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The Mechatronic Approach in the Mathematical Modelling And Simulation To Control The Water Hammer In Hydraulic Facilities

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Abstract – The physical phenomenon of water hammer causes considerable damage and even the destruction of hydraulic installations, so this research aims to develop a method of analysis and synthesis of this phenomenon that allows maintaining the transient water hammer overpressure within technically acceptable values. acceptable to improve the operational reliability and safety of hydraulic systems. To contribute to the solution of this problem, a mathematical model is developed, using the mechatronic approach, for which the water hammer phenomenon is represented with state variables, considering the hydromechanical system as a mechanical plant to be controlled, with multiple inputs and outputs. This allows the mathematical model of water hammer to be general for various hydraulic installations, that is, the mathematical model and the numerical simulation are valid for various fluids, pipes and fittings with different materials, diameters, lengths, wall thicknesses, and for valves of different characteristics. Conventional methods do not consider very important parameters and coefficients of the fluid and the pipe that significantly affect the dynamics of the water hammer, which can obviously produce serious errors, instead, the mathematical model obtained with the mechatronic approach is general, consistent, and repeatable. In addition, with the method developed in this research, the difference between the critical valve closure time (operating time) and the total valve closure time is evident, however, with the classical methods these values can be confused with dangerous consequences. It has been proven that the mechatronic approach allows in a very effective way the analysis and synthesis of the hydraulic system dynamics, including all the parameters, and coefficients in a clear and complete way. The mechatronic method allows to maintain the transient overpressure within technically acceptable values, that is, hydraulic installations with high operational reliability.

***Keywords*:** water hammer; dynamic; transient; celerity; mechatronic.

1. Introduction

Water hammer is a very dangerous and destructive phenomenon in fluid transmission and distribution lines; in applications such as: drinking water systems, irrigation pipes, oil pipelines, hydroelectric plants, pumping stations and others. It is caused by the rapid closing or opening of a hydraulic valve that produces sudden changes in the acceleration of the fluid, which generates an overpressure or transient wave that travels at very high speeds and can hit and destroy the components of the hydraulic installation [1].

Water hammer depends on several parameters such as the material of the pipe, the length of the pipe, the viscosity of the fluid and mainly the closing time of the valve. Also, the valve loss factor, which varies with the degree of opening, plays a key role. The most widely used method in engineering to control the overpressure wave due to water hammer is to increase the valve closing time.

The traditional equations used in engineering to calculate the closing time of valves contain many empirical coefficients that cannot be generalized for all cases, so it is necessary to use devices such as: compensation tanks, air chambers, air valves, valve testing, among others, to ensure operational safety [2].

Several numerical methods have been developed to analyze the phenomenon of water hammer, the mathematical model presents a formulation of the time-dependent characteristic equations that model the dynamics of water hammer, the numerical simulation of the mathematical model allows obtaining as a result the temporal response of the parameters, mainly the magnitude of the surge or transient wave [3] [4] [5].

In references [6] and [7] developed by the team of researchers presenting this proposal, the mechatronic approach has been used for modeling and numerical simulation of thermal and thermodynamic processes, to analyze the dynamic behavior of solar heating of water and air respectively. The results obtained in these investigations indicate that the use of the holistic and synergistic approach of mechatronics, to model and simulate the dynamic behavior of systems, simplifies, and systematizes their analysis. This has motivated the use of the mechatronic approach in other systems, in this case, to model and simulate the behavior of the water hammer phenomenon in hydraulic systems.

Mechatronic system design combines the classic disciplines of mechanical engineering such as statics, dynamics, thermodynamics, fluid mechanics, metrology, and tribology with subjects such as electronics, software design/engineering, and optics. This combination is much more than the sum of these disciplines, rather in mechatronics, they are integrated synergistically and holistically.

Holistic is used as a synonym for systemic, holism emphasizes the importance of the whole, which is greater than the sum of the parts, and gives importance to the interdependence of these. Synergy is the action of two or more causes whose effect is greater than the sum of the individual effects. The holistic and synergistic integration of mechatronics combines the joint action of the disciplines: mechanics, electronics, and software; to analyze and synthesize systems in an integral way, giving importance to the interdependence between their components, which allows the design of current industrial products and processes, which are highly complex [8] [9].

In this research, the mechatronic approach has been used to develop the mathematical model of the water hammer dynamics to then simulate numerically and obtain the graphic and numerical results that will allow the analysis and synthesis of the dynamic behavior of the overpressure transient of the water hammer. with the aim of controlling with great precision the opening and mainly the closing of the valves to keep the overpressure transient within safe limits to avoid damage and guarantee a safe operation of the hydraulic installations.

