Model-Based Determination of Process Force in Multi-Axis Milling

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Abstract - Milling is a machining process in which productivity is highly dependent on the process force between the milling tool and the workpiece. Therefore, force control strategies are introduced to reduce the production time while ensuring a desired process force. In order to produce more complex geometries, the machine tool is provided with additional rotational degrees of freedom of the machine table or the milling tool, which is referred to as multi-axis milling. In a machine tool with a rotating machine table, inertial and gravitational forces disturb the measurement of the process force provided by the table dynamometer.

This contribution introduces a model-based determination of the process force. Gravitational and inertial forces are calculated and subtracted from the measurement. Furthermore, the measured process force is transformed to align the measured process force with the process force determined by numerical engagement simulation. Experiments with and without a milling operation are conducted to validate the model quality. In the conducted experiments, the process force determined in a milling operation with a rotated machine table has a relative error of less than 13.3 % in x- and y-direction compared to a milling operation without rotation and same engagement conditions. In contrast, the process force in z-direction has a relative error of up to 38.3 % due to unmodelled disturbances. For a force control strategy, mainly the determination of the process force in the x- and y-direction is relevant. Therefore, the model is sufficiently accurate in order to be tested in a milling process with a force control strategy.

Keywords: multi-axis milling, table dynamometer, gravity compensation, inertia compensation, force control

1. Introduction

Milling is a machining process characterized by high precision and flexibility in the geometries produced. A major objective in milling is to increase productivity while maintaining the required tolerances of the workpiece and preventing tool failure. One approach to increase the productivity is to introduce force control strategies. By controlling the process force, productivity can be increased while preventing excessive tool wear and hereby early tool failure. [1].

In order to measure the process force, a table dynamometer is used in many force control strategies. The dynamometer is mounted on the machine table and the workpiece itself is mounted on the dynamometer. Frequent changes in the engagement between the milling tool and the workpiece characterize milling operations affecting the dynamic behavior of the process force. Therefore, control strategies need to be able to adapt to these changing conditions in order to ensure high control performance [1].

A popular approach to achieve a desired process force is the use of adaptive control. The control parameters are continuously updated in order to minimize the control error. Adaptive controller achieve high control performance for milling operations with constant engagement. However, these approaches cannot avoid an overshoot of the process force after a rapid change in engagement conditions due to the delayed adaptation of the controller to the new conditions. [2–7]

In contrast, a novel approach is the use of model predictive control to ensure a desired process force. The engagement conditions can be determined beforehand using numerical engagement simulation. Therefore, model predictive controller adapt the feed rate before a change in the engagement conditions occurs in order to avoid overshoot of the process force. [8].

In the previously mentioned studies, force control strategies for 3-axis milling are considered. Since the machine table is aligned horizontal during milling, small disturbances act on the measurement. Therefore, the measured force maps the process force accurately. The production of more complex geometries requires additional rotational degrees of freedom (DOF) of the machine tool, which is referred to as multi-axis milling. This contribution focuses on a multi-axis milling process using three translational DOF to move the milling tool and two rotational DOF to move the machine table. Rotating the machine table while the milling tool is engaged is referred to as 5-axis milling. When applying the alignment of the machine table between the milling tool engagements, the milling process is referred to as 3+2-axis milling.

By rotating the machine table and thus the attached table dynamometer, the force measurement is disturbed by gravitational and inertial forces. A first study suggests to rotate the gravitational force in the direction of the dynamometer and lump all the inertia effects in a single disturbance torque in order to determine the process force [9]. The inertial forces are based on the velocity and acceleration of the machine table. Therefore, it is inaccurate to lump inertial forces in a moment. A second study specifies the rotation's influence as a combination of linear, centrifugal, Euler and Coriolis forces [10]. In both approaches the process force remains oriented in the direction of the measurement coordinate system (MCS). However, to apply model predictive control to multi-axis milling, the measured process force needs to be rotated into the direction of the machine tool coordinate system (MTCS). This allows the prediction of the process force by the engagement simulation. Besides unmodelled process behavior, the presented models have not been validated in a milling operation [9, 10].

The paper is organized as follows. In section 2, the force measurement model in multi-axis milling using a table dynamometer is introduced. The experimental setup for identification and validation is described in section 3. In section 4, the effective mass of the table dynamometer influencing the measurement is identified. The model's quality to describe the gravitational force, inertial forces and the rotation of the process force is evaluated in section 5.

