and mineralogy of the particles transported by the fluid. Thus, for high concentrations of solids, the particles with smaller size and the water behave as an equivalent non-Newtonian fluid which is the carrier of the larger particles. Studies considering non-Newtonian carriers are comparatively much less than those that considers water or a Newtonian transport fluid [2]. The objective of this article is to report the results of an experimental study regarding the concentration distribution of the solid particles transported by a pseudoplastic fluid in laminar regime and interpret them considering the diffusive fluxes due to gradient of concentration and gradient of viscosity.

2. Experimental Set-up and Materials

2.1. Experimental set-up

A sketch of the experimental set-up used in the research is shown in Fig. 1. From the head tank, the slurry is pumped towards the test section by means of two centrifugal pumps EBARA DWO400 connected in series and controlled by a frequency inverter SEW EURODRIVE model MOVITRAC B. The test section consisted in a 12 m long transparent PVC pipe with an inner diameter of 5.08 cm. Along the pipe, 3 pressure transducers were installed to get continuous pressure records. Near the end of the pipe, an electrical resistance tomography (ERT) sensor with a data acquisition system model P2+ made by ITS was installed in order to record the concentration distribution of solids. The software used for data processing was ITS System 2000. The slurry returned to the head tank by means of a pipe of 2.54 cm diameter. A refrigerating system was installed along this pipe that kept the slurry at a constant temperature. Discharge was measured with a magnetic flowmeter SIEMENS 3100 and a signal transmitter SIEMENS MAG-500.

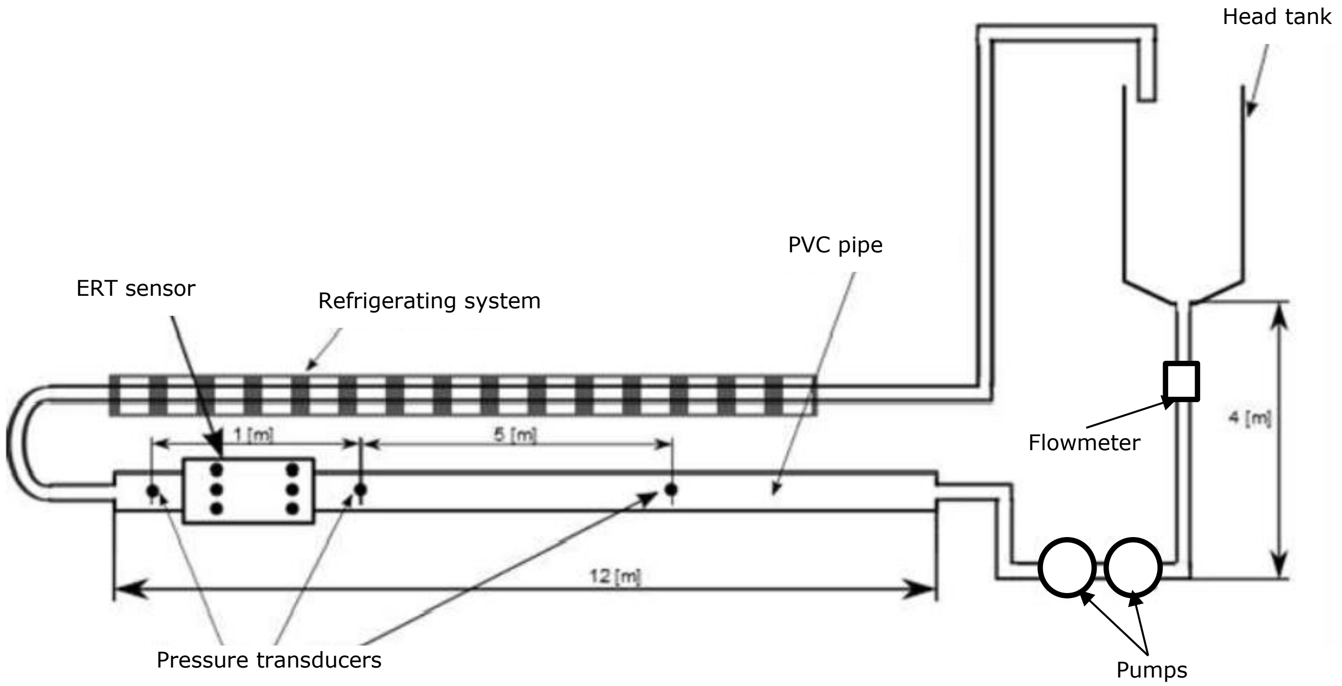

Fig. 1: Experimental set-up.

