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Modelling and Analysis of Vortex-Induced Vibrations for Flexible Cylinders Conveying Two-Phase Slug Flows

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Abstract - Hydrocarbon flows in a marine riser may appear in multiple phases with varying flow patterns, among which, slug flows are known to exhibit complex flow characteristics due to fluctuations in multiphase mass, velocities, and pressure changes. This flow regime can therefore cause significant vibrations and cyclic stresses on the pipeline. The fatigue life of a marine riser can also be significantly reduced when subjected to external excitations due to vortex-induced vibrations (VIV). Bulk of the literatures concerning two-phase flow induced vibrations have largely focused on the use of a simplified linearized tensioned beam model. Fundamental understanding of the vibration phenomena in three-dimensional space and time considering geometric and hydrodynamic non-linearities is still lacking. In this study, a long flexible cylinder subjected to external excitations by vortex-induced vibrations (VIV) in combination with internal slug flow-induced vibrations (SIV) is investigated. A three-dimensional semi-empirical model is presented which comprises of nonlinear structural equations of coupled cross-flow, in-line and axial motions along with equations involving centrifugal force and Coriolis force capable of predicting vibrations due to hydrodynamic slug flow. The model describes variations in lift and drag coefficients using two coupled wake oscillator models and is capable of capturing mass variations in the slug-flow regime. A numerical space-time finite difference approach in conjunction with frequency domain and modal analysis is used for analysing the highly non-linear partial differential equations. The presented model is validated through comparisons with published experimental results pertaining to internal SIV and external VIV for horizontal and vertical pipes. Similar trends in modal participation and oscillation amplitudes were observed in the validation results, indicating that the model has quantitative similarities with published experimental results for both VIV and SIV.

***Keywords*:** VIV, Flexible cylinder, Slug-flow, 3-D response

1. Introduction

In the last few decades, with the abundant growth in energy demands and the depletion of hydrocarbon sources in shallow waters have led oil-industry pioneers to explore hydrocarbon reserves in deep-water oceans. One main component used during oil and gas production in deep-water oceans are marine risers, which are long flexible cylindrical structures providing a link between the subsea wells and the offshore floating unit. As risers are long flexible structures, they can undergo significant flow-induced vibrations (FIV) when exposed to unavoidable environmental loads such as currents, waves, and fluctuations in internal flow [1]. Vortex-induced vibrations (VIV) caused due to ocean currents are one of the major threats for structural integrity. Many researchers in the past through numerical and experimental methods have extensively studied VIV of rigid and flexible cylinders in various external flow conditions [2]. For example, when most studies neglected the effect of in-line oscillations due to low amplitudes, Trim et al. [3] showed there is a direct co-relation between in-line and cross-flow oscillation amplitudes and fatigue damages caused due to in-line oscillations were as important as cross-flow responses. Furthermore, Srinil [4] explored vibration responses of flexible curved/straight cylinders in the in-line and axial direction by taking into consideration the geometric and hydrodynamic non-linearities. Zanganeh & Srinil [5] developed a three-dimensional VIV model to demonstrate the significant effects of axial dynamics and mean drag magnifications for flexible circular cylinders. Also, the effect of boundary conditions on key VIV parameters were in detail investigated by Xu et al. [6] and Sayed-Aghazadeh et al. [7]. Along with numerical and experimental methods, computational fluid dynamics (CFD) techniques have also been used to predict and simulate VIV.