2. Force measurement model

In order to determine the process force in a milling operation, the measured forces and the machine table's motion can be acquired in 3D space. Due to the rotation of the machine table the force measurement is disturbed by gravitational and inertial forces. The setup for measuring the process force in multi-axis milling is shown in Fig. 1. The machine table is rotated resulting in a different orientation of the MCS than the MTCS.

A model is developed in order to determine the process force based on the disturbed force measurement and the table dynamometers motion. In [11] a rotation model is developed which describes the orientation of the machine table based on the machine table's rotation angles. Hence, in this contribution the rotation model is used to transform forces from the MTCS in the MCS as illustrated in Fig. 1. The rotation is described by the rotation ϑ_x in x-direction and ϑ_z in z-direction. The dynamometer measures the force $F_M \in \mathbb{R}^3$ in the MCS. In order to determine the process force $F_P \in \mathbb{R}^3$ in the MTCS, the forces are rotated using the rotation matrix ${}^{\mathrm{m}}\mathbf{T}_{\mathrm{mt}} \in \mathbb{R}^{3\times 3}$. The denotation ${}^{\mathrm{m}}\mathbf{T}_{\mathrm{mt}}$ is used to transform the force ${}^{\mathrm{m}}\mathbf{F}$ orientated in direction of the MTCS. The rotation matrix

$${}^{\mathrm{m}}\mathbf{T}_{\mathrm{mt}}(\vartheta_{\mathrm{x}},\vartheta_{\mathrm{z}}) = \mathbf{T}_{z}(\vartheta_{\mathrm{z}}) \,\mathbf{T}_{x}(\vartheta_{\mathrm{x}}) \tag{1}$$

is defined with

$$\mathbf{T}_{z}(\vartheta_{z}) = \begin{pmatrix} \cos(\vartheta_{z}) & -\sin(\vartheta_{z}) & 0\\ \sin(\vartheta_{z}) & \cos(\vartheta_{z}) & 0\\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathbf{T}_{x}(\vartheta_{x}) = \begin{pmatrix} 1 & 0 & 0\\ 0 & \cos(\vartheta_{x}) & -\sin(\vartheta_{x})\\ 0 & \sin(\vartheta_{x}) & \cos(\vartheta_{x}) \end{pmatrix}.$$
(2)

The measured force ${}^{\mathrm{m}}F_{\mathrm{M}}$ reads

$${}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{M}} = {}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{P}} + {}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{G}} + {}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{CF}} + {}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{E}},\tag{3}$$

considering the process force ${}^{m}F_{P}$, the gravitational force ${}^{m}F_{G} \in \mathbb{R}^{3}$, the centrifugal force ${}^{m}F_{CF} \in \mathbb{R}^{3}$ and the Euler force ${}^{m}F_{E} \in \mathbb{R}^{3}$. Since the machine table has only rotatory DOF, inertial force components caused by linear movement of the machine table are not considered. Apart from the process force ${}^{m}F_{P}$, the other forces acting on the measured force ${}^{m}F_{M}$ depend on a resulting mass *m* influencing the measurement. The table dynamometer influences its measurement with a part of its weight as illustrated by the red frame in Fig. 1. In addition, the workpiece and the fixing aid influence the measurement. Therefore, the resulting mass *m* is given as

$$m = m_{\rm td} + m_{\rm fix} + m_{\rm wp},\tag{4}$$

with the table dynamometers effective mass m_{td} , the mass m_{fix} of the fixing aid and the mass m_{wp} of the workpiece. The mass m_{wp} of the workpiece decreases during a milling operation due to the removal of material. In [12], a material removal model is introduced to describe the material removal based on geometric information. The gravitational force is expressed using the acceleration of gravity g and the measured mass m. Before milling, the workpiece is mounted on the machine table and the dynamometer is tared. Thus, the initial resulting mass m_0 influences the gravitational force ${}^{m}F_{G}$ according to

$${}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{G}} = {}^{\mathrm{m}}\mathbf{T}_{\mathrm{mt}} {}^{\mathrm{mt}}\boldsymbol{F}_{\mathrm{G}} - \begin{pmatrix} 0\\0\\m_{0} g \end{pmatrix} \qquad \text{with } {}^{\mathrm{mt}}\boldsymbol{F}_{\mathrm{G}} = \begin{pmatrix} 0\\0\\m g \end{pmatrix}.$$
(5)