To achieve this goal, the phenomenon of water hammer is represented with state variables, considering the hydraulic installation as a mechanical plant, with various inputs and outputs; considering in a general way all the coefficients and parameters that intervene in this phenomenon, for which, the mathematical model of the transient overpressure of the water hammer is represented in the space of states. The matrix model is solved by computational simulation and the results are compared with those obtained using empirical and conventional methods to determine the advantages and disadvantages of using this method of analysis and synthesis with a mechatronic approach.

The mechatronic approach allows to represent the dynamics of thermal, thermodynamic, hydromechanical, pneumatic and other systems, as if they were mechanical plants of the mass-spring-damper type, which helps in a very effective way for the analysis and synthesis of the dynamics of the systems; and, it also allows to include all the parameters, variables, and coefficients in a clear way.

2. Material and Methods

In the bibliographic review, it has not been possible to find the use of the mechatronic approach for the analysis and synthesis of the dynamic behavior of the transient pressure of the water hammer in hydraulic installations, much less its use to define a high-precision control of the valves. hydraulic systems that allow this phenomenon to be kept within limits that prevent the destruction of the hydraulic systems and their correct operation. This research, based on previous work, wants to precisely demonstrate the great advantages of using this method, compared to conventional methods and, therefore, can have a great impact on the improvement and reliability of hydraulic installation designs, which they are largely infrastructure works of great strategic importance for the countries.

The method must be consistent and repeatable for different fluids, different materials of pipelines, different dimensions, and forms of installations, and even, it must consider disturbances such as: temperature variations, component wear, etc. Therefore, the mathematical model in the state space must be general, considering the different characteristics, and properties of the hydromechanical installations.

2.1. Mathematical Modelling

Conventional methods for water hammer control use the Joukowski, Allievi and Michaud equations [10], which are also the basis of international design standards and engineering calculations for hydromechanical systems (AWWA, ASME and API). These equations are used worldwide, including in specialized software packages for the design of hydraulic installations, they are presented below.

The Joukowski equation, in Eq. (1), allows calculating the celerity (speed of the transient) (a) in m/s, depending on: density of the fluid (ρ) in kg/m^3), modulus of compressibility (B) of the fluid in Pa, modulus of elasticity de Young (E) for for the pipe material in Pa, thickness of the pipe wall (e) in m and the diameter of the hydraulic pipe (d) in m:

$$a = \sqrt{\frac{\frac{B}{\rho}}{1+B*\frac{d}{E*e}}} \tag{1}$$

Allievi's equation, in Eq. (2), allows calculating the critical valve closing time for water hammer (tc) in s as a function of the length of the pipe (L) in m and the celerity:

$$tc = \frac{2*L}{a} \tag{2}$$

From the Allievi equation it can also be determined in Eq. (3), the value of the critical transitory overpressure (Ptop) using the nominal flow speed (v) in m/s, the density, and the speed:

$$Ptop = \rho * a * v \tag{3}$$

With the Michaud equation, in Eq. (4), the operating time (to) can be calculated based on the length of the pipe, flow speed, gravity in m/s^2 and piezometric height (ΔH) in m:

$$to = \frac{2*L*v}{0.4*\Delta H*g} \tag{4}$$

To represent the mathematical model of water hammer transients in state space, it is necessary to determine the inputs, outputs, and disturbances of the hydro-agricultural installation, which is illustrated in Figure 1, in which it is presented as the state vector includes the outputs of this target process ΔP and ΔQ and the input vector includes only ΔV.

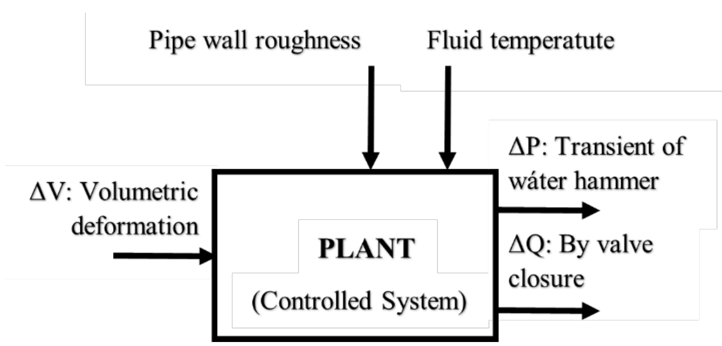

Fig. 1: Inputs, outputs, and disturbances of the hydromechanical installation.