2.2. Materials

Glass microspheres of three different size distributions and density (ρ) equal to 2500 kg/m^3 were the solid particles used in the experiments. Characteristic diameters of each size distribution are given in Table 1.

Table 1: Characteristic diameters of the glass microspheres used in the experiments.

Type	d_{20} (µm)	d_{50} (µm)	d_{80} (µm)
I	100	120	150
II	205	300	380
III	435	600	740

The pseudoplastic fluid used as a carrier was generated dissolving sodium carboxymethyl cellulose (CMC) in water. When the solids particles are added to the fluid, the finest particles and the fluid behaves as an equivalent fluid and the rheology to this mixture needs to be known. Three different concentrations (weight/weight) of CMC in water were considered: 0.5%, 0.6% and 0.8%, and concentrations (volume/volume) of solids in the CMC and water mixture ranging from nominal 17% to 32%. Thus, a total of 26 different conditions associated to the materials were defined. The largest particles of the different types of solids present a process of sedimentation during the rheology determination, and it was found that particles with sizes smaller than 60-70 µm do not sediment through the rheological tests. The characteristic rheology corresponds to the mixture of fines particles and the pseudoplastic aqueous CMC solution. Rheology was determined with a rheometer made by Anton Paar model Rheolab QC, with a peltier temperature control. The rheograms fitted well to a Ostwald–de Waele (or power law) model, $\tau = K\dot{\gamma}^n$, where τ is the shear stress, $\dot{\gamma}$ is the strain rate, K is the consistency coefficient and n is the flow index. Density (fine particles+fluid) was measured with was measured by a Gamma RTM Dr J Ambrus densimeter. A summary of the parameter characterizing the mixtures and the solid concentrations (ϕ) are presented in Table 2.

2.3. Flow discharge

The volumetric discharge Q of the mixtures ranged between 0.030 and 1.748 L/s depending on each specific the mixture, and it is given in Table 2.

Table 2: Rheological parameters and concentration of solids in the experiments.

Mixture No.	d_{50} (μm)	ϕ (%, vol/vol)	ρ (kg/m^3)	K (Pa·s^n)	n	Q (L/s)
1	120	19.278 – 27.221	1005.025	0.29	0.69	0.538 – 1.636
2	120	16.767 – 23.411	1005.025	0.13	0.83	0.470 – 1.396
3	120	18.292 – 28.648	1005.025	0.44	0.66	0.302 – 1.258
4	120	21.813 – 25.524	1006.899	0.81	0.61	0.323 – 0.996
5	120	21.979 – 31.643	1006.899	1.12	0.58	0.108 – 0.968
6	120	23.709 – 31.078	1006.899	1.31	0.54	0.085 – 0.762
7	120	17.355 – 20.595	1008.065	2.94	0.46	0.197 –1.149
8	120	22.718 – 24.273	1008.065	3.34	0.47	0.083 – 1.020
9	300	18.330 – 41.107	1005.025	0.36	0.66	0.069 – 1.748
10	300	24.385 – 47.449	1005.025	0.43	0.64	0.050 – 1.584
11	300	29.368 – 44.152	1005.025	0.73	0.58	0.099 – 1.677
12	300	19.614 – 41.622	1006.899	0.81	0.56	0.050 – 1.287
13	300	23.841 – 43.549	1009.899	0.65	0.59	0.114 – 1.735
14	300	27.680 – 43.138	1006.899	0.92	0.57	0.091 – 1.497
15	300	17.008 – 32.324	1008.065	3.63	0.45	0.030 – 0.473
16	300	24.551 – 41.744	1008.065	3.26	0.44	0.076 – 1.338
17	300	30.508 – 46.760	1008.065	3.52	0.42	0.030 – 1.162
18	600	22.251 – 38.988	1005.025	0.22	0.71	0.069 – 1.748
19	600	26.137 – 41.421	1005.025	0.21	0.71	0.159 – 1.584
20	600	29.928 – 44.486	1005.025	0.14	0.77	0.099 – 1.677
21	600	20.897 – 33.029	1006.899	0.31	0.68	0.358 – 1.400
22	600	27.317 – 42.342	1006.899	0.32	0.67	0.237 – 1.197
23	600	28.687 – 41.318	1006.899	0.23	0.71	0.231 – 1.340
24	600	25.000 – 43.511	1008.065	1.14	0.55	0.030 – 0.819
25	600	26.600 – 43.387	1008.065	0.77	0.59	0.107 – 1.132
26	600	32.189 – 52.278	1008.065	0.38	0.68	0.139 – 1.233

