

particles, and it has to be modified for an Ostwal-de Waele fluid and settling particles. According to Phillips et al model, the transport of concentration ϕ is given by:

$$\frac{D\phi}{Dt} = -\nabla \cdot (N_C + N_\mu) \quad (1)$$

where $D\phi/Dt$ is the material derivative of the concentration ϕ . N_C and N_μ are the fluxes of particles due to the gradient of concentration and to the gradient of viscosity, respectively, and they depend on dimensionless diffusion coefficients D_C and D_μ that were determined experimentally by Phillips et al. The fluxes are given by:

$$N_C = -D_C a^2 \phi \nabla(\dot{\gamma} \phi) = -D_C a^2 (\phi^2 \nabla \dot{\gamma} + \dot{\gamma} \phi \nabla \phi) \quad (2)$$

$$N_\mu = -D_\mu \dot{\gamma} \phi^2 \left(\frac{a^2}{\mu_m} \right) \nabla \mu_m \quad (3)$$

In the above equations, a is the particle diameter and μ_m the viscosity of the mixture, which depends on the solids concentration. Note that the flux due to viscosity gradient cannot exist in the flow of a pure Newtonian fluid.

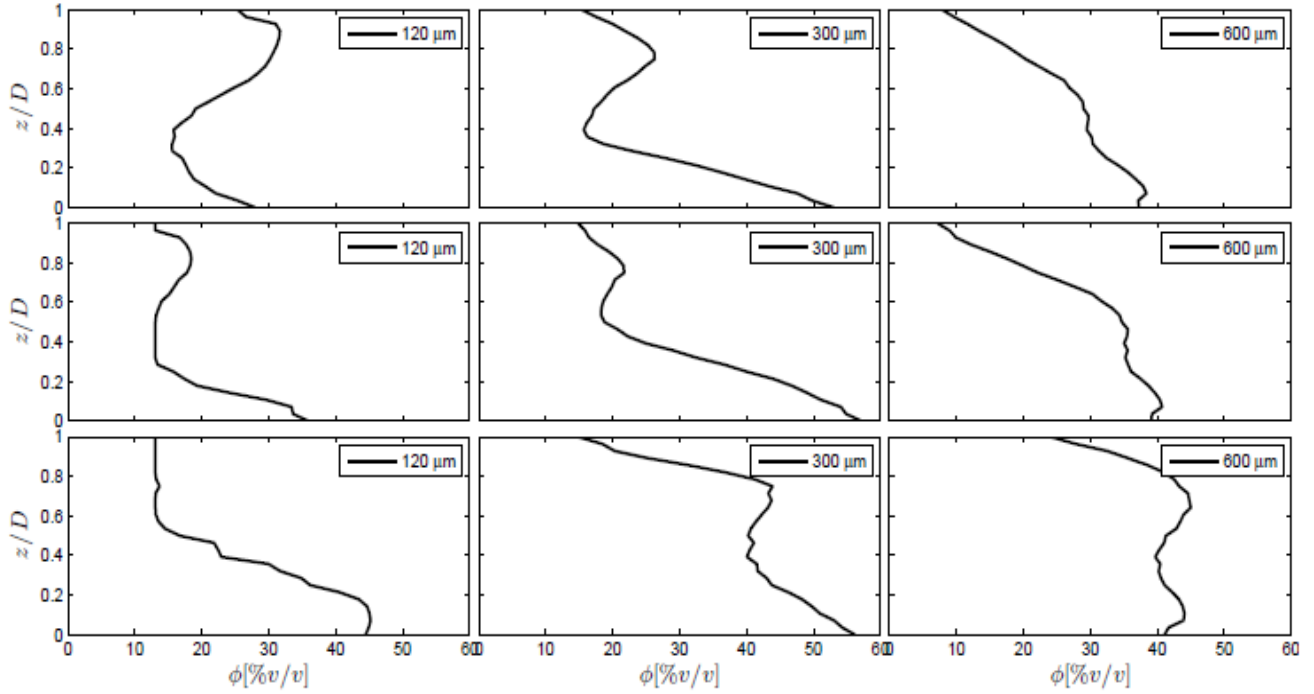


Fig. 3: Distribution of the horizontal averaged concentration. Each column from left to right corresponds to $d_{50} = 120 \mu\text{m}$, $300 \mu\text{m}$ and $600 \mu\text{m}$. Each file from bottom to top corresponds to the nominal discharges $Q = 0.3 \text{ L/s}$, 0.8 L/s and 1.4 L/s .