Researchers in the recent past have noticed that risers are capable of flow-induced oscillations due to internal single and multiphase flow. It is known that hydrocarbon flows in marine risers mostly occur in multiphase flow, through simultaneous flow of gas and liquid. This in turn can lead to complex hydrodynamic behavior because of various space-time varying multiphase flow variables. Of the various multiphase flow regimes, the slug flow regime may induce large vibrations and cyclic stresses along the pipeline due to chaotic fluctuations in mass distributions and pressure variations [1]. Therefore, it becomes equally important for the sake of structural integrity to analyze and study structural motions of marine risers due to simultaneous VIV and internal slug flows. Hara [8] showed two-phase flow-induced vibrations in flexible cylinders mainly occur due to mass periodic fluctuations, centrifugal force and Coriolis force exerted on the inner walls of the cylinder. Blanco & Casanova [9] developed a fluid structure model to examine the fatigue life of submarine pipelines exposed to internal slug flow-induced vibrations (SIV) and external uniform currents. The study proved that the estimated fatigue life can be different from those predicted when internal and external flows are considered separately. Knudsen et al. [10] performed a time-domain numerical analysis using linear structural model to investigate cross-flow responses of a truncated marine riser subjected to combined VIV and internal slug flows. The study showed that slug lengths and slug velocity contributed to the overall riser oscillation amplitudes and frequency. Safrendyo & Srinil [11] studied the effects of combined VIV-SIV for steady and random slug excitations. Multifrequency oscillations and increased response amplitudes were noticed during random slug excitation and the author advices to consider the combined effects of combined VIV-SIV during the design of marine risers conveying multiphase flow. Similarly, Meng et al. [12] developed a novel approach to detect the intermittent feature of internal slug flows for flexible risers. The study concludes by stating that internal two-phase slug flows effects on riser VIV responses are much larger than single-phase flows. Also, new natural modes and beating phenomenon were observed.

However, the bulk of literatures concerning flow-induced vibrations previously mentioned have mostly focused on the structural dynamics of a riser subjected to external hydrodynamic loads (VIV) with few accounting for internal two-phase flow induced vibrations (SIV). Also, literature concerning flow-induced vibrations have largely focused on the use of a simplified linearized tensioned beam model. Fundamental understanding of the SIV phenomena in three-dimensional space and time considering in-line and axial dynamics is still lacking. Therefore, the present study aims at overcoming such model limitations by considering structural and hydrodynamic non-linearities to investigate two-phase slug flow induced vibrations (SIV) of flexible cylinder in conjunction with vortex-induced vibrations (VIV).

2. VIV-SIV Phenomenological Model

Marine risers with high aspect ratios (length-diameter ratio, *L/D*) are usually employed for deep-water explorations which are susceptible to high amplitude oscillations due to vortex-shedding. Figure 1 shows a schematic 3 D model of a long flexible circular cylinder subjected to simultaneous external uniform flow and internal multiphase flow. The cylinder is assumed to be fully submerged with internal flows in the axial direction and external uniform flow in the in-line X direction. Different boundary conditions can be applied to the model, however pinned-pinned condition applies in this study as it is common for marine risers and other experimental studies. The flexible cylinder is assumed to be linearly elastic with constant Young's modulus (E) and have uniform structural properties such as mass (m), diameter (D), cross-sectional area (A_r), damping coefficient (C), bending stiffness (EI), axial stiffness (EA_r) and moment of Inertia (I). Also, due to mean drag force, the cylinder will displace itself from its original XY position and form a stable curved configuration. The cylinder is capable of further displacements in the X direction due to non-linear coupling terms in the equation. Flexible cylinders in horizontal and near horizontal positions are likely to bend a from a curved shape in the XZ direction due to the mass of internal flow and gravitational force. Also, for realistic predictions of VIV for offshore operations, it is crucial to take into consideration three-dimensional vibration response and geometric non-linearities [5]. In reality, two-phase flows may develop into different flow patterns in a long riser based on the pipe-fluid properties [17]. The flow patterns may be further modified during pipe oscillation, in this study the slug-flow regime is assumed to be undisturbed by pipe oscillations. The slug flow is modelled as a fluid with time-varying mass in the form of a rectangular pulse train using Matlab code and functions [18].