2.4. Flow regime

To define the flow regime, the criterion by Mishra and Tripathi [3] was used. According to that criterion, the flow regime is laminar when the Reynolds number, as defined by Metzner and Reed [4], $Re = \frac{1}{8^{n-1}}\left(\frac{4n}{3n+1}\right)^n \frac{\rho U^{2-n} D^n}{K}$, is less than a critical value given by $Re_C = \frac{(4n+2)(5n+3)}{3(3n+1)^2} 2100$. D and U are the pipe diameter and flow velocity, respectively. For all the mixtures, the minimum value of Re_C is 2186 and the maximum value of the Reynolds numbers for all the flow conditions is Re = 1287, ensuring the laminar regime.

3. Experimental Results

A total of 241 concentration distributions of the solid particles (for equal number of flow conditions) were obtained with the electrical resistance tomography sensor. Grouped according to the size d_{50} of the particles, the number of concentration distributions is: 78 for d_{50} = 120 μm, 81 for d_{50} = 300 μm and 82 for d_{50} = 600 μm. Given the impossibility to show the measurements for all the experiments, 9 were chosen to be presented in this paper and they are shown in Fig. 2. (The complete set of measurements can be found in the appendix of the Master Thesis of the second author [5], downloadable from http://repositorio.uchile.cl/handle/2250/132978). Associated to the tomographic images presented in Fig. 2 are the concentration profiles along the vertical axis z that are shown in Fig. 3. The value of the concentration for each location z corresponds to the horizontal average.