The inertial force consists of the centrifugal force ${}^{m}F_{CF}$ and the Euler force ${}^{m}F_{E}$. The centrifugal force depends on the angular velocity $\omega \in \mathbb{R}^{3}$ of the machine table and the distance between the center of mass and the origin of the rotation $r \in \mathbb{R}^{3}$. Respectively, the Euler force depends on the angular acceleration $\dot{\omega} \in \mathbb{R}^{3}$ and the distance r. Using the denotation × describing the cross product of two vectors, the two inertial force reads

$${}^{m}\boldsymbol{F}_{CF} = m\left(\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})\right), \tag{6}$$
$${}^{m}\boldsymbol{F}_{E} = m\left(\boldsymbol{\omega} \times \boldsymbol{r}\right). \tag{7}$$



Fig. 1: Setup of machine table, table dynamometer, fixing aid and workpiece in multi-axis milling

3. Experimental setup

For the experimental validation a 5-axis simultaneous machining center of the type Mazak VARIAXIS i-600 is used. The machine tool is equipped with a Siemens SINUMERIK 840 D SL numerical control unit. The internal machine tool signals are acquired via the Siemens axis data stream (ADAS) in real time, using the sample rate of the interpolation cycle of the position controller (500 Hz). For the evaluation of the measurement model, the position, velocity and acceleration of all axes are acquired. The force measurement is realized using a force dynamometer of type Kistler 9255B at a sample rate of 5000 Hz. In order to reduce the measurement noise, a low pass filter at a cut-off frequency of 2000 Hz is applied.

For the milling operations, a two fluted solid carbide end mill by Seco Tools GmbH, Erkrath, Germany, of the type JSE512100D2C.0Z2 SIRA is used. The workpiece material is steel 1.7225 (42CrMo4). In order to cover different engagement conditions and engagement changes, a geometry is chosen, which consists of different width and depth of cut as well as abrupt and continuous changes in the geometrical engagement [13]. The dynamometer is tared before rotating the machine table. The machined geometry and therefore the engagement conditions are kept equal in both rotated and not

rotated machine tool table setup. Hence, a comparison between of the process force in a 3-axis milling and 3+2-axis milling process is possible. Fig. 2 shows the machined geometry for both scenarios. Effects caused by the tool wear are neglected due to the small amount of cuts. Due to technical limitations, a 5-axis milling operation with equal engagement conditions is not possible. Therefore, the validation for determine the process force holds for 3+2-axis milling even though the model can be applied for a 5-axis milling process. The cutting speed is set to $v_c = 80 \frac{m}{min}$ and the feed to f = 0.1 mm.



Fig. 2: Machined geometry considered for the milling operation

4. Identification of the dynamometers effective mass

The effective mass m_{td} of the table dynamometer cannot be determined precisely using geometric information and the dynamometers total weight. Therefore, it needs to be identified based on measurement data. The table dynamometer is mounted on the machine table, tared and the machine table is rotated. After the decay of dynamic effects, only the gravitational force affects the measurement. Using (3) - (5), the mass m_{td} can be determined based on the measured force ${}^{m}F_{M}$ in each direction using a least square approach. It is assumed that modelling the effective mass m_{td} as a constant value is sufficient. This assumption is valid if the identification indicates the same effective mass in each measured direction and between measurement with different rotation angles and an attached workpiece.

For the identification, the machine table is rotated with the rotation $\vartheta_x = \{-60^\circ, -30^\circ, 30^\circ\}$. For each rotation ϑ_x a rotation with $\vartheta_z = \{0^\circ, 30^\circ, 60^\circ, 90^\circ\}$ is investigated. For the rotation $\vartheta_x = \{-60^\circ, -30^\circ\}$ two rotations are conducted for each rotation ϑ_z . In one experiment solely the table dynamometer is used and in the second a workpiece with the mass $m_{\text{fix}} + m_{\text{wp}} = m_{\text{fwp}} = 12.3$ kg is attached. The values of the identified effective mass m_{td} for each measurement direction are shown in Fig. 3.



Fig. 3: Identified effective mass m_{td} of the table dynamometer

The identification of the effective mass m_{td} based on the measurement in x- and y-direction has a small variance. In x-direction, the relative error with respect to the median is 0.6 % for the 75 percentile and 0.9 % for the 25 percentile.

Respectively, the relative error is 0.8 % and 0.4 % in y-direction. Therefore, the assumption of a constant effective mass m_{td} is confirmed. The effective mass is modelled based on the median of all identified values as $m_{td} = 20$ kg. Compared to the x- and y-direction, the relative error in z-direction is high with values of 2.8 % and 2.5 %. This indicates a larger uncertainty in the measurement in z-direction.