The matrix equation in state space is represented as follows:

$$\begin{bmatrix} \dot{\Delta P} \\ \dot{\Delta Q} \end{bmatrix} = [A] * \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} + [B] * [\Delta V]$$

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = [C] * \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} + [D] * [\Delta V]$$

Where A is the coefficient matrix of the plant outputs, B is the vector of coefficients of the output of s results of the target process, C is the scale matrix, D is the stereo matrix, and y1, y2 are the mirror variables.

Applying the mechatronic approach, a hydraulic installation is represented as a mechanical target process that has a mass, two springs, and viscous damping, as it is presented in the following Figure 2.

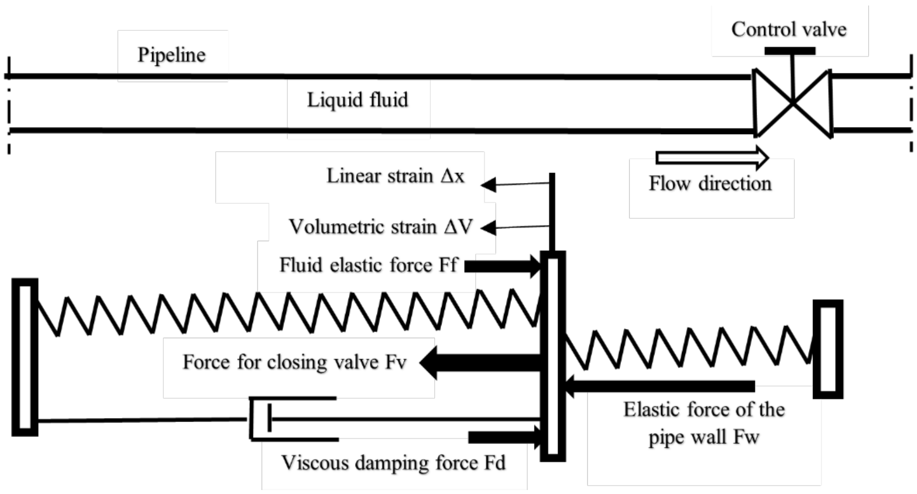

Fig. 2: Representation of a hydraulic installation as a mechanical target process.

Developing the free body diagram and using the equation of Newton's second law, Eq. (5):

$$m * \ddot{\Delta x} = Fv - Ff + Fw - Fd \tag{5}$$

Where, m is the fluid mass in kg, Δx is the fluid deformation in m, Fv is the valve closing force in N, Ff is the fluid elastic force in N, Fw is the wall elastic force of the pipe in N and Fd is the viscous damping force in N.

The term of the inertia of the fluid can be represented as follows, in Eq. (6):

$$m * \ddot{\Delta x} = \rho * L * \dot{\Delta Q} \tag{6}$$

Where, ΔQ is the flow rate variation due to valve operation in m^3/s.

The force caused by the closing of the valve Fv is replaced, in Eq. (7), by:

$$Fv = \Delta P * S \tag{7}$$

Where, ΔP is the transient water hammer pressure in Pa and S is the crossflow area in m^2.

The elastic reaction force of the fluid Ff due to the deformation of the fluid is calculated as follows, in Eq. (8), [11]:

$$Ff = \frac{B}{L} * \Delta V \tag{8}$$

Where ΔV is the volumetric strain of the fluid in m^3.

The elastic force of the pipe wall is determined using the Laplace-Young equation, in Eq. (9), [12]:

$$Fw = \frac{4*E*e}{d*L} * \Delta V \tag{9}$$

The damping force due to the viscosity of the fluid is calculated with the Poiseuille equation, in Eq. (10), [13]:

$$Fd = 40.74 * \frac{\mu * S * L}{d^4} * \Delta Q \tag{10}$$

Where, μ is the dynamic viscosity of the fluid in Pa.s.

Deriving Eq. (8) and replacing Eqs. (6) - (7) - (9) - (10) in Eq. (5) allow to obtain a system of two linear ordinary differential equations that are presented in Eqs. (11) - (12) below:

$$\dot{\Delta P} = -\frac{B}{S * L} * \Delta Q \tag{11}$$

$$\dot{\Delta Q} = \frac{S}{\rho * L} * \Delta P - \frac{B}{\rho * L^2} * \Delta V + \frac{4 * E * e}{\rho * d * L^2} * \Delta V - 40.74 * \frac{\mu * S}{\rho * d^4} * \Delta Q \tag{12}$$

Then, the mathematical model in the state-space representation of the transient caused by water hammer phenomenon is as follows:

$$\begin{bmatrix} \dot{\Delta P} \\ \dot{\Delta Q} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{B}{S * L} \\ \frac{S}{\rho * L} & -40.74 * \frac{\mu * S}{\rho * d^4} \end{bmatrix} * \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{B}{\rho * L^2} + \frac{4 * E * e}{\rho * d * L^2} \end{bmatrix} * [\Delta V]$$

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} * [\Delta V]$$

3. Computational Simulation and Results

To develop the computational simulation, it is necessary to evaluate the coefficients and parameters of a specific hydraulic installation, for this case the real data of a pressure pipe with the protection valve of a hydroelectric plant located in a mountainous area of Ecuador will be used, these values are presented in Table 1.

