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Alternate Formulation for Entropy Minimization in Forced Internal Convection

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Abstract **-** In this paper, we present an alternate formulation for minimizing entropy generation in forced internal convection. Whereas most existing approaches focus on finding an optimum Reynolds number that minimizes entropy generation for a given mass flow and heat transfer rate, we explore the effects of changing flowrate for fixed channel dimensions, as well as changing flowrate and channel dimensions simultaneously. For turbulent flow with fixed channel dimensions, the minimum entropy generation is realized at different optimum Reynolds number than that for fixed mass flow. When varying both flowrate and channel dimensions, no absolute minimum exists, and entropy generation continues to decrease with increasing mass flow and channel dimensions. In this case, minimizing entropy corresponds either to optimizing the channel dimensions for the maximum available flowrate, or vice versa.

Keywords**:** Entropy generation minimization, forced internal convection

1. Introduction

Minimizing entropy generation represents an essential step towards achieving sustainable development and preserving natural resources for future generations. In systems involving forced internal convection, optimized flow conditions achieved by minimizing entropy generation can reduce useful energy losses, improve heat transfer rates, and/or minimize pressure drops, all of which leads to greater efficiency, reduced costs, and improved system reliability.

Bejan [1] was among the first investigators to produce an analysis to minimize entropy generation in turbulent flow in round ducts for constant heat flux conditions. Later authors [2-4] extended the analysis to include laminar flow, the effects of different channel geometries, and variations in conditions the axial direction. More recent authors have produced analyses for more specific applications, such as forced convection in microchannels [5-6]. In all these cases, however, the baseline conditions are those of known heat transfer and mass flowrate, so that the parameter variation resulting in optimal conditions corresponds to changing duct cross sectional dimension. An alternate analysis in which channel size remains constant (channel size being a measure of capital cost) and mass flow is optimized instead, is generally lacking in the literature. Furthermore, a method in which both mass flow and channel dimension are treated as independent parameters to be simultaneously optimized appears not to have been developed.

2. Derivation of Governing Equations

The generic system serving as the basis for analysis consists of a flow passage subject to convective heat transfer as shown in Fig. 1. Modelling assumptions include that

- steady-state conditions are maintained,
- the fluid is incompressible, and
- the cross-section geometry does not change in the flow direction.

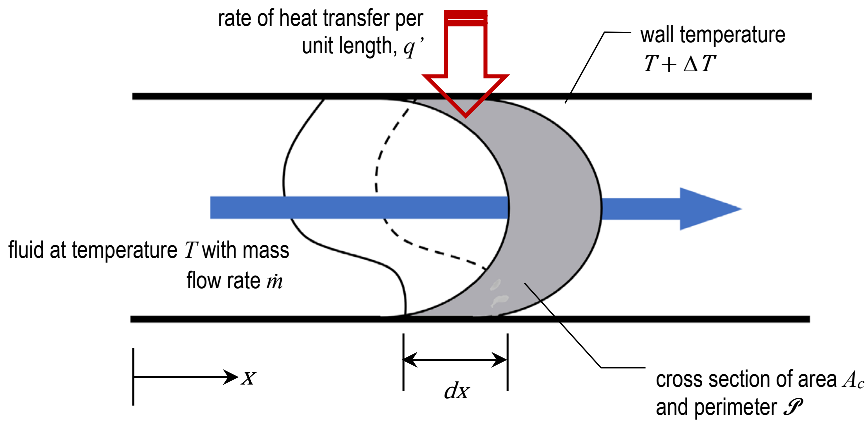

Fig. 1: Duct of arbitrary cross section subject to heat transfer (After Bejan [1]).

