

# Supplier Evaluation and Selection Using AHP Method and Uncertainty Theory

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**Abstract** - Supplier evaluation and selection is an important decision making problem. It is extremely difficult to maintain the performance of manufacturer without considering a suitable set of suppliers. One of the key problems in SCM is supplier evaluation and selection that is to find the best supplier among several alternatives according to various criteria, such as quality, technology capability, pollution control, environmental management, green production and green competency. With respect to the identification of criteria, selection of criteria and calculation of priority weights should be considered.

Generally speaking, supplier evaluation and selection is a multiple attribute decision making (MADM) problem. And the criteria for supplier selection depend highly on individual companies and industries. Because different companies have different organizational structure, management strategy, enterprise culture and others, all of these influence the determination of supplier selection criteria. So, the identification of supplier selection criteria is on the basis of specific environments, and requires domain experts' assessment and judgement.

In order to find the best supplier, integrating experts' assessments is very important after identifying supplier selection criteria. Many methods have been applied to supplier selection, such as the analytic hierarchy process (AHP), fuzzy set theory, TOPSIS method, Dempster-Shafer evidence theory and others. By the way supplier evaluation and selection problem is associated with recognitive and stochastic uncertainties. So it is necessary to make a more effective supplier evaluation and selection method that can consider recognitive and stochastic uncertainties simultaneously.

Analytic hierarchy process (AHP) is popular and extensively used to deal with complex decision problem due to its simplicity in concept and convenience in operation. AHP is structured technique for handling complex decision problems. Both qualitative and quantitative factors are combined by using AHP in the decision making process. Chief executive officer's subjective judgement yields recognitive uncertainties and another uncertainty is referring to stochastic nature of various decision parameters. Various researchers proposed supplier selection model deal with stochastic parameters. Recognitive uncertainty is due to lack of knowledge or incomplete information. Fuzzy based technique is widely used to handle the recognitive uncertainty.

This paper proposes an integrated method for dealing with supplier evaluation and selection problem using a combined AHP and uncertainty theory. The objective of this paper is to develop framework for reducing the purchasing risks associated with suppliers. First, the solution methodology of AHP is used to select the supplier with lots of factors, second, the fuzzy set ranking methodologies is used to integrate the special multi attribute decision problems, and then computer program is developed to demonstrate a methodology for the supplier evaluation and selection based decision support system. An illustrative example is presented to demonstrate the effectiveness of the proposed model.

**Keywords:** Supplier Evaluation, Supplier Selection, Analytic Hierarchy Process, Fuzzy Inference System, Uncertainty Theory

## 1. Introduction

Supply selection in supply chain is critical strategic decision for organization's success and has attracted much attention of both academic researchers and practitioners. The supplier performance analysis problem is one of the multiple attribute decision making (MADM) problems. The main objective of supplier selection process is to reduce purchase risk, maximize overall value to the purchaser, and develop closeness and long-term relationships between buyer and suppliers in today's competitive scenario. One of the key problems in SCM is supplier evaluation and selection that is a complicated multi-criteria decision making problem. The identification of criteria and integration of experts' assessments are two components of the supplier selection problem.

Supplier evaluation and selection is a multiple attribute decision making (MADM) problem. And the criteria for supplier selection depend highly on individual companies and industries. Because different companies have different organizational structure, management strategy, enterprise culture and others, all of these influence the determination of supplier selection criteria. So, the identification of supplier selection criteria is on the basis of specific environments, and requires domain experts' assessment and judgement.

To find the best supplier, integrating experts' assessments is very important after identifying supplier selection criteria. Many methods have been applied to supplier selection, such as the analytic hierarchy process (AHP), fuzzy set theory, TOPSIS method, Dempster-Shafer evidence theory and others. It is necessary to consider cognitive and stochastic uncertainties simultaneously because supplier evaluation and selection problem is associated with cognitive and stochastic uncertainties.