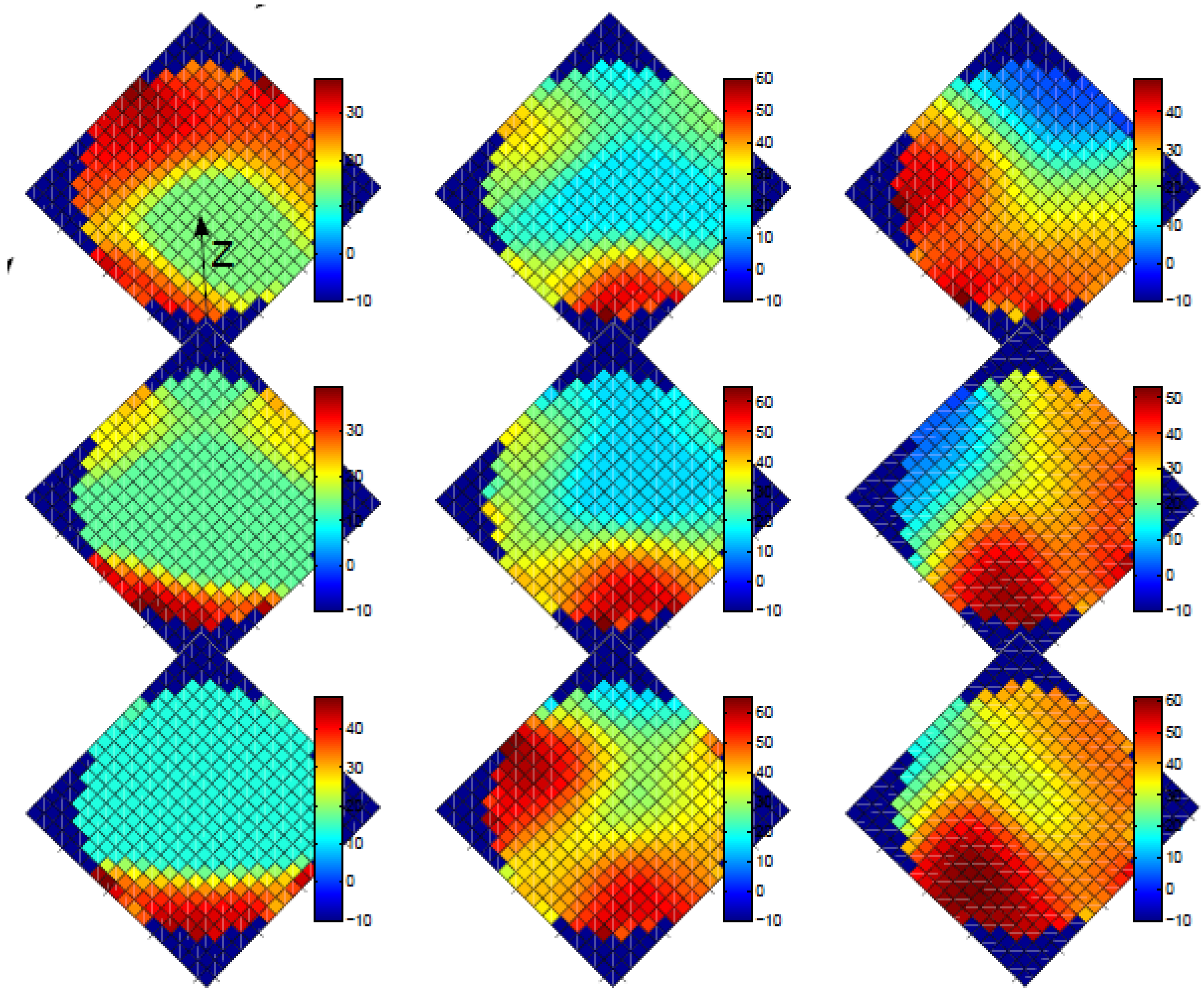

Fig. 2: Concentration distributions obtained with the ETR sensor. Each column from left to right corresponds to d_{50} = 120 μm, 300 μm and 600 μm. Each file from bottom to top corresponds to the nominal discharges Q = 0.3 L/s, 0.8 L/s and 1.4 L/s. Concentration below 0% does not have physical sense. The value of -10% was chosen to delimit the pipe boundaries.

The measured concentration presented unexpected distributions (at least for the authors), showing lower values near the centre of the pipe and sometimes with the highest values near the top of the pipe, as can be seen in Fig. 3. After reviewing the hardware, the data acquisition and data analysis protocols it was discarded that they were measurements artefacts or data misprocessing, and the results should be analysed considering the dynamics of a non-Newtonian fluid flow and its interaction with the solid particles.

4. Analysis of experimental results

The measurements show that the particles with d_{50} equal to 120 and 300 μm present a concentration distribution with two local maxima, one located in the upper half of the pipe and the second one in the lower half, with a region of low concentration in the centre of the conduit (under some conditions, the concentration at the centre of the pipe was zero). The distribution for the particles with d_{50} = 600 μm follows a more common tendency, with higher values near the bottom, although lower values are still found near the centre of the pipe. Another important issue was that, although the solid particles are negatively buoyant, they are kept in suspension. Particles with d_{50} = 300 and 600 μm do not settle, even for flows with Reynolds numbers as low as Re = 100.

The hypothesis that the concentration distribution of the solid particles and its suspension in laminar regime is the result of fluxes deriving from hydrodynamic diffusion process [6, 7] and the non-Newtonian characteristic of the fluid arose as an explanation. A qualitative analysis based on the diffusive model by Phillips et al. [8] was used to explain the shape of the concentration distribution of solids. It has to be noted that that model considers a Newtonian fluid with neutrally buoyant particles, and it has to be modified for an Ostwal-de Waele fluid and settling particles. According to Phillips et al model, the transport of concentration ϕ is given by:

$$\frac{D\phi}{Dt} = -\nabla \cdot \left(N_C + N_\mu\right) \tag{1}$$

where $D\phi/Dt$ is the material derivative of the concentration ϕ. N_C and N_μ are the fluxes of particles due to the gradient of concentration and to the gradient of viscosity, respectively, and they depend on dimensionless diffusion coefficients D_C and D_μ that were determined experimentally by Phillips et al. The fluxes are given by:

$$N_C = -D_C a^2 \phi \nabla(\dot{\gamma}\phi) = -D_C a^2 \left(\phi^2 \nabla\dot{\gamma} + \dot{\gamma}\phi\nabla\phi\right) \tag{2}$$

$$N_\mu = -D_\mu \dot{\gamma} \phi^2 \left(\frac{a^2}{\mu_m}\right) \nabla \mu_m \tag{3}$$

In the above equations, a is the particle diameter and μ_m the viscosity of the mixture, which depends on the solids concentration. Note that the flux due to viscosity gradient cannot exist in the flow of a pure Newtonian fluid.