Applying conservation of energy and the accounting of entropy (the second law of thermodynamics), respectively, yields

$$0 = q'dx - \dot{m}dh \tag{1}$$

and

$$\dot{S}'_{gen} = \frac{d\dot{S}_{gen}}{dx} = \frac{\Delta T}{T^2}q' - \frac{\dot{m}}{T\rho}\frac{dP}{dx}, \tag{2}$$

where $\dot{S}_{gen}$ is the rate of entropy generation and h is the specific enthalpy of the fluid. Eliminating h via the thermodynamic relation $Tds = dh - dP/\rho$ and assuming that the wall to fluid temperature difference ΔT is much smaller than the absolute fluid temperature T, Eqs. (1) and (2) combine to give the entropy generation per unit length, $\dot{S}'_{gen}$ as

$$\dot{S}'_{gen} = \frac{d\dot{S}_{gen}}{dx} = \frac{\Delta T}{T^2}q' - \frac{\dot{m}}{T\rho}\frac{dP}{dx}. \tag{3}$$

Making use of the definitions of Nusselt number and Darcy friction factor to eliminate ΔT and dP/dx yields, after much manipulation,

$$\dot{S}'_{gen} = \frac{q'^2}{\chi k T^2 Nu} + \frac{\chi^3 \mu^5 f Re^5}{128\rho^2\dot{m}^2 T}, \tag{4}$$

where $\chi = \mathscr{P}/D_h$, the ratio of the perimeter to hydraulic diameter. Termed the shape ratio, χ is a constant for any given cross-sectional geometry [2, 4]. In the case of a round duct, $\chi = \pi$.

The first term on the right-hand side of Eq. (4) represents the entropy generation rate per unit length due to heat transfer whereas the second term is due to friction pressure drop. Given the functional dependence of the Nusselt number and friction factor on Reynolds number, for most flows the heat transfer irreversibility will tend to decrease as the pressure drop irreversibility increases and vice versa. Hence, an optimal thermodynamic condition often exists, one which and can be found by minimizing the value of $\dot{S}'_{gen}$ in Eq. (4).

3. Entropy minimization strategies

Equations of the form of Eq. (4) have served as the basis for many schemes for entropy generation minimization over the past several decades. Most notably, Bejan [1] first found the minimum entropy generation in turbulent flow in a round duct for a fixed heat transfer and mass flowrate. With known values of those two parameters, setting the derivative of Eq. (4) with respect to Reynolds number equal to zero results in the optimum Reynolds number that minimizes the entropy generation. Bejan made use of a simplified expression for Darcy friction factor in smooth tubes attributed to McAdams [7] and the well-known Nusselt correlation of Dittus and Boelter [8].

Later authors [2-4] have generalised this strategy, allowing for correlations for friction factor (f) and Nusselt number (Nu) of the generic forms

$$f = C_1/Re^p \tag{5}$$

and

$$Nu = C_2 Re^m Pr^n \tag{6}$$

to be incorporated into the analysis. Here, Re and Pr are the Reynolds number based on hydraulics diameter and Prandtl number, respectively, and C_1 and C_2 are constants. Incorporating these relations gives the dimensionless entropy generation per unit length to be

$$N_S = \frac{\dot{S}'_{gen}}{q'^2/kT^2} = \frac{1}{\chi C_2 Re^m Pr^n} + \frac{\chi^3 C_1 Re^{5-p}}{128 B_0^2}, \tag{7}$$

where the variable B_0 is deemed a "duty parameter," fixed by the required values of $\dot{m}$ and q':

$$B_0 = \frac{\rho \dot{m} q'}{\mu^{5/2}(kT)^{1/2}}. \tag{8}$$

Minimum entropy generation occurs at an optimum Reynolds number of

$$Re_{opt} = \left(\frac{128m}{\chi^4 C_1 C_2 (5-p)}\right)^{1/(5-p+m)} B_0^{2/(5-p+m)} Pr^{-n/(5-p+m)}. \tag{9}$$

Figure 2 shows the general shape of the entropy generation for turbulent flow in circular ducts as predicted by Eqs. (7)-(9). As seen in the figure, the region for which $Re < Re_{opt}$ is characterized by entropy generation due to heat transfer whereas for $Re > Re_{opt}$, pressure drop irreversibility dominates.

The above scheme in which $\dot{m}$ and q' are constant has only one degree of freedom, Reynolds number. A subtlety that can easily be lost in the analysis is that this is equivalent to varying the channel perimeter $\mathscr{P}$, and furthermore, that increasing perimeter *decreases* Reynolds number:

$$Re = \frac{4\dot{m}}{\mathscr{P}\mu}. \tag{10}$$

Additionally, practical considerations may make the physical size of the channel cross section the limiting factor rather than the necessity of a given flowrate. These issues are examined next in order to develop a more generally applicable entropy minimization strategy.