This paper proposes an integrated method for dealing with supplier evaluation and selection problem using a combined AHP and uncertainty theory. The objective of this paper is to develop framework for reducing the purchasing risks associated with suppliers. First, the solution methodology of AHP is used to select the supplier with lots of factors, second, the fuzzy set ranking methodologies is used to integrate the special multi attribute decision problems, and then computer program is developed to demonstrate a methodology for the supplier evaluation and selection based decision support system. The numerical example is presented to demonstrate the effectiveness of the proposed model.

## 2. Literature Review

Weber et al. [1] reviewed, annotated, and classified related articles which have been appeared since 1966. There are two issues in supplier evaluation and selection. Regarding the development of supplier selection model, many decision models have been suggested for supporting the supplier selection process along its main steps. Simple linear weighting models have been adapted to deal with uncertainty in decision making deriving from incomplete and qualitative data and unstructured purchasing situations. Some innovative approaches, based on artificial intelligence techniques such as Fuzzy Logic match very well with decision making situations where supplier's evaluation is also perceptive.

Regarding the second issue, tendency to enlarge the set of evaluative attributes, vendors focused on a technical output evaluation, in terms of quality, delivery speed and reliability, price offered, but when the relationship becomes closer and longer, the number of selection criteria increase and vendors are selected on their global performances.

Tung and Torng [2] presented a fuzzy decision making approach to deal with the supplier selection problem in supply chain system. Then a hierarchy multiple criteria decision making model based on fuzzy-sets theory is proposed to deal with the supplier selection problems in the supply chain system.

Lewis [3] suggested that of all the responsibilities that related to purchasing, none was more important than the selection of a proper source. When the supplier relationship management concept is concerned, producers are trying to build long-term and profitable relationships with supplier.

Humphreys et al [4] presented a framework for integrating environmental factors into the supplier selection process. Producers consider factors like quality, flexibility, price, delivery when evaluating supplier performance. As environmental pressure is increasing, many producers begin to consider environmental issues and the measurement of their suppliers' environmental performance

## 3. Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process is a theory of measurement through pairwise comparisons and relies on the judgements of experts to derive priority scales. It is these scales that measure intangibles in relative terms. The comparisons are made using a scale of absolute judgements that represents, how much more, one element dominates another with respect to a given attribute.

The judgements may be inconsistent, and how to measure inconsistency and improve the judgements, when possible to obtain better consistency is a concern of the AHP. The derived priority scales are synthesised by multiplying them by the priority of their parent nodes and adding for all such nodes.

AHP Saaty (1980) is a structured technique for handling complex decision problems. Both qualitative factors are combined by using AHP in the decision making process. The process of applying AHP has three steps. First, establish a hierarchical structure by recursively decomposing the decision problem. Second, construct the pairwise comparison matrix to indicate the relative importance of alternatives. A numerical rating including nine rank scales is suggested, as shown in

Table 1. Third, calculate the priority weights of alternatives according to the pairwise comparison matrix by the following equation:

$$Aw = \lambda_{max}w, \quad w = (w_1, w_2, \dots, w_n)^T \quad (1)$$

Where A is a n dimensional comparison matrix,  $\lambda_{max}$  is the largest eigenvalue of A and w is the eigenvector corresponding to  $\lambda_{max}$

To measure the inconsistency within the pairwise comparison matrix A we define a consistency index (C.I.) by the following equation.

$$C. I. = \frac{\lambda_{max} - n}{n - 1} \quad (2)$$

We use the consistency ratio C.R. to measure the degree of C.I. by the following equation.

$$C. R. = \frac{C. I.}{R. I.} \quad (3)$$

Where R.I. is the random consistency index, its value is related to the dimension of the matrix as in Table 2.

If  $C.R. < 0.10$  then the inconsistency degree of the comparison matrix A is consider acceptable and the eigenvector w is used as the weighting vector after normalization. Otherwise, the comparison matrix needs to be adjusted.

Table 1: Numerical rating in the AHP method.