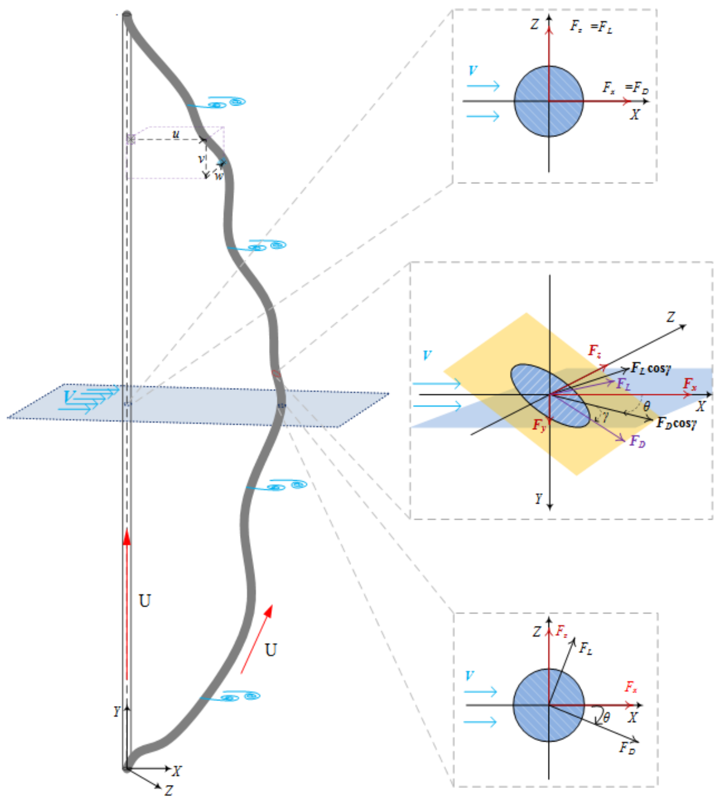


Figure1. Schematic 3-D model of a flexible cylinder undergoing simultaneous VIV and SIV.

2.1 Non-linear riser structural model

Dimensionless terms such as $U = u/D$, $V = v/D$, $W = w/D$, $Y = y/L$, $T = \omega_n t$, are introduced to the developed equations for better understanding during parametric studies. The 3-D dimensionless non-linear partial differential equations of coupled cross-flow (CF), in-line (IL), and axial (AX) motions combined with the equation of motion for two-phase internal flow for a long flexible cylinder can be expressed as [4,5]:

$$(1+M)\ddot{u} + 2\zeta\dot{u} + E_I u^{(IV)} + 2U_r M\dot{u}' + U_r^2 M u'' + (\dot{m}\dot{u} + U_r\dot{m}u' + U_r m'\dot{u} + U_r^2 m'u') - (T_n u')' = \frac{E_A}{L_D}(v''u' + v'u'') + \frac{1}{2}\frac{E_A}{L_D}\left(3u''u'^2 + u''v'^2 + \frac{2}{L_D}v''v'u' + u''w'^2 + \frac{2}{L_D}w''w'u'\right) + F_x \tag{1}$$

$$(1+M)\ddot{v} + 2\zeta\dot{v} + E_I v^{(IV)} + 2U_r M\dot{v}' + U_r^2 M v'' + (\dot{m}\dot{v} + U_r\dot{m}v' + U_r m'\dot{v} + U_r^2 m'w') - (T_n v')' = E_A v'' + 2\frac{E_A}{L_D}v''v' + \frac{E_A}{L_D}(u''u' + v''v' + w''w') + \frac{1}{2}\frac{E_A}{L_D}\left(v''u'^2 + \frac{2}{L_D}u''u'v' + 3v''v'^2 + v''w'^2 + \frac{2}{L_D}w''w'v'\right) + F_y \tag{1}$$

$$(1+M)\ddot{w} + 2\zeta\dot{w} + E_I w^{(IV)} + 2U_r M\dot{w}' + U_r^2 M w'' + (\dot{m}\dot{w} + U_r\dot{m}w' + U_r m'\dot{w} + U_r^2 m'w') - (T_n w')' = \frac{E_A}{L_D}(v''w' + v'w'') + \frac{1}{2}\frac{E_A}{L_D}\left(w''v'^2 + w''v'^2 + \frac{2}{L_D}u''u'w' + \frac{2}{L_D}v''v'w' + 3w''w'^2\right) + F_z \tag{3}$$