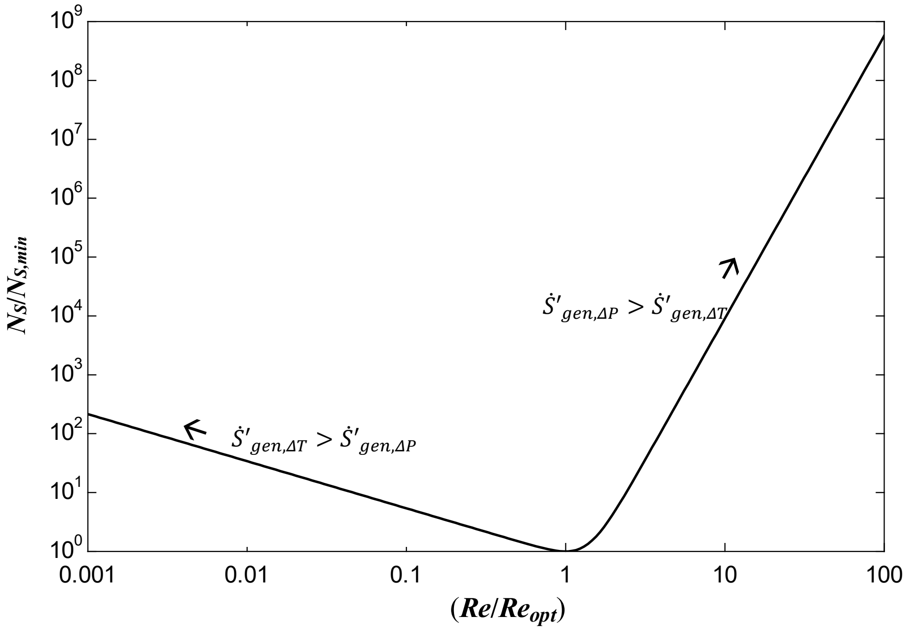

Fig 2: Entropy generation number for turbulent flow in a round duct subject to constant wall heat flux, The values of the constants in Eqs. (5) and (6) are those found in McAdams [7] and Dittus and Boelter [8].

3.1. Minimization of Entropy for Variable Flow Rate

The previous analysis investigates the trends of entropy generation in forced internal convection while holding heat transfer and mass flowrate constant. Practically, then, variation of Reynolds number amounts to changing channel dimension. These criteria may suffice for the initial design of a piece of equipment, but once a certain design is realized, optimizing its performance under different operating conditions would obviously not admit changing dimensions of the flow passage. Moreover, it is easy to imagine that mass flow rate is of only secondary importance to heat transfer in many cases and that the physical size of the equipment is more of a limiting factor. Hence, a scheme to minimize the rate of entropy generation for a fixed heat transfer and channel cross section is warranted.

Eliminating $\dot{m}$ between Eqs. (4) and (10) allows perimeter $\mathscr{P}$ to appear explicitly:

$$\dot{S}'_{gen} = \frac{q'^2}{\chi k T^2 Nu} + \frac{\chi^3 \mu^3 f Re^3}{8\rho^2 \mathscr{P}^2 T}. \tag{11}$$

The variation of Re in Eq. (11) now represents a changing flowrate rather than channel dimension. The same procedure used to generate Eq. (7) gives the rate of dimensionless entropy generation per unit length as

$$N_S = \frac{\dot{S}'_{gen}}{q'^2/kT^2} = \frac{1}{\chi C_2 Re^m Pr^n} + \frac{\chi^3 C_1 Re^{3-p}}{8B_1^2}. \tag{12}$$

A newly defined dimensionless group B_1 appears in Eq. (12). The analogue of B_0, B_1 is fixed by the required values of $\mathscr{P}$ and q':

$$B_1 = \frac{\rho \mathscr{P} q'}{\mu^{3/2}(kT)^{1/2}}. \tag{13}$$

Minimum entropy generation under these conditions occurs at an optimum Reynolds number of

$$Re_{opt} = \left(\frac{8m}{\chi^4 C_1 C_2 (3-p)}\right)^{1/(3-p+m)} B_1^{2/(3-p+m)} Pr^{-n/(3-p+m)}. \tag{14}$$

Figure 3 gives the entropy generation for variable flow rate as predicted by Eqs. (12)-(14). For comparison, entropy generation for constant flowrate is shown as well.