Table 1: Coefficients and constants of the mathematical model.

Symbol	Coefficient - Constant	Value	Units
Po	Rated hydraulic pressure	$1.48 \times 10^{+6}$	Pa
Qo	Nominal flow	0.6	m^3/s
d	Hydraulic pipe diameter	0.5	m
L	Pipe length	100	m
ρ	Water density	1000	Kg/m^3
g	Gravity	9.81	m/s^2
μ	Dynamic viscosity of water	0.001	Pa.s
B	Bulk compressibility modulus of water	$2.2 \times 10^{+9}$	Pa
E	Young's modulus of elasticity for pipe wall material (steel)	$2.05 \times 10^{+11}$	Pa
e	Thickness of conduit wall	0.007	m

With these values the model is simulated iteratively until the water hammer transient remains below the ramp formed by the flow gradient by valve closure ΔQwh with valve operation time, as illustrated in Figure 3.

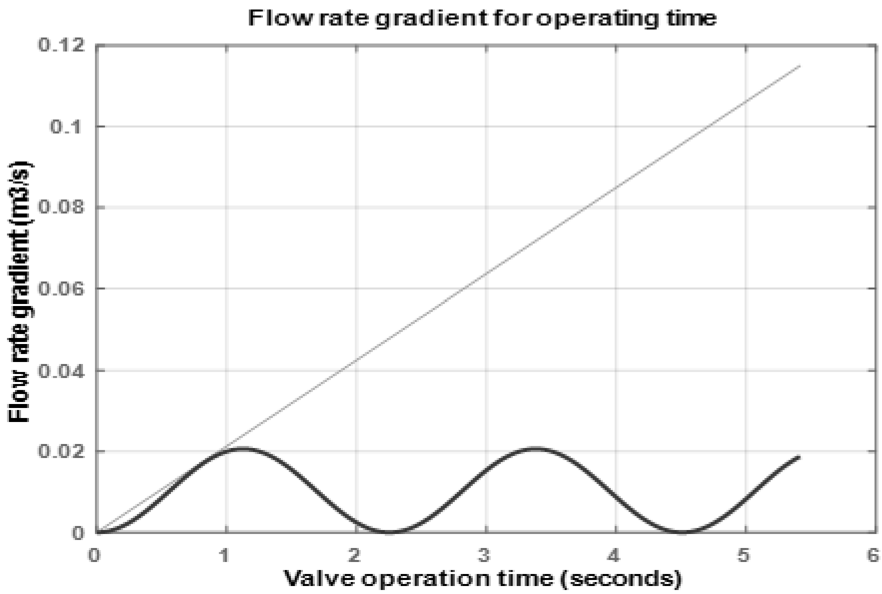

Fig. 3: Flow rate gradient for operating time.

The numerical results of the conventional equations for water hammer control and the computational simulation are presented in Table 2.

Table 2. Numerical results of computational simulation

Symbol	Results	Value	Units
a	Celerity (speed of the transient wave)	1,116	m/s
tc	Critical valve closing time	0.953	s
Ptop	Critical transient overpressure of the full water hammer	$3.41 \times 10^{+6}$	Pa
to	Operating time	5.415	s
ΔQwh	Flow gradient by valve closure	0.1150	m^3/s
tt	Total valve closing time (valve closure law)	28.255	s

The mathematical model of transient water hammer, using the mechatronic approach, considers the properties, coefficients, and constants of both the fluid and the pipe, so it is a general model that can be used for fluids with different properties, for pipes and fittings with very different materials, diameters, lengths, wall thicknesses, and for valves of different characteristics. Therefore, the simulation performed is for a specific case, but it is very easy to change the parameters and coefficients to simulate a wide variety of different hydraulic installations.

After iteratively carrying out several numerical simulations, in which the value of the flow gradient has been varied sequentially, it has been possible to determine the limit value such that the water hammer wave is less than the ramp formed between the flow gradient and the maneuver time, this limit is the maximum allowed value of the flow gradient that allows controlling the transient water hammer overpressure.