5. Model validation

5.1. Model validation without a milling operation

To determine the error on the process force when the gravitational and inertial force are subtracted from the measurement, the model is validated in an experiment without milling. For this purpose, a cuboid workpiece with a mass $m_{\text{fwp}} = 10.1 \text{ kg}$ is mounted on the dynamometer. The machine table is rotated varying the angular velocity in the range of $\|\boldsymbol{\omega}\| \leq 2000 \frac{\circ}{\min}$. Hereby, the gravitational and inertial forces influence the measured force as shown in Fig. 4. An overline on a variable \bar{a} denotes that the table dynamometer or the sensors of the machine tool has measured the variable a.



Fig. 4: Angular velocity and measured forces of a rotation experiment without a milling operation

The distance between the combined center of mass and the center of rotation r is determined using the measurement of the workpiece's and the dynamometer's relative position in the machine tool. The aid for fixing the workpiece on the dynamometer is light compared to the mass of the workpiece. Therefore, it is assumed that the fixing aid does not influence the center of mass. Considering the individual centers of mass, the distance between the combined center of mass and the center of rotation results in $r = (0.02 \ 0.065 \ 0.035)^{T}$ (in m). In order to remove the influence of measurement noise on the evaluation of the model quality, all signals are filtered phase-free using a zero-phase filter. Based on (3), (5), (6) and (7) the expected measured force is calculated and compared to the measured force in the experiment. The results are shown in Table 1. The minimum and maximum value for both the measured force and the calculated forces is determined in order to assess the order of magnitude of the force components, referred to as force range. All three measurement directions show a similar result. Therefore, the evaluation is shown exemplarily in x-direction. The calculated force fits the measurement with a high quality. The relative mean absolute error (MAE) with respect to the measured force range is 0.3 %. The dominant force acting on the table dynamometer is the gravitational force. It describes 255.4 N of the total 298.7 N which is the calculated measured force range. In comparison, the centrifugal force is neglectable for common angular velocities in multi-axis milling and a small distance between the origin of rotation and the center of mass. The Euler force influence the measurement significantly. For the setup in this contribution, the influence is still smaller compared to the gravitational force.

Overall, the MAE is low with 0.9 % or less. To validate the gravitational force model, the maximum difference between the experimental and the calculated measured force is determined when the machine table stands still (denoted as static). The largest difference is observed in y-direction with a maximum static error of 3.8 %. Hence, the gravitational model is regarded as high quality. While rotating the machine table, the error increases (denoted as dynamic). A maximum difference of 8.3 % is observed. The distance r is approximated using geometric information. Further research needs to be conducted to determine the distance r which fits the measurement more accurately.

Force range (in N)				Model quality with respect to ${}^{m}\overline{F}_{i,M}$ in the MCS (in %)			
	F _x	$F_{\mathbf{y}}$	$F_{\rm z}$		$F_{\rm x,M}$	$F_{y,M}$	$F_{z,M}$
${}^{\mathrm{m}}\overline{F}_{\mathrm{M}}$	315.3	216.8	167.6	MAE	0.3	0.3	0.9
^m F _M	298.7	209.0	175.5	max. difference static	1.4	3.8	2.1
${}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{G}}$	255.4	207.6	147.6	max. difference dynamic	6.7	4.5	8.3
${}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{E}}$	49.8	4.2	35.1				
${}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{CF}}$	0.3	0.7	0.2				

Table 1: Force range (difference between maximum and minimum value of a force) and model quality for describing gravitational and inertial forces

5.2. Model validation with a milling operation

The main objective of the measurement model is to determine the process force ${}^{mt}F_{P}$ in a multi-axis milling operation based on the force measurement ${}^{m}\overline{F}_{M}$. To validate the model quality, milling operations with a rotated machine table are compared with milling operations without a rotation of the machine table. In a 3+2-axis milling process, the measured force is disturbed by the gravitational force and the measured process force is orientated in the MCS. Using (2) - (5), the process force ${}^{mt}F_{P}$ is given as

$${}^{\mathrm{mt}}\boldsymbol{F}_{\mathrm{P}} = {}^{\mathrm{m}}\boldsymbol{T}_{\mathrm{mt}}^{-1} {}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{P}} = {}^{\mathrm{m}}\boldsymbol{T}_{\mathrm{mt}}^{-1} ({}^{\mathrm{m}}\boldsymbol{\overline{F}}_{\mathrm{M}} - {}^{\mathrm{m}}\boldsymbol{F}_{\mathrm{G}}).$$
(8)