Phillips et al. model does not consider the downward flux resulting from the action of gravity. In the range of Stokes, this flux is given by

$$N_g = -\frac{2}{9} f g \phi a^2 \frac{\rho_s - \rho}{\mu_{eff}} \quad (4)$$

where f is a hindering function:

$$f = (1 - \phi)^\beta \quad (5)$$

with β obtained from

$$\frac{4.8 - \beta}{\beta - 2.4} = 0.0365 \left(C_D Re_p^{2/(2-n)} \right)^{0.57} \left[1 - 2.4 \left(\frac{d}{D} \right)^{0.27} \right] \quad (6)$$

where $Re_p = \rho V^{2-n} a^n / K$ is the particle Reynolds number (V is the particle velocity) and C_D the drag coefficient that can be computed from any available relationship, like that by Dhole et al. [9] which is valid in the range $5 \leq Re_p \leq 500$: $C_D = (24/Re_p) \left(1 + 0.148 Re_p^{2.35n/(2.42n+0.918)} \right)$.

In order to carry out a qualitative analysis, the flow in a cylindrical pipe will be simplified to a two dimensional Pouseuille flow, where the z direction is along the diameters of the pipe (Fig. 4). As gravity acts in the vertical direction, N_g will be projected along z in the analysis that follows. Assuming a steady state flow with no secondary currents, i.e. only with the component u of the velocity in the x direction, the momentum equation is reduced to $0 = -\partial P / \partial x + \partial \tau_{zx} / \partial z$, where z is the coordinate normal to x , P is the pressure and τ_{zx} the shear stress that is reduced to $\tau_{zx} = K(\partial u / \partial z)^n$. To simplify the notation, it is defined $P_x = \partial P / \partial x$ and $\dot{\gamma} = \partial u / \partial z$. Integrating the momentum equation with respect to z , $\dot{\gamma} = ((P_x z + C_1) / K)^{1/n}$ is obtained, with C_1 a constant of integration. The mixture of fine particles and the pseudoplastic carrier behave as an equivalent pseudoplastic fluid, characterized by a mixture consistency coefficient, K_m , which is a function of the volumetric concentration of solids, ϕ . It can be estimated according to the relationships of Kawase and Ulbrecht (1983) [10]. Thus, the deformation shear rate of the mixture can be written as

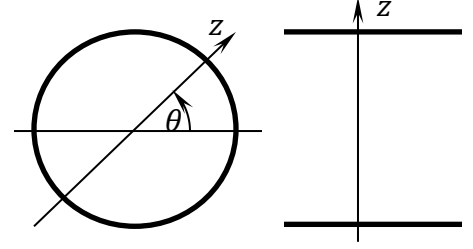


Fig. 4: Simplification of the cylindrical geometry to the two dimensional Poiseuille flow.

$$\dot{\gamma}_m = \left(\frac{P_x z + C_1}{K_m} \right)^{1/n} \quad (7)$$

An effective viscosity of the mixture is defined as

$$\mu_m = K_m |\dot{\gamma}_m|^{n-1} = K_m \left| \frac{P_x z + C_1}{K_m} \right|^{\frac{n-1}{n}} \quad (8)$$

Using $\dot{\gamma}_m$ instead of $\dot{\gamma}$, and μ_m in the expressions for the fluxes, and considering two dimensional Pouseuille flow, it is possible to get more manageable relationships that will allow us to know the flux of solid particles in the 2D pipe. Thus, the fluxes associated to concentration gradient and viscosity gradient are reduced to:

$$N_C = -D_C a^2 \left(\phi^2 \frac{d|\dot{\gamma}_m|}{dz} + \phi |\dot{\gamma}_m| \frac{d\phi}{dz} \right) \quad (9)$$

$$N_\mu = -D_\mu |\dot{\gamma}_m| \phi^2 \left(\frac{a^2}{\mu_m} \right) \frac{d\mu_m}{dz} \quad (10)$$

$$\frac{d\mu_m}{dz} = \frac{dK_m}{d\phi} \frac{d\phi}{dz} \quad (11)$$

With respect to the diffusion coefficients, $D_C = 0.43$ [11] and $D_\mu = D_C / (0.01042\phi + 0.142)$ [12] were used. It is worth to stress the important role played by the energy loss P_x through the mixture viscosity in all the fluxes. The absolute value of the shear rate should be used in the equations because the diffusive model is based on the frequency of particle collisions which scales with $|\dot{\gamma}_m|$, according to the model of Leighton and Acrivos [7, 8].