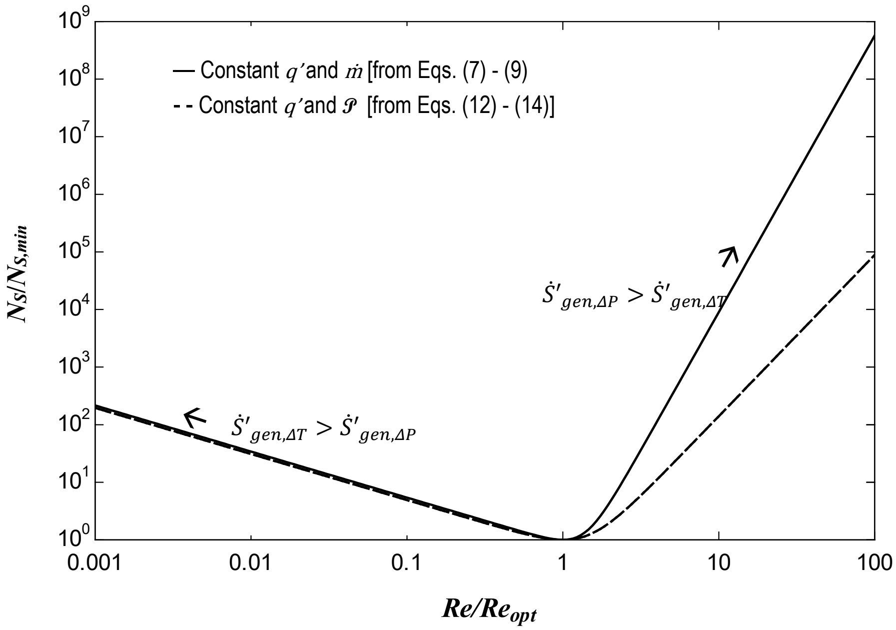

Fig 3: Comparison of entropy generation numbers for turbulent flow with constant q' for a round duct. The solid line is for constant q' and $\dot{m}$ whereas the dashed line is for constant q' and $\mathcal{P}$.

From the figure we see that in the region where heat transfer entropy generation dominates ($Re < Re_{opt}$,), there is only a slight difference between the constant mass flow and constant perimeter constraints. For $Re > Re_{opt}$, however, N_S shows less sensitivity to deviation from optimal Reynolds number when channel dimensions are held constant. Nonetheless, in cases where entropy generation due to pressure drop dominates, Fig. 3 suggests that significant decreases in entropy generation are still possible by decreasing *Re* via smaller flowrates.

In should also be kept in mind that the optimum Reynolds for a given value of B_1 is different than that for constant B_0. That is, adjusting mass flow to minimize entropy generation for a required channel dimension does not result in the same Reynolds number that minimizes entropy by adjusting channel size for a required mass flow.

3.2. Minimization of Entropy for Variable Flow Rate and Channel Dimension

In many practical situations, a specified rate of heat transfer represents the most important task and the design of a system to achieve that goal may include the selection of both channel size and flowrate. The question therefore naturally arises as to how to minimize entropy generation for these two degrees of freedom. The task is not a simple one, however, chiefly due to the fact that both flow rate and channel dimension appear in the Reynolds number. A different formulation is therefore required.