A high model quality is ensured, if the process force ${}^{mt}F_{P}$ is equal for all milling operations. The mass loss for the machined geometry introduced in section 3 is around 2 g. Thus, the resulting mass m is assumed to be constant for the milling operation. Due to the very frequent intermittent engagement of the cutting edges, a comparison of the time series of the process force between all individual milling operations would show large deviations, as the force pattern exhibits stochastic deviations. Therefore, the process force ${}^{mt}F_{i,P}$ in i-th direction is enveloped. Both the top envelope denoted as ${}^{mt}_{t}F_{i,P}$ and the bottom envelope denoted as ${}^{mt}_{b}F_{i,P}$ are considered for the evaluation. The rotation angles of the milling operations are shown in Table 2. Three milling operations with an unrotated machine table ($\vartheta_x = \vartheta_z = 0^\circ$) were conducted. The mean of those milling operations is used as the ground truth. The mean difference between the minimum and maximum envelope of the process force is determined in each direction to assess the expected range of the process force. The MAE for the milling operation with an unrotated machine table is evaluated with respect to all three measurements. For the determination of the MAE in milling operations with a rotated machine table a single milling operation is used. All evaluations are conducted while the milling tool is engaged maximally for each cut. Hereby, a movement of the milling tool between cuts and the entry and exit of the milling tool in a cut is neglected in the evaluation. The MAE is calculated with respect to the process force range detailed in Table 2. Equal milling operations are assumed to have similar engagement conditions and disturbances acting on the measurement. Still, differences in the measurement of the process force with an unrotated machine table were observed with a relative MAE of less than 3.7 %. The evaluation of the relative MAE for the milling operations with a rotated machine table will indicate a good model quality, if the relative MAE is in a similar range to the milling operations with an unrotated machine table. The process force from the milling operations with unrotated and rotated machine table are shown in Fig. 5. The process force in x- and y- directions are the mostly relevant in a force control strategy. Therefore the two directions are evaluated primary. In the experimental setup, the process force in negative x-direction is comparably small to the process force in the positive x- and in the y-direction. In the rotated experiments, mainly the gravitational force influences the lower limit of the measured force in x-direction. The small relative MAE of 1.7 % respectively 1.6 % evaluating the lower envelope in x-direction indicates that the gravitational force is compensated adequately. When evaluating the rotation of the measured force in x- and y-direction with great process forces, the relative MAE increases with a value up to 13.3 %. The rotation of the process force results in an error which cannot be explained by the deviation between the experiments. Therefore it is concluded, that additional disturbances act on the measurement, which are not modelled. Additionally, the assumption that the small amount of cuts does not influence the process force or that the engagement conditions are similar in the experiments need to be further investigated. Overall, the results in x- and y- direction indicate that the model is accurate enough to evaluate the control performance of a force control strategy using the model-based determination of the process force.

	process force range (in N)	Relative MAE with respect to process force range (in %)				
		$artheta_{\scriptscriptstyle X}=0^\circ$, $artheta_{\scriptscriptstyle Z}=0^\circ$	$\vartheta_x = -20^\circ$, $\vartheta_z = 31^\circ$	$\vartheta_x = -35^\circ$, $\vartheta_z = -57^\circ$		
${}^{\rm mt}_{b}F_{\rm x,P}$	200.0	0.9	1.7	1.6		
${}^{\mathrm{mt}}_{\mathrm{t}}F_{\mathrm{x},\mathrm{P}}$	308.9	1.5	7.0	11.1		
${}^{\rm mt}_{b}F_{\rm y,P}$	201.0	2.2	11.7	13.3		
${}^{\mathrm{mt}}_{\mathrm{t}}F_{\mathrm{y},\mathrm{P}}$	291.0	1.2	5.6	6.4		
${}^{\rm mt}_{b}F_{z,P}$	72.0	1.9	8.4	14.1		
${}^{\mathrm{mt}}_{\mathrm{t}}F_{\mathrm{z},\mathrm{P}}$	72.0	3.7	38.3	15.9		

Table 2: MAE of the process force from different milling operations with respect to the process force range

The rotation in z-direction shows a higher relative MAE compared to the other directions. Especially the top envelope of the experiment with the rotation $\vartheta_x = -20^\circ$, $\vartheta_z = 31^\circ$ has a large difference compared to the unrotated experiment with a relative MAE of 38.3 %. Further investigations have shown a temperature dependency of the force measurement, especially in z-direction. In the milling process, the milling tool cuts of hot chips from the workpiece which fall on the dynamometer. Therefore, the dynamometer is heated up which leads to a drift in the force measurement observable especially in z-direction. Further research will be conducted to reduce the influence of the temperature increase on the measurement.