It is easy to see that the direction of the fluxes N_C and N_μ is defined by the sign of $d\phi/dz$, $d|\dot{\gamma}_m|/dz$ and $d\mu_m/dz$. The flux of particles due to gravity, N_g , depends only on $(\rho_s - \rho)$ and it is always downwards for negatively buoyant particles. The net flux of particles along the z direction is $N_T = N_C + N_\mu + N_g \sin \theta$. The result of an analysis for different conditions is given below for some particular cases.

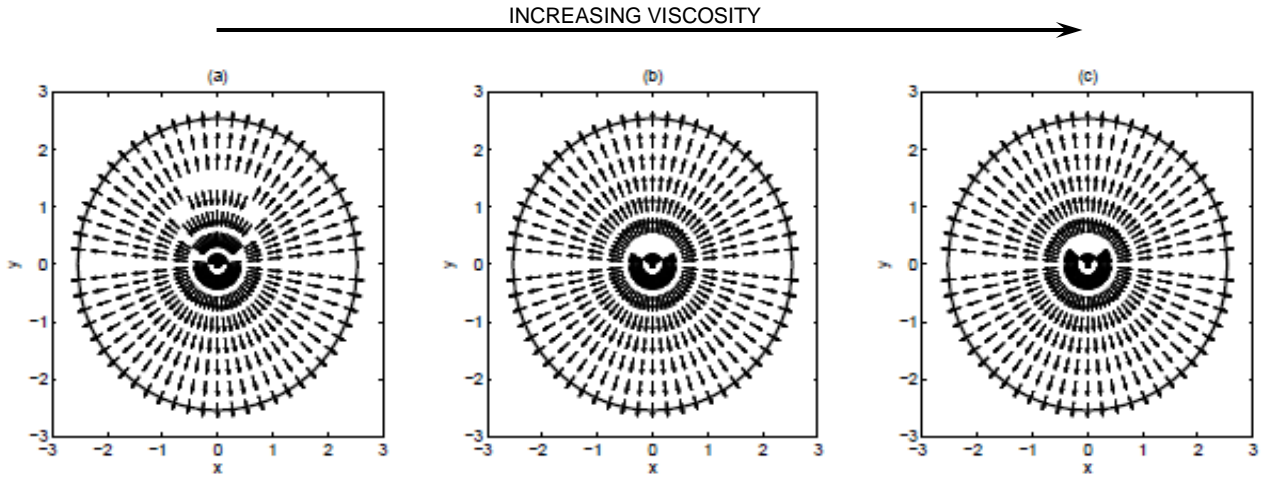


Fig. 5: Effect of viscosity in the direction of the flux of particles. $Q \sim 1$ L/s and $d_{50} = 600 \mu\text{m}$.

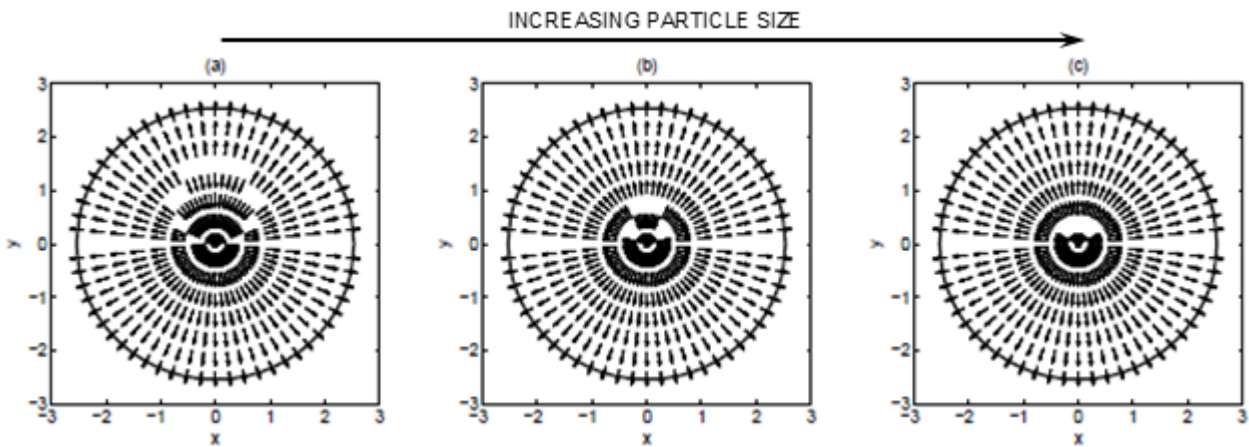


Fig. 6: Effect of particle size in the direction of the flux of particles. $Q \sim 1$ L/s.