where u, v, and w represent structural displacements in in-line (X), axial (Y) and cross-flow (Z) directions and F_x, F_y, and F_z are the corresponding hydrodynamic forces. The overdot and prime in the equations denote derivatives with respect to time t and space Y, m is the structure mass per unit length, E is the modulus of elasticity, A_r is the cross-sectional area, I is the moment of inertia and T represent tension in space. In non-dimensional form the structure mass (M) is defined as the ratio of ratio of fluid mass (m_f) and the total structural mass of submerged cylinder (m_o). ζ is the damping ratio and is described as $\frac{1}{m_0\omega_n}c = 2\zeta$. Bending stiffness of the cylinder E_I in dimensionless form is defined as $E_I = EI\frac{1}{m_0\omega_n^2L^4}$. Likewise, axial stiffness is expressed as the normalized product of modulus of elasticity and cross-sectional area $E_A = \frac{EAr}{m_0\omega_n^2L^2}$. The internal flow velocity U_r is expressed in dimensional form as $U_r = \frac{U}{\omega_n L}$. For risers with high inclination angles, variation of tension along the length of the riser is expressed in dimensionless form as:

$$T_n = T_{n0} - G_n(1+M)(1+Y)sin\theta + G_n\frac{\pi}{4\mu}(1+C_m)(1-Y)sin\theta \tag{4}$$

Where $T_{n0} = \frac{T}{m_0\omega_n^2L^2}$ and C_m is the added mass coefficient. Non-dimensional tension equation gives rise to a unique parameter G_n, $G_n = \frac{g}{wn^2 * L}$, where g is the gravitational force. The overall structure geometric non-linearities, mean displacements and 3-D vibrations are accounted for through quadratic and cubic coupling terms in the equation (1-3). The semi-empirical model also takes into account the centrifugal force and the Coriolis force along with inertial and momentum change effects to emulate vibrations due to hydrodynamic slug flows which have been found to be key in emulating two-phase flow-induced vibrations [8].

2.2 Non-linear hydrodynamic forces

Vortices are shed when external current flows past a circular cylinder causing oscillatory drag and lift forces with frequencies $2\Omega_f$ and Ω_f respectively. The respective drag and lift forces cause the cylinder to oscillate in the IL and CF directions and are denoted as F_x and F_z. The oscillations in the IL and CF directions induce oscillations in the axial direction (Eq. 2) due to non-linear coupling terms in the equation. The fluctuating and self-exciting drag and lift forces can be modeled using the van der Pol wake oscillator equation by introducing wake variables $p = 2C_D/Cd_o$ and $q = 2C_L/Cl_o$, where Cd_o and Cl_o are drag and lift coefficients of a stationary cylinder [13]. The wake oscillator in dimensionless form can be expressed as:

$$\frac{d^2p}{dT^2} + 2\varepsilon_u\Omega(p^2-1)\left(\frac{dp}{dT}\right) + 4\Omega^2 p = A_u\left(\frac{d^2U}{dT^2}\right) \tag{5}$$

$$\frac{d^2q}{dT^2} + \varepsilon_w\Omega(q^2-1)\left(\frac{dq}{dT}\right) + \Omega_f^2 q = \Lambda_w\left(\frac{d^2W}{dT^2}\right) \tag{6}$$