Combining Eqs. (4) and (10) to make mass flow and perimeter appear explicitly gives

$$N_S = \frac{\dot{S}'_{gen}}{q'^2/kT^2} = \frac{1}{\chi Nu} + \frac{8\chi^3 kTf\dot{m}^3}{\rho^2 q'^2 \mathcal{P}^5} = \frac{1}{\chi Nu} + \frac{8\chi^3 f B_0^3}{B_1^5}. \tag{15}$$

In this formulation, B_0 and B_1 are proxies for $\mathcal{P}$ and $\dot{m}$, respectively. Making use of Eqs. (5) and (6) and realizing that Reynolds number is related to B_0 and B_1 by

$$Re = \frac{4B_0}{B_1}, \tag{16}$$

we get

$$N_S = \frac{\dot{S}'_{gen}}{q'^2/kT^2} = \frac{B_1^m}{4^m \chi C_2 Pr^n B_0^m} + \frac{8\chi^3 C_1 B_0^{3-p}}{4^p B_1^{5-p}}. \tag{17}$$

Figure 4 shows the dependence of N_S on B_0 and B_1 for a round duct subject to turbulent forced convection. We see that N_S continues to decrease with ever increasing B_0 (~mass flow) and B_1 (~channel dimension) without limit. This is consistent with the form of Eq. (17), for which no values of B_0 and B_1 exist that can satisfy the condition $\partial N_S/\partial B_0 = \partial N_S/\partial B_1 = 0$.

Physically, as B_0 (and thus $\dot{m}$) becomes very large, it enhances the heat transfer process so that q' eventually occurs at a vanishingly small wall-to-fluid temperature difference, ΔT. Hence, the entropy generation due to heat transfer represented by the first term on the right-hand side of Eq. (17) approaches zero. For a fixed channel cross section, the increased mass flow would be accompanied by an increased fluid velocity as well. Thus, the fluid shear rate would increase, and a larger friction with it. However, increasing channel dimension simultaneously with mass flowrate allows for this to be accomplished with *smaller* fluid velocities. This is evidenced by large values of B_1 (~ $\mathcal{P}$) causing entropy generation due to friction pressure drop to become very small, as seen in the second term on the right-hand side of Eq. (17). In short, increased mass flow with a concomitant increase in channel cross sectional dimension can lead to smaller entropy generations than those associated with varying either parameter by itself.

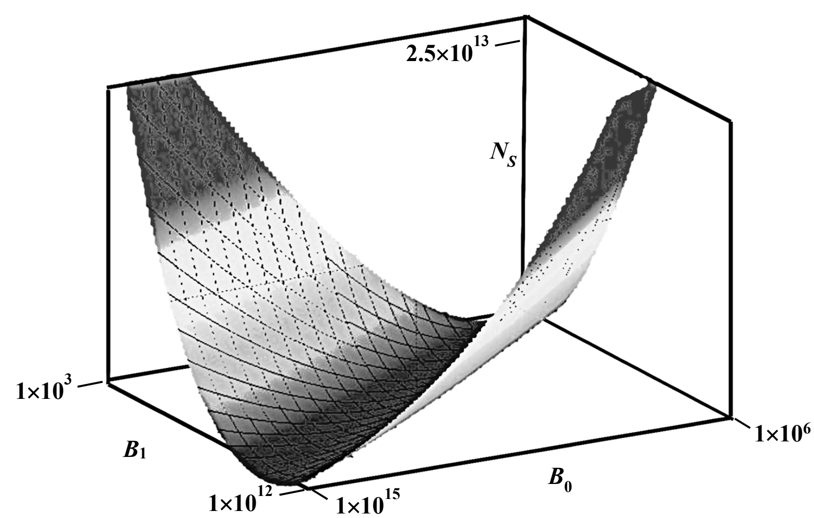

Fig 4: Dependence of N_S on B_0 and B_1.for turbulent flow in a round channel for a fluid with Pr = 0.7. Smaller entropy generations are possible by making $\dot{m}$ and $\mathcal{P}$ progressively larger.

All of this may seem to suggest that minimizing entropy generation in forced internal convection can only be achieved at the most impractical of conditions, that of an infinite flowrate and an infinitely large cross section. Nonetheless, optimal conditions can still be found when varying both channel dimension and mass flow. For continuous functions defined by closed bounded regions, an absolute maximum and minimum must exist [9]. With no values of B_0 and B_1 that meet the partial derivative requirements for the existence of relative extrema, the minimum N_S point must therefore appear on the boundary of a given range of B_0 and B_1. To find this point, we simply find the values of N_S from Eq. (17) corresponding to the largest values of B_0 and B_1 that can be achieved in practice for a given situation. The smaller of the two N_S values determines the optimum operating point. The process is illustrated in Fig. 5.