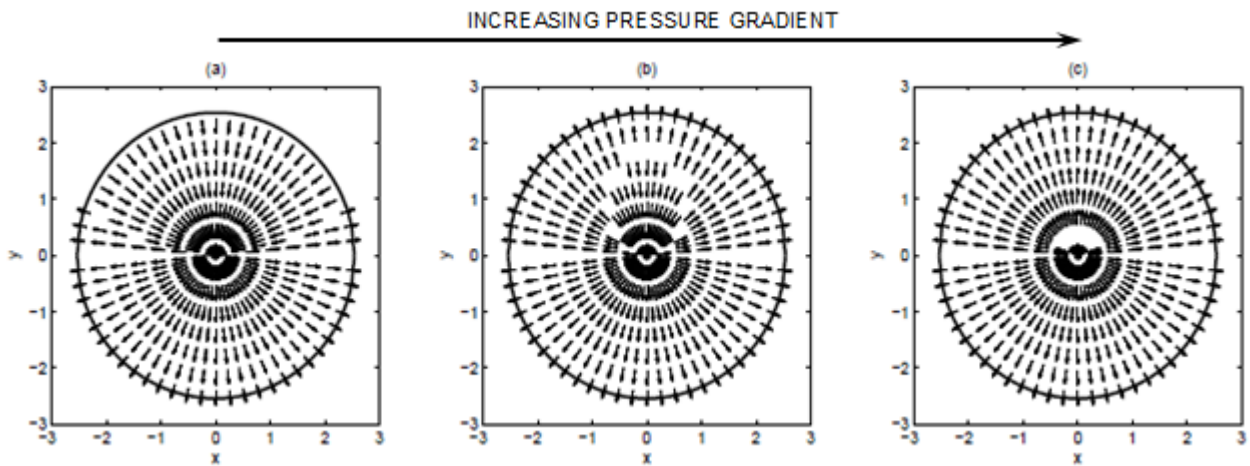


Fig. 7: Effect of the head loss in the direction of the flux of particles. $Q \sim 1.6$ L/s and $d_{50} = 120 \mu\text{m}$.

For a relatively high discharge ($Q \sim 1$ L/s) and $d_{50} = 600 \mu\text{m}$, it is found that $|N_g \sin \theta| > |N_c + N_\mu|$ only for the less viscous mixtures. Radial fluxes (along z) for this condition are presented in Fig. 5, in which viscosity increases from left to right. A similar analysis shows that for the same viscosity and discharge, $|N_g \sin \theta| > |N_c + N_\mu|$ in the centre of the pipe. Fig. 6 corresponds to the cases in which discharge, viscosity and pressure gradient are kept constant, changing the particle size. It is observed that for the two largest sizes of particles (d_{50} equal to 300 and 600 μm) gravity fluxes dominates only near the center of the pipe, with strong fluxes towards the walls due to the gradient of concentration and viscosity. The effect

of the head loss is presented in Fig. 7, where the flux directions are shown for $Q \sim 1.6$ L/s, $d_{50} = 600$ μm , $K = 0.29$ Pa·sⁿ, and $n = 0.60$, and three pressure gradients: $P_x/\rho_m g = 0.10$ m, 0.25 m, 0.30 m. It is observed that at higher head loss per unit length, diffusive fluxes overcome the gravitational one.

4. Conclusion

The qualitative analysis of fluxes of solid particles due to gradient of concentration and viscosity indicates that the concentration of particles carried by a pseudoplastic fluid in laminar regime can present minimum values near the centre and higher close to the walls. The analysis, although highly simplified, preserves the most important physical mechanisms that govern the migration of the solid particles. Thus, it was explained why larger particles did not settle in the experiments. It was found that the pressure gradient (head loss) controls the fluxes through the effective viscosity of the mixture formed by the solid particles and the pseudoplastic fluid.

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