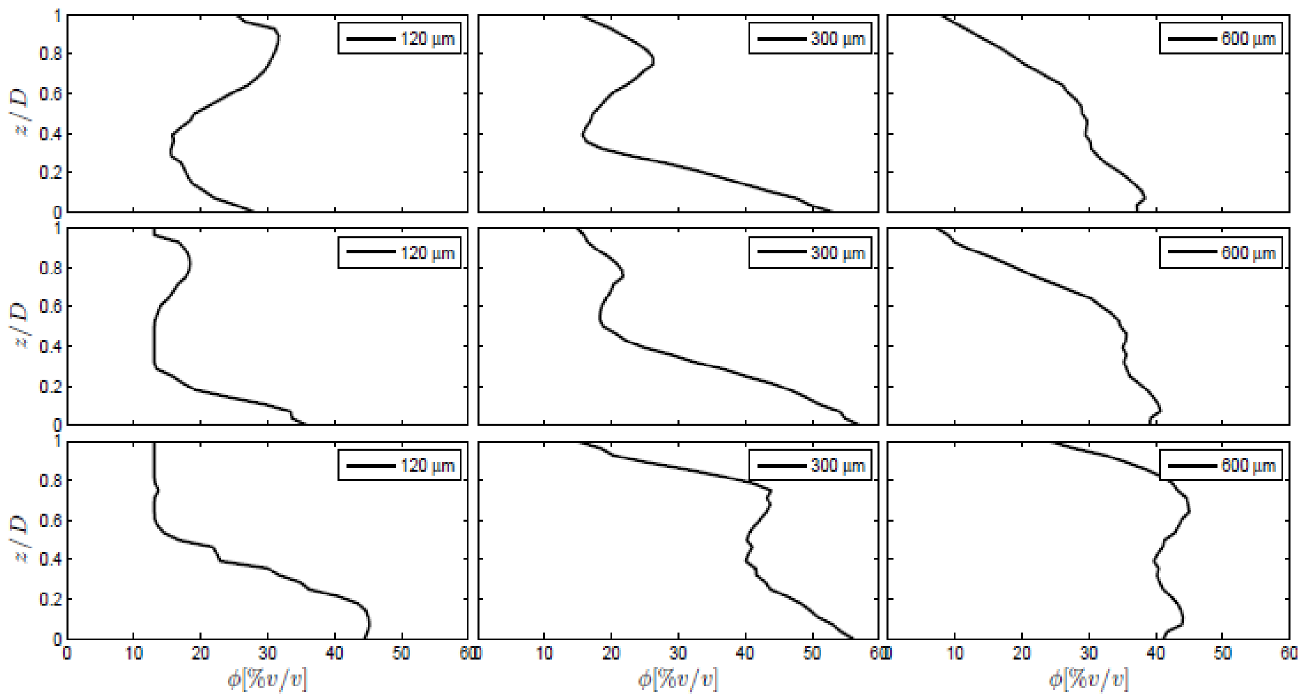

Fig. 3: Distribution of the horizontal averaged concentration. Each column from left to right corresponds to d_{50} = 120 μm, 300 μm and 600 μm. Each file from bottom to top corresponds to the nominal discharges Q = 0.3 L/s, 0.8 L/s and 1.4 L/s.