Where $\Omega_f = 2\pi stV/D$, St is the Strouhal number assumed to be 0.18, ε_u, ε_w, A_u, A_w are empirical coefficients calibrated with previous experimental literature [14]. During cylinder oscillations, the drag and lift forces become random and no longer correspond to the X and Z direction due to the cylinder relative motion. By following Zanganeh & Srinil [5] and applying non-dimensional terms as mentioned in the previous section, the total hydrodynamic force equation can be expressed as:

$$F_x = \frac{1}{4\mu}V'_{rel}C_{l0}q\dot{W} + \frac{1}{4\mu}V'_{rel}C_{d0}p\left(\frac{V_r}{2\pi} - \dot{U}\right) + \frac{1}{2\mu}V'_{rel}\bar{C}_d\left(\frac{V_r}{2\pi} - \dot{U}\right) \tag{7}$$

$$F_y = \frac{1}{4\mu} V'_{rel} C_{l0} q \dot{V} + \frac{1}{4\mu} V'_{rel} C_{d0} p \dot{V} + \frac{1}{2\mu} V'_{rel} \bar{C}_d \dot{V} \tag{8}$$

$$F_z = \frac{1}{4\mu} V'_{rel} C_{l0} q \left(\frac{V_r}{2\pi} - \dot{U} \right) - \frac{1}{4\mu} V'_{rel} C_{d0} p \dot{W} - \frac{1}{2\mu} V'_{rel} \bar{C}_d \dot{W} \tag{9}$$

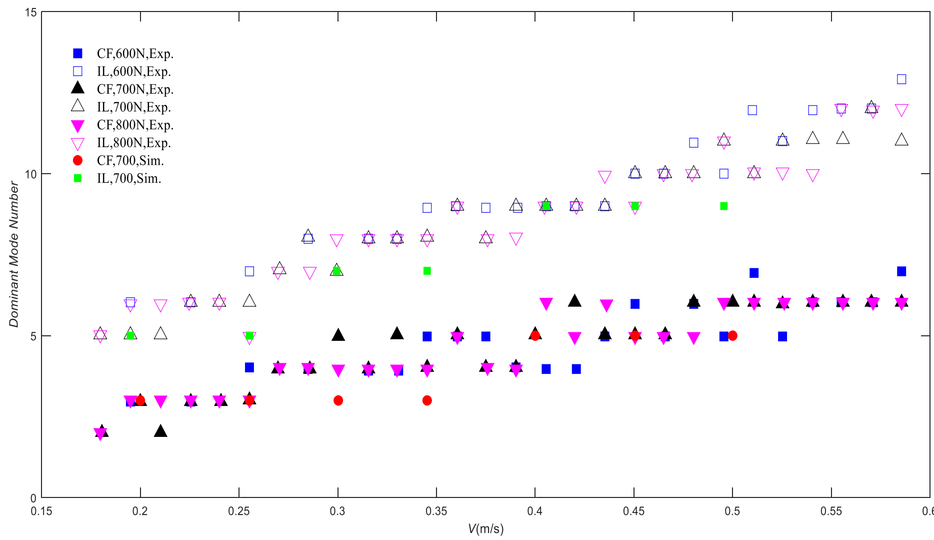
Where, $\bar{C}_d$ is the mean drag coefficient and V'_{rel} is the relative velocity of the oscillating cylinder expressed as:

$$V'_{rel} = \sqrt{\left(\frac{V_r}{2\pi} - \dot{U} \right)^2 + \left(\dot{V} \right)^2 + \left(\dot{W} \right)^2} \tag{10}$$