Scale	Meaning
1	Equal importance
3	Moderate importance
5	Strong importance
7	Demonstrated importance
9	Extreme importance
2,4,6,8	Intermediate values

Table 2: Random consistency index R.I.

n	1	2	3	4	5	6	7	8	9	10
R.I.	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

#### 4. Linear programming models

For a multi attribute decision making problem, let  $A = [A_1, A_2, \dots, A_n]$  be a set of alternatives,  $G = [G_1, G_2, \dots, G_n]$  be a set of attributes.  $w = (w_1, w_2, \dots, w_n)^T \in H$  be the weight vector of attributes,  $w_j \geq 0, \sum_{j=1}^m w_j = 1$ , H is the set of the information about attribute weights, which can be unknown completely, or expressed as intuitionistic fuzzy number or constructed in the following forms:

- 1) A weak ranking:  $[w_i \geq w_j]$
- 2) A strict ranking:  $[w_i - w_j \geq \alpha_i]$
- 3) A ranking with multiples:  $[w_i \geq \alpha_i w_j]$
- 4) An internal form:  $[\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i]$
- 5) A ranking of differences:  $[w_i - w_j \geq w_k - w_l]$

Let the decision matrix  $R = (r_{ij})_{n \times m}$  be an intuitionistic fuzzy decision matrix, where  $r_{ij} = (\alpha_{ij}, \beta_{ij})$ , whose interval equivalent form is  $r_{ij} = [\alpha_{ij}, 1 - \beta_{ij}]$ ,  $\alpha_{ij}$  denotes the degree of the alternative  $A_i$  satisfies the attribute  $G_j$

Consider that the score of an intuitionistic fuzzy number can measure the magnitude of an intuitionistic fuzzy number, the score matrix  $S = (s(r_{ij}))_{n \times m}$  of the intuitionistic fuzzy decision matrix R, where  $s(r_{ij}) = s(\alpha_{ij}, \beta_{ij}) = \alpha_{ij} - \beta_{ij}$ ,  $s(r_{ij}) \in [-1.1], i = 1, 2, \dots, n; k = 1, 2, \dots, m$

We can establish the following linear programming models:

$$\begin{aligned}
 & \min \sum_{i=1}^{n-1} \sum_{j=i+1}^n (d_{ijl} + d_{iju}) \\
 \text{s.t. } & 0.5(\sum_{k=1}^m w_k (\bar{s}(r_{ik}) - \bar{s}(r_{jk})) + 1) + d_{ijl} \geq \mu_{ij} \\
 & \quad i = 1, 2, \dots, n-1; j = 1, 2, \dots, n \\
 & 0.5(\sum_{k=1}^m w_k (\bar{s}(r_{ik}) - \bar{s}(r_{jk})) + 1) - d_{iju} \leq 1 - v_{ij} \\
 & \quad i = 1, 2, \dots, n-1; j = 1, 2, \dots, n \\
 & \quad w = (w_1, w_2, \dots, w_n)^T \in H \\
 & \quad w_i \geq 0, i = 1, 2, \dots, n, \sum_{i=1}^n w_i = 1 \\
 & \quad d_{ijl}, d_{iju} \geq 0, i = 1, 2, \dots, n-1; j = i+1, \dots, n
 \end{aligned}$$

## 5. Supplier evaluation and selection

Based on the intuitionistic fuzzy decision matrix and the obtained weight intervals of attributes, the decision model is built to derive the weight vector of attributes, and then determine the most desirable alternative. The algorithm has 5 steps.