Phillips et al. model does not consider the downward flux resulting from the action of gravity. In the range of Stokes, this flux is given by

$$N_g = -\frac{2}{9} f g \phi a^2 \frac{\rho_S - \rho}{\mu_{eff}} \tag{4}$$

where f is a hindering function:

$$f = (1-\phi)^\beta \tag{5}$$

with β obtained from

$$\frac{4.8-\beta}{\beta-2.4} = 0.0365\left(C_D Re_P^{2/(2-n)}\right)^{0.57}\left[1-2.4\left(\frac{d}{D}\right)^{0.27}\right] \tag{6}$$

where $Re_P = \rho V^{2-n} a^n / K$ is the particle Reynolds number (V is the particle velocity) and C_D the drag coefficient that can be computed from any available relationship, like that by Dhole et al. [9] which is valid in the range 5≤ Re_P ≤500 : $C_D = (24/Re_P)\left(1 + 0.148 Re_P^{2.35n/(2.42n+0.918)}\right)$.

In order to carry out a qualitative analysis, the flow in a cylindrical pipe will be simplified to a two dimensional Pouseille flow, where the z direction is along the diameters of the pipe (Fig. 4). As gravity acts in the vertical direction, N_g will be projected along z in the analysis that follows. Assuming a steady state flow with no secondary currents, i.e. only with the component u of the velocity in the x direction, the momentum equation is reduced to $0 = -\partial P/\partial x + \partial \tau_{zx}/\partial z$, where z is the coordinate normal to x, P is the pressure and τ_{zx} the shear stress that is reduced to $\tau_{zx} = K(\partial u/\partial z)^n$. To simplify the notation, it is defined $P_x = \partial P/\partial x$ and $\dot{\gamma} = \partial u/\partial z$. Integrating the momentum equation with respect to , $\dot{\gamma} = ((P_x z + C_1)/K)^{1/n}$ is obtained, with C_1 a constant of integration. The mixture of fine particles and the psudoplastic carrier behave as an equivalent pseudoplastic fluid, characterized by a mixture consistency coefficient, K_m , which is a function of the volumetric concentration of solids, ϕ. It can be estimated according to the relationships of Kawase and Ulbrecht (1983) [10]. Thus, the deformation shear rate of the mixture can be written as

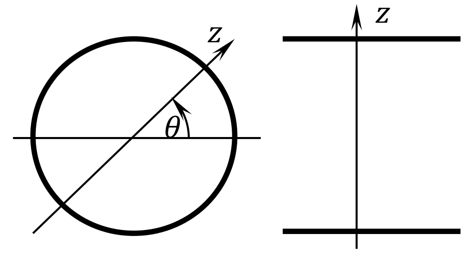

Fig. 4: Simplification of the cylindrical geometry to the two dimensional Poiseulille flow.

$$\dot{\gamma}_m = \left(\frac{P_x z + C_1}{K_m}\right)^{1/n} \tag{7}$$

An effective viscosity of the mixture is defined as

$$\mu_m = K_m |\dot{\gamma}_m|^{n-1} = K_m \left|\frac{P_x z + C_1}{K_m}\right|^{\frac{n-1}{n}} \tag{8}$$

Using $\dot{\gamma}_m$ instead of $\dot{\gamma}$, and μ_m in the expressions for the fluxes, and considering two dimensional Pouseuille flow, it is possible to get more manageable relationships that will allow us to know the flux of solid particles in the 2D pipe. Thus, the fluxes associated to concentration gradient and viscosity gradient are reduced to:

$$N_C = -D_C a^2\left(\phi^2 \frac{d|\dot{\gamma}_m|}{dz} + \phi|\dot{\gamma}_m|\frac{d\phi}{dz}\right) \tag{9}$$

$$N_\mu = -D_\mu |\dot{\gamma}_m| \phi^2 \left(\frac{a^2}{\mu_m}\right)\frac{d\mu_m}{dz} \tag{10}$$

$$\frac{d\mu_m}{dz} = \frac{dK_m}{d\phi}\frac{d\phi}{dz} \tag{11}$$

With respect to the diffusion coefficients, D_C = 0.43 [11] and $D_\mu = D_C/(0.01042\phi + 0.142)$ [12] were used. It is worth to stress the important role played by the energy loss P_x through the mixture viscosity in all the fluxes. The absolute value of the shear rate should be used in the equations because the diffusive model is based on the frequency of particle collisions which scales with $|\dot{\gamma}_m|$, according to the model of Leighton and Acrivos [7, 8].