The curve in Fig. 5 showing the optimum value of B_1 for a given B_0 is found by setting $\partial N_S/\partial B_0 = 0$, whereas the optimum B_0 curve is found by setting $\partial N_S/\partial B_1 = 0$. The results are

$$B_{1,opt} = \left(\frac{4^{m-p} \cdot 8\chi^4 C_1 C_2 (5-p) Pr^n}{m}\right)^{1/(5-p+m)} B_0^{(3+m-p)/(5-p+m)} \tag{18}$$

and

$$B_{0,opt} = \left(\frac{m}{4^{m-p} \cdot 8\chi^4 C_1 C_2 (3-p) Pr^n}\right)^{1/(3-p+m)} B_1^{(5+m-p)/(3-p+m)}. \tag{19}$$

Equations (18) and (19) for $B_{1,opt}$ and $B_{0,opt}$ correspond to the optimum Reynolds numbers given in Eqs. (9) and (14), respectively.

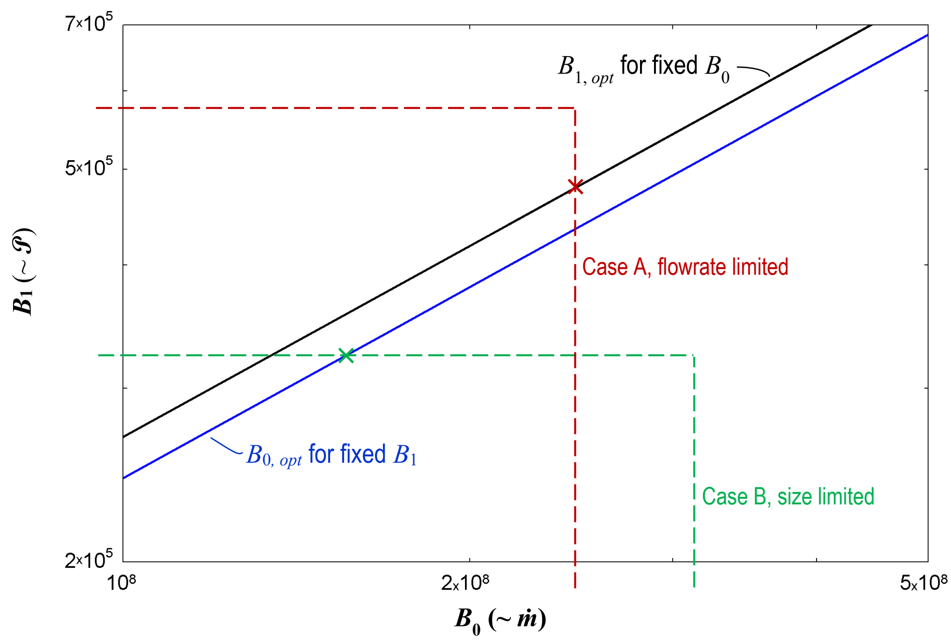

Fig 5: Determination of minimum entropy generation point for varying flowrate and channel dimension simultaneously. The dashed lines indicate the range of available flowrates and channel dimensions for two hypothetical cases. In Case A, the minimum entropy generation occurs at the maximum available flowrate and a smaller perimeter than is available. In Case B, the maximum channel size limits the entropy generation, the minimum occurring at a smaller flowrate than is achievable.

4. Conclusion

We have presented an alternative formulation to minimize entropy generation for the forced internal convention of an incompressible fluid. Instead of minimizing entropy generation only for the case of required rates of heat transfer and mass flow, in which case an optimum channel dimension is sought, we have developed a method for optimizing mass flow for a fixed channel dimension. The trends in entropy generation as Reynolds number deviates from its optimal value are similar whether optimizing based on channel dimension or flow rate. The value of the optimum Reynolds number itself, however, and therefore the specific flow conditions, are different depending on whether designing for specified flow rate or specified channel dimension. When allowing both channel dimension and flow rate to be changed simultaneously, no absolute minimum entropy generation condition exists for a given heat transfer rate. In this case, the optimal conditions are found by either making use of the maximum available channel dimension and optimizing the mass flow, or vice versa, whichever results in the smaller entropy generation.

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