Step 1) uses the intuitionistic fuzzy decision matrix and weight intervals to establish the following linear programming model:  $\varphi^* = \max \sum_{i=1}^n \sum_{j=1}^m (1 - \beta_{ij} - \alpha_{ij}) w_j$

s.t.  $w = (w_1, w_2, \dots, w_m)^T \in \Delta$

And then determine the optimal weight vector  $w^* = (w_1^*, w_2^*, \dots, w_m^*)^T$  of attributes by solving the model

Step 2) calculate the overall attribute values  $z_i(w^*)$  of all the alternatives:  $A_i$  ( $i = 1, 2, \dots, m$ ), where  $z_{il}(w^*) = \sum_{j=1}^m w_j^* \alpha_{ij}$  for lower bound and  $z_{iu}(w^*) = \sum_{j=1}^m w_j^* (1 - \beta_{ij})$  for upper bound

Step 3) uses the possibility degree formula:

$$p(z_i(w^*) \geq z_j(w^*)) = \max \left[ 1 - \max \left[ \frac{z_{ju}(w^*) - z_{il}(w^*)}{z_{iu}(w^*) - z_{il}(w^*) + z_{ju}(w^*) - z_{jl}(w^*)}, 0 \right], 0 \right]$$

To calculate the possibility degree:  $p_{ij} = p(z_i(w^*) \geq z_j(w^*))$  and then construct the possibility degree matrix  $P = (p_{ij})$

Step 4) uses the following formula:  $w_i = \frac{1}{n(n-1)} (\sum_{j=1}^n p_{ij} + \frac{n}{2} - 1)$

To derive the priority vector  $w = (w_1, w_2, \dots, w_n)^T$  of the complementary judgement matrix  $P$

Step 5) rank and select the alternatives:  $A_i$  ( $i = 1, 2, \dots, n$ ) according to the priority vector  $w = (w_1, w_2, \dots, w_n)^T$

## 6. Numerical example

There is a customer who intends to buy a car. Five types of cars  $A_i$  ( $i = 1, 2, \dots, 5$ ) are available. The customer takes into account six attributes to decide which car to buy:  $G_1$ - safety,  $G_2$ - car performance,  $G_3$ - structure,  $G_4$ - reliability,  $G_5$ - economy,  $G_6$ - beauty. The membership degree  $\alpha_{ij}$  and the non-membership degree  $\beta_{ij}$  of the alternative  $A_i$  to  $G_i$  can be obtained and denoted by  $r_{ij} = (\alpha_{ij}, \beta_{ij})$ ,  $i = 1, 2, \dots, 5; j = 1, 2, \dots, 6$  which are listed in Table 3

Table 3: Intuitionistic fuzzy decision matrix R.

	$G_1$	$G_2$	$G_3$	$G_4$	$G_5$	$G_6$
$A_1$	(0.3, 0.5)	(0.6, 0.3)	(0.6, 0.4)	(0.8, 0.2)	(0.4, 0.5)	(0.5, 0.3)
$A_2$	(0.7, 0.3)	(0.5, 0.3)	(0.7, 0.2)	(0.7, 0.1)	(0.5, 0.4)	(0.4, 0.1)
$A_3$	(0.4, 0.3)	(0.7, 0.2)	(0.5, 0.4)	(0.6, 0.3)	(0.4, 0.3)	(0.3, 0.2)
$A_4$	(0.6, 0.2)	(0.5, 0.4)	(0.7, 0.2)	(0.3, 0.2)	(0.5, 0.4)	(0.7, 0.3)

$A_5$	(0.5, 0.3)	(0.3, 0.5)	(0.6, 0.3)	(0.6, 0.2)	(0.6, 0.2)	(0.5, 0.2)
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The information about attribute weights is as follows:

$H = [w_1 \leq 0.3, 0.2 \leq w_3 \leq 0.5, w_2 \leq 0.2, w_3 - w_2 \geq w_5 - w_4, 0.1 \leq w_5 \leq 0.4, w_4 \leq w_1, w_4 \leq 0.1, w_6 \geq 0.2]$ , the evaluator compares each pair of cars  $A_i$  ( $i = 1, 2, \dots, 5$ ), and constructs the intuitionistic judgement matrix B.

Table 4: Intuitionistic judgement matrix B.