It is easy to see that the direction of the fluxes N_C and N_μ is defined by the sign of $d\phi/dz$, $d|\dot{\gamma}_m|/dz$ and $d\mu_m/dz$. The flux of particles due to gravity, N_g, depends only on $(\rho_S - \rho)$ and it is always downwards for negatively buoyant particles. The net flux of particles along the z direction is $N_T = N_C + N_\mu + N_g \sin\theta$. The result of an analysis for different conditions is given below for some particular cases.

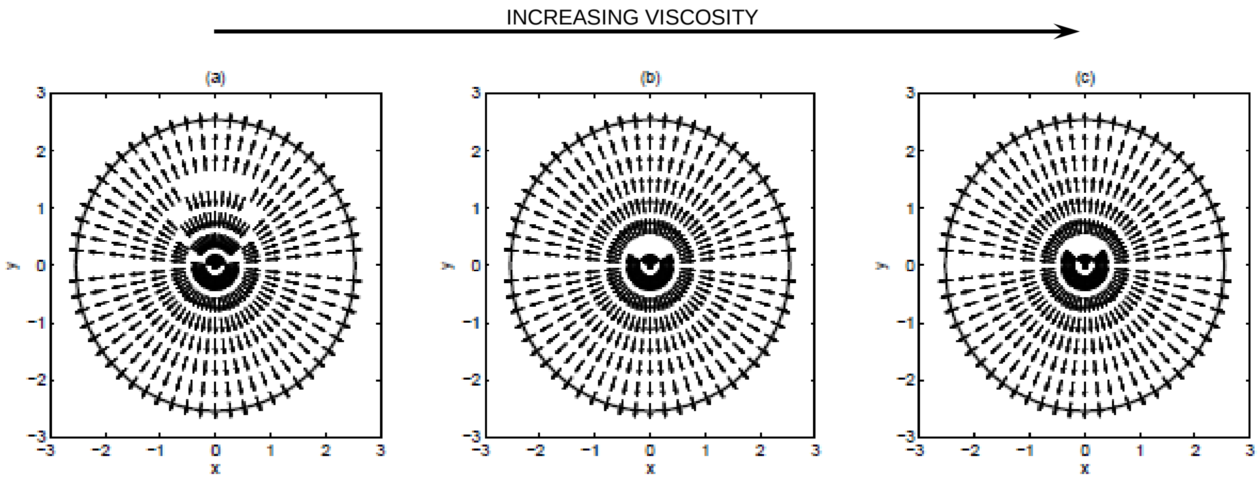

Fig. 5: Effect of viscosity in the direction of the flux of particles. $Q \sim 1$ L/s and $d_{50} = 600$ μm.

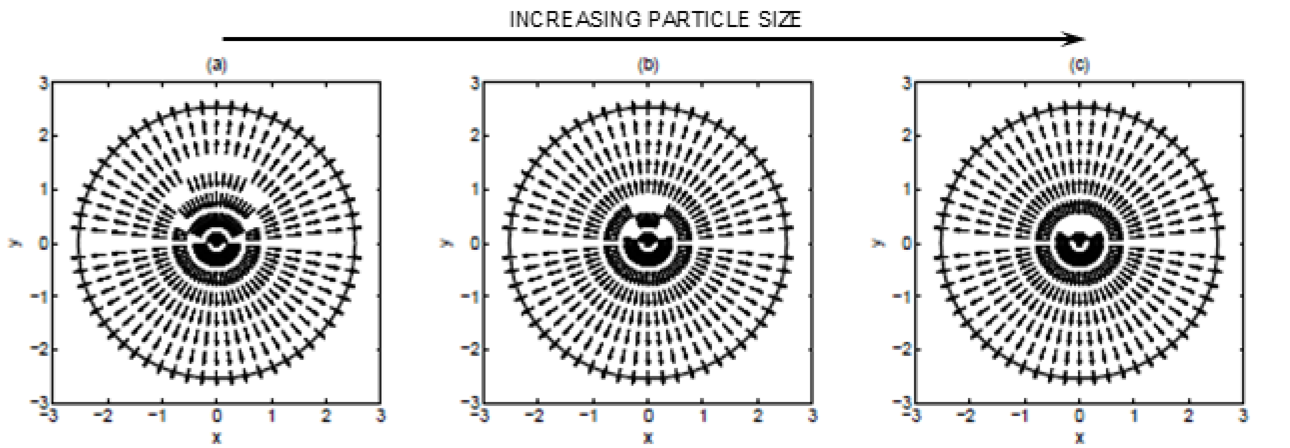

Fig. 6: Effect of particle size in the direction of the flux of particles. $Q \sim 1$ L/s.