Fig. 5: Comparison between the process force in milling operations with different rotation angle of the machine table in each measurement direction

6. Conclusion

This contribution presents a model to determine the process force in multi-axis milling using a table dynamometer. Model parameters are chosen based on geometric information acquired from the machining center and on experimental data. The validation is conducted in two steps. Firstly, the measured force for rotating the machine table without a milling operation is calculated and compared to the measurement. Secondly, 3+2-axis milling operations are performed in order to compare the determined process force. The relative MAE in determining the process force in rotated experiments is less than 13.3 % in x- and y-direction (absolute 38.7 N). In [8] a total process force in x- and y- direction of 500 N is desired. Therefore the model quality is sufficiently high for the use in a force control strategy. Further research needs to be conducted to assess the effect of inaccuracies on the control performance in a force control strategy using the model-based determination of the process force. This investigation will reveal whether the relative error is caused by unmodelled disturbances acting on the dynamometer or by a difference in the engagement and tool wear between the experiments conducted.

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8.References

- [1] A. G. Ulsoy and Y. Koren, "Control of Machining Processes," *Journal of Dynamic Systems, Measurement, and Control*, vol. 115, pp. 301–308, 1993, doi: 10.1115/1.2899070.
- [2] T. Watanabe and S. Iwai, "A Control System to Improve the Accuracy of Finished Surfaces in Milling," *Journal of Dynamic Systems, Measurement, and Control*, vol. 105, no. 3, pp. 192–199, 1983, doi: 10.1115/1.3140655.
- [3] C. Ma and Y. Altintas, "Direct adaptive cutting force control of milling processes," *Automatica*, vol. 26, no. 5, pp. 899–902, 1990, doi: 10.1016/0005-1098(90)90006-4.
- [4] L. K. Lauderbaugh and A. G. Ulsoy, "Model Reference Adaptive Force Control in Milling," *Journal of Engineering for Industry*, vol. 111, no. 1, pp. 13–21, 1989, doi: 10.1115/1.3188726.
- [5] Y. Koren, O. Masory, and S. Malkin, "Adaptive Control with Process Estimation," *CIRP Annals*, vol. 30, no. 1, pp. 373–376, 1981, doi: 10.1016/S0007-8506(07)60960-3.
- [6] Jun-Ho Oh and M. Tomizuka, "Model Reference Adaptive Control of the Milling Process," 1983.
- [7] Y. Altintaş, "Direct adaptive control of end milling process," *International Journal of Machine Tools and Manufacture*, vol. 34, no. 4, pp. 461–472, 1994, doi: 10.1016/0890-6955(94)90078-7.
- [8] S. Stemmler, M. Ay, M. Schwenzer, D. Abel, and T. Bergs, "Model-based Predictive Force Control in Milling," in 2019 18th European Control Conference (ECC), Naples, Italy, 2019, pp. 4313–4318.
- [9] F. Klocke, M. Blattner, O. Adams, M. Brockmann, and D. Veselovac, "Compensation of Disturbances on Force Signals for Five-axis Milling Processes," *Proceedia CIRP*, vol. 14, pp. 472–477, 2014, doi: 10.1016/j.procir.2014.03.098.
- [10] S. Rekers, *Correction of Systematic Errors in Piezoelectric Cutting Force Measurement*. Aachen: Apprimus Wissenschaftsverlag, 2019.
- [11] C. D. Yan, W.-H. Chieng, and S.-L. Jeng, "Five-Axis Slerp for Tool-Orientation Planning in a Five-Axis CNC Machine", *Journal of the Chinese Society of Mechanical Engineers*, vol. 39, pp. 565–574, 2018.
- [12] A. K. Rüppel, P. Ochudlo, M. Bickel, S. Stemmler, T. Bergs, and D. Abel, "Modeling Mass Loss and Displacement of the Center of Mass in Milling Processes," *56th CIRP Conference on Manufacturing Systems*, 2023.
- [13] M. Schwenzer, S. Stemmler, M. Ay, A. K. Rüppel, T. Bergs, and D. Abel, "Model predictive force control in milling based on an ensemble Kalman filter", *Journal of Intelligent Manufacturing*, pp. 1–13, 2022, doi: 10.1007/s10845-022-01931-2.