(0.5, 0.5)	(0.2, 0.7)	(0.4, 0.6)	(0.2, 0.3)	(0.3, 0.4)
(0.7, 0.2)	(0.5, 0.5)	(0.6, 0.2)	(0.5, 0.4)	(0.6, 0.4)
(0.6, 0.4)	(0.2, 0.6)	(0.5, 0.5)	(0.3, 0.6)	(0.4, 0.6)
(0.3, 0.2)	(0.4, 0.5)	(0.6, 0.3)	(0.5, 0.5)	(0.6, 0.3)
(0.4, 0.3)	(0.4, 0.6)	(0.6, 0.4)	(0.3, 0.6)	(0.5, 0.5)

The score matrix S of the intuitionistic fuzzy decision matrix R is calculated

Table 5: Score matrix S.

	$G_1$	$G_2$	$G_3$	$G_4$	$G_5$	$G_6$
$A_1$	-0.2	0.3	0.2	0.6	-0.1	0.2
$A_2$	0.4	0.2	0.5	0.6	0.1	0.3
$A_3$	0.1	0.5	0.1	0.3	0.1	0.1
$A_4$	0.4	0.1	0.5	0.1	0.1	0.4
$A_5$	0.2	-0.2	0.3	0.4	0.4	0.3

The score matrix S can be normalized into  $\bar{S}$

Table 6: Normalized score matrix  $\bar{S}$ .

	$G_1$	$G_2$	$G_3$	$G_4$	$G_5$	$G_6$
$A_1$	0	0.714	0.25	1	0	0.33
$A_2$	1	0.571	1	1	0.4	0.667
$A_3$	0.5	1	0	0.4	0.4	0
$A_4$	1	0.429	1	0	0.4	1
$A_5$	0.667	0	0.5	0.6	1	0.667

Optimal solution by solving linear programming is as follows.  $\varphi^* = 0.0907$

$w_1 \in [0.1197, 0.2440]$ ,  $w_2 \in [0.1605, 0.1998]$ ,  $w_3 \in [0.2000, 0.2788]$

$w_4 \in [0.0614, 0.0899]$ ,  $w_5 \in [0.1062, 0.1474]$ ,  $w_6 \in [0.2000, 0.2472]$

Optimal weight vector based on fuzzy decision matrix R is founded.

$w^* = (0.1962, 0.1605, 0.2000, 0.0899, 0.1062, 0.2472)^T$  And the attribute values of alternatives are as follows:

$z_1(w^*) = [0.5132, 0.6285]$ ,  $z_2(w^*) = [0.5725, 0.7768]$ ,  $z_3(w^*) = [0.4614, 0.7208]$ ,

$z_4(w^*) = [0.5911, 0.7219]$ ,  $z_5(w^*) = [0.5075, 0.7122]$ ,

Finally the possibility degree matrix P is founded:

Table 7: Possibility degree matrix P

0.5	0.2078	0.4835	0.2268	0.4637
0.7922	0.5	0.6897	0.5752	0.6916
0.5165	0.3103	0.5	0.3429	0.4877
0.7732	0.4248	0.6571	0.5	0.6563
0.5363	0.3084	0.5123	0.3437	0.5

And the priority vector of P is:  $w = (0.1691, 0.2374, 0.1829, 0.2256, 0.1850)^T$

So, we rank the cars  $A_i$  ( $i = 1, 2, \dots, 5$ ) in accordance with  $w$ :  $A_2 \succ A_4 \succ A_5 \succ A_3 \succ A_1$ . And the best one is  $A_2$ .

## 7. Conclusion

This paper proposes an integrated method for dealing with supplier evaluation and selection problem using a combined AHP and uncertainty theory. This paper suggest framework for reducing the purchasing risks associated with suppliers. And the solution methodology of AHP is used to select the supplier with lots of factors. The fuzzy set ranking methodologies is used to integrate the special multi attribute decision problems, and then computer program is developed to demonstrate a methodology for the supplier evaluation and selection based decision support system.

Numerical example shows us the effectiveness of the proposed model. But for the problems of data collecting and its analysis in hierarchical decision structures, the DHP(Delphic Hierarchy Process) method can be used in future study

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