

Shaking Force and Shaking Moment Balancing in Planar Serial Manipulators

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Abstract - High-speed mechanical systems are recognized as key sources of vibration excitation. Fast-moving manipulators, in particular, can generate substantial fluctuating forces and moments. As a result, balancing shaking forces and shaking moments, which arise from the inertial forces of the links, becomes critically important. The quality of mass balancing not only affects vibration levels but also influences the lifespan, reliability, and accuracy of manipulators. In addition to the negative effects mentioned, vibrations contribute to environmental pollution, energy loss, and may lead to various health issues. Therefore, improving mass balancing quality holds not only technical, technological, and economic significance but also social implications. This paper discusses the complete balancing of shaking forces and moments in planar serial manipulators. To achieve this, a combination of two approaches is utilized. First, dynamic decoupling and linearization of the motion equations are performed. Following this, the shaking forces acting on the frame become constant and equal to the sum of the gravitational forces of the links, while the shaking moment becomes proportional to the acceleration of the first link attached to the frame. This enables the addition of a pair of identical gears, mounted on the manipulator's frame and first link, to cancel the shaking moment. The proposed solution is demonstrated using a planar two-degree-of-freedom serial manipulator and validated through numerical simulations conducted with ADAMS software.

Keywords: shaking forces balancing; shaking moments balancing, effort minimization on the frame; decoupling of motion equations, robot design; planar serial manipulators; dynamics of robots

1. Introduction

Balancing linkages is a fundamental aspect of mechanism design. Despite its long history, the theory of mechanism balancing continues to evolve, with researchers regularly introducing new approaches and solutions. This ongoing innovation underscores the enduring interest in balancing problems within the research community. Laboratories worldwide