In accordance with International Standards, the components of a hydraulic installation are designed to resist up to a maximum value of overpressure (ΔPwh) caused by transient water hammer of 40% of the nominal pressure in the system. This value will be used as the control setpoint, in this case it has a value of ΔPwh= $5.92 \times 10^{+5}$ Pa.

A hydraulic installation can be controlled with servo-valves that hydraulically feedback the pressure set point, this being a very slow process that could not control the transient due to water hammer, for which, solely with the aim of demonstrating that the control of the flow gradient in the maneuver time is also limited, a proportional-integral-derivative controller (PID controller) is used, as shown in Figure 4.

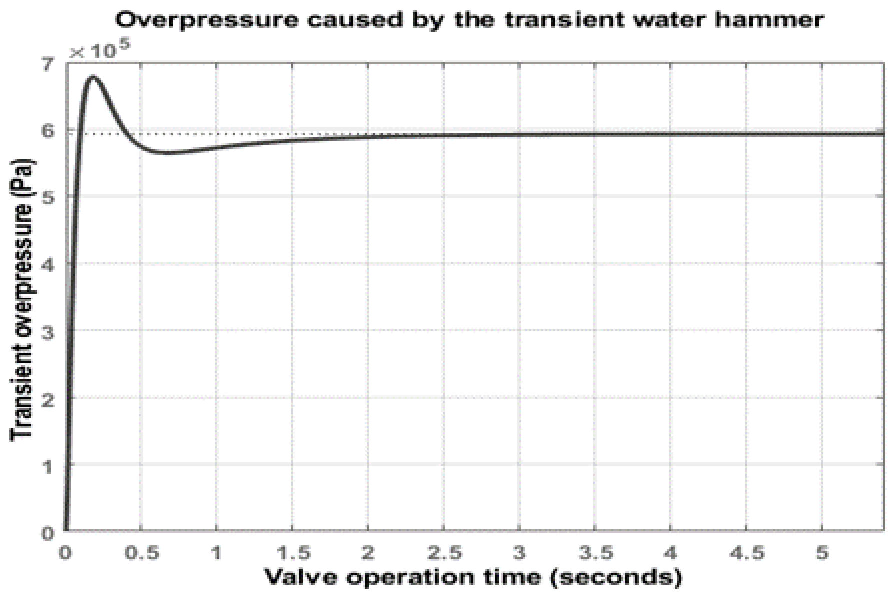

Fig. 4: Overpressure caused by the transient water hammer.

Figure 4 shows how, when a very fast control input is used, the water hammer overpressure also increases in a very short time, exceeding ΔPwh and causing a brief overshoot on this value, to then be controlled by the PID, which allows keep the overpressure within the required limits. The overshoot is due to the step type control input which, as indicated, is not relevant for this case because the water hammer control must be done in open loop with a ramp type control input. Therefore, in this simulation it is shown that it is possible to stabilize the ΔPwh within the operating time.

Currently, it is very advantageous to install bypasses in the system, with water hammer anticipation valves, which have a pilot that, upon detecting overpressure in the system, opens very quickly, relieving water hammer; but equally the closure of these valves must comply with the total valve closure time (valve closure law).

4. Conclusions

A mathematical model was developed using the mechatronic approach in which the water hammer phenomenon is represented with state variables, considering the hydromechanical system as an objective process to control, with multiple inputs and outputs. This mathematical model and the developed numerical simulation of transient water hammer is general, consistent, and repeatable, so it can be used for various hydraulic installations, for fluids with different properties, for pipes and accessories with very different materials, diameters, lengths, and wall thicknesses, and for valves of different types and characteristics.

Conventional equations do not consider very important fluid and pipe parameters and coefficients that significantly affect water hammer dynamics, which obviously can produce serious errors. With the model obtained with the mechatronic approach, there is a clear difference between the critical valve closure time (operating time) and the total valve closure time, while with classical methods these values can be confused with dangerous consequences.

It has been verified that the mechatronic approach allows to represent the dynamics of hydraulic systems, as if it were a mechanical plant of the mass-spring-damper type, allowing in a very effective way analysis and synthesis of the dynamics of the hydraulic system; and, it has also allowed to include all the parameters, variables, and coefficients in a clear and complete way.

Due to the above, it is concluded that the mechatronic method used in this investigation, for the control of the transient water hammer, effectively allows to maintain the overpressure of the transient water hammer within technically acceptable values and, therefore, to develop and install safe hydraulic systems with high operational reliability.

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