

# Bubbles, Drops, and Particles in Nonlinear Flows

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## Extended Abstract

Many industrial products take the form of foams, emulsions, and suspensions, in which bubbles, drops, and solid particles are suspended in a liquid. Many times, these two-phase systems are processed in rotating mixing equipment generating shear or extensional flows. A bubble, drop, or particle subjected to these types of flows may translate, deform, rotate, and even break. Thus, fluid mechanics with bubbles, drops, and particles in shear and extensional flows has a long history and important place in science since the pioneering work of GI Taylor [1-2] almost a century ago.

Almost all of studies in the literature assume that, in these processes, the continuous phase is subjected to a linear flow, such as axisymmetric extensional flow, hyperbolic flow, or simple shear flow. Yet very little is dedicated to nonlinear flows, which are neglected, but can explain better (than the linear flow) the real flow which occurs in rotating mixing devices. For example, a linear hyperbolic flow can describe the flow close to the centre of Taylor's four-roller mill apparatus, but it cannot describe the flow near the rollers (Antanovskii [3]). Nonlinear flows can also be found in microfluidic devices, where the size of the drop is comparable to that of the apparatus (Pozrikidis [4], Dimitrakopoulos [5]).

The report presented here explores small deformations of bubbles, drops, and particles in two nonlinear creeping motions. The first example is an axisymmetric nonlinear cubic extensional flow, first suggested by Sherwood [6] and explored by Favelukis [7-8] and Liu et al. [9]. Here the flow outside the drop may contain closed circulations or separating surfaces, while the flow inside the drop may double the number of internal circulations. The second example is a two-directional nonlinear cubic shear flow, recently suggested by Favelukis [10]. In this flow the streamlines reveal unusual shapes, which sometimes are accompanied by many stagnation points. The deformation of the drop is presented in terms of the capillary number ( $Ca$ ), the viscosity ratio ( $\lambda$ ), the nonlinear intensity of the flow ( $E$ ), and the linear velocity ratio ( $G$ ).

## References

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