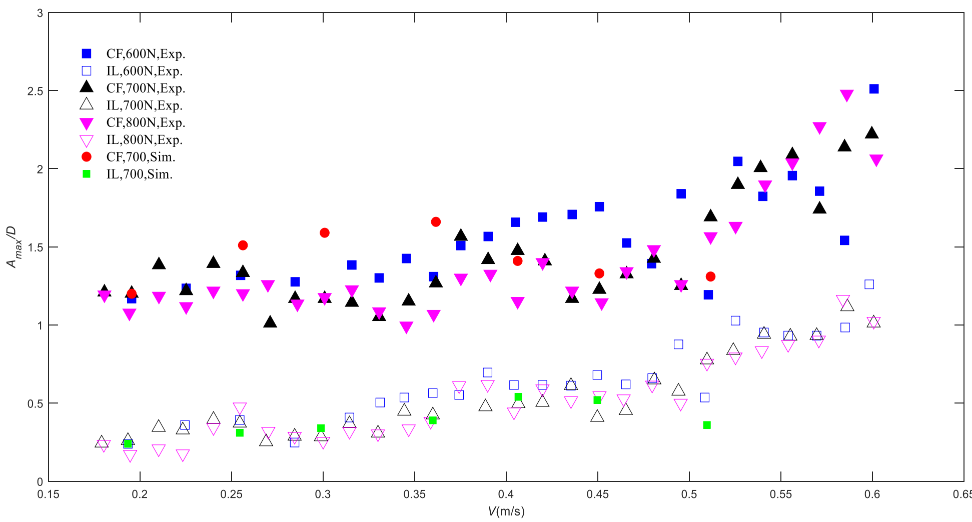
The corresponding values for $C_{d0}, C_{l0}, \bar{C}_d$, ε_u, ε_w, A_u, A_w are based on calibration results from past literatures and are 0.2,0.3 1.2, 0.3,0 0.3, 12 and 12 respectively [5,15]. The highly non-linear partial differential equations are numerically solved through central-finite difference scheme using Matlab codes in both time and space domain.

3. Results and Discussion

As mentioned previously, experimental results available for flexible cylinders subjected to the combined effects of VIV and internal two-phase flows are very limited, therefore, the developed model is validated with experimental results for cylinders subjected to VIV and SIV separately. For VIV validation, numerical and experimental results are compared by employing laboratory tests studied by Song et al. [14], where a long flexible cylinder with properties E=201*GPa*, L=28.04m, OD=0.016m, ID=0.015m, density=7930kg/m^3 was considered with pre-tension T=600N,700N,800N and subjected to uniform external flow velocities *V,* ranging from 0.18 to 0.6 m/s. The cylinder inclination is assigned to be 0 degrees, which indicates it to be horizontal similar to the experiment tests. Internal gas-liquid flow is neglected in this case. The equations are numerically solved in Matlab with a time step of $\Delta t = 0.001$s and the cylinder is discretized spatially as $\Delta y = 10$. This discretization in time and space allows for convergence of steady-state results. Figure 2(a) illustrates the comparison between numerical and experimental results of obtained dominant modes for external flow velocity ranging from 0.2 – 0.6 m/s and pretension 700 N. The figure shows that the numerical simulation follows similar trends of modal participation showing increasing dominant mode numbers with increase in external flow velocity following Strouhal rule. It can also be seen from the figure that at current velocities 0.2 m/s, 0.25 m/s, 0.4 m/s and 0.45 m/s, simulation results exactly coincide with experimental results by displaying exact dominant mode numbers while other velocities show reasonable similarities in dominant mode numbers. To further validate the model the maximum cross-flow and in-line oscillation amplitudes are compared with experimental results. Figure 2(b) illustrates the varying cross-flow and in-line amplitudes for external flow velocities 0.2-0.6 m/s. Similar to modal comparisons, the figure shows a similar trend of increasing amplitudes with increasing current velocities following a zig-zag progression. Such variations are also highlighted by Zanganeh & Srinil [5]. Overall, it can be concluded from the figures that the predicted VIV results provide quantitative similarities to previous experimental results.