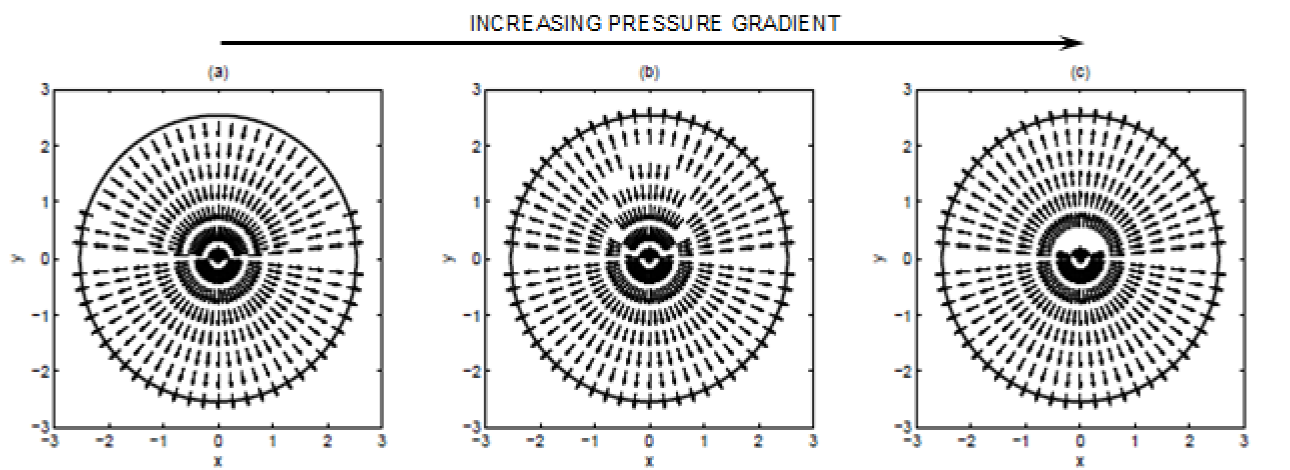

Fig. 7: Effect of the head loss in the direction of the fluxof particles. $Q \sim 1.6$ L/s and $d_{50} = 120$ μm.

For a relatively high discharge ($Q \sim 1$ L/s) and $d_{50} = 600$ μm, it is found that $|N_g \sin\theta| > |N_C + N_\mu|$ only for the less viscous mixtures. Radial fluxes (along z) for this condition are presented in Fig. 5, in which viscosity increases from left to right. A similar analysis shows that for the same viscosity and discharge, $|N_g \sin\theta| > |N_C + N_\mu|$ in the centre of the pipe. Fig. 6 corresponds to the cases in which discharge, viscosity and pressure gradient are kept constant, changing the particle size. It is observed that for the two largest sizes of particles (d_{50} equal to 300 and 600 μm) gravity fluxes dominates only near the center of the pipe, with strong fluxes towards the walls due to the gradient of concentartion and viscosity. The effect of the head loss is presented in Fig. 7, where the flux directions are shown for $Q \sim 1.6$ L/s, $d_{50} = 600$ μm, $K = 0.29$ Pa·s^n, and $n = 0.60$, and three pressure gradients: $P_x/\rho_m g = 0.10$ m, 0.25 m, 0.30 m. It is observed that at higher head loss per unit length, diffusive fluxes overcome the gravitational one.

4. Conclusion

The qualitative analysis of fluxes of solid particles due to gradient of concentration and viscosity indicates that the concentration of particles carried by a pseudoplastic fluid in laminar regime can present minimum values near the centre and higher close to the walls. The analysis, although highly simplified, preserves the most important physical mechanisms that govern the migration of the solid particles. Thus, it was explained why larger particles did not settle in the experiments. It was found that the pressure gradient (head loss) controls the fluxes through the effective viscosity of the mixture formed by the solid particles and the pseudoplasic fluid.

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