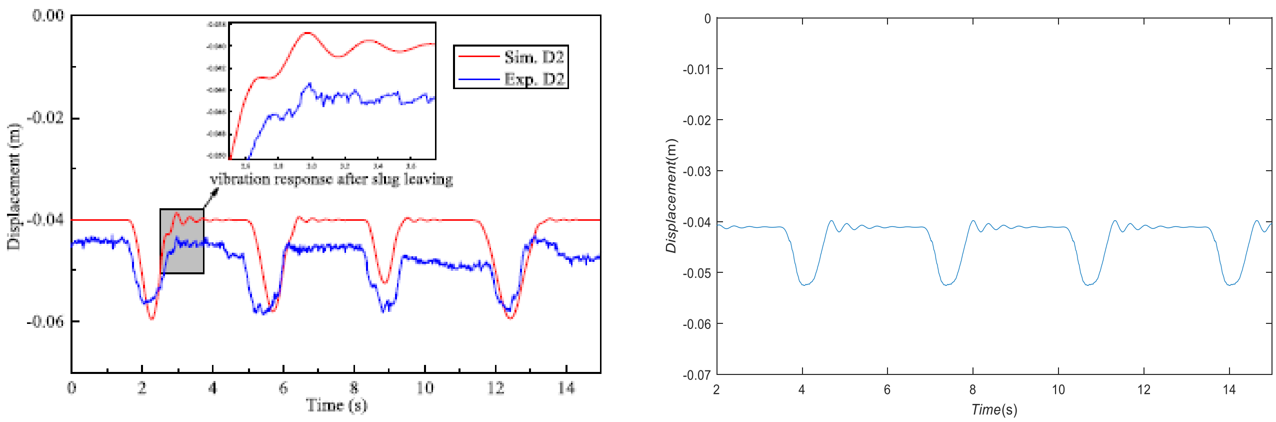
(a)



(b)

Figure 2. Comparison between numerical and experimental results (a) dominant modes (b) cross-flow and in-line amplitudes.

SIV predictions of the model has also been validated through comparisons from previous experimental tests conducted by Wang et al. [16] for long flexible cylinders conveying two-phase slug flows. The cylinder is surrounded by air with properties OD=0.063m, L=3.81m, Elastic modulus=750MPa, density=926kg/m^3 with internal liquid density=1000kg/m^3 and viscosity=0.001Pas. An idealized slug flow model is created using Matlab functions where a single slug unit contains liquid slugs and elongated gas bubbles over a thin film of liquid. The developed slug model is capable of describing different slug properties such as slug length, L_s, gas bubble length, L_f, liquid holdup, HL_s, and translational velocity, V_t. The slug flow model follows a sine wave with flat peaks, where the area under the flat peaks indicate liquid slug length L_s, and the remaining describe gas bubble length, L_f. For validation the values of L_s, L_f, and V_t were obtained from the literature as 1.19m, 10.53m and 3.5 m/s respectively. Figure 3 compares the time history graphs for amplitude responses of numerical simulation and experiment results at the midpoint of the cylinder. Similar trends can be observed in the graph obtained from numerical simulation (3b) and the amplitude of displacement obtained as 0.01 m is the same with experiment results. However, discrepancies were found for the mean deflection value due to the lack of information of HL_s and HL_f values in the literature and therefore a value of 0.03 has been subtracted from the total amplitude response for visual comparison.



(a) (b)

Figure 3. Comparison between time history graphs of numerical and experimental results for SIV validation: (a) Experiment [16] (b) Numerical simulation.

4. Conclusion

In this study a three-dimensional phenomenological model has been developed for long flexible cylinders subjected to external vortex-induced vibrations (VIV) and internal slug-flow induced vibrations (SIV). The model describes variations in lift and drag coefficients using two coupled wake oscillator models and is capable of capturing mass variations in the slug-flow regime. A space-time numerical finite-difference approach has been employed to solve the highly non-linear three-dimensional VIV-SIV response. Results from VIV model validation show that model follows similar trends of increasing modal participation and oscillation amplitudes with increase in external flow velocity. Similarly, SIV validation results demonstrate similar trends of oscillation amplitudes showing the developed model has quantitative similarities with published experimental results for both VIV and SIV. The presented three-dimensional model with less simplifications considering geometric and hydrodynamic non-linearities will be beneficial for designers for practical VIV-SIV vibration prediction in offshore risers and mooring cables during the early stages of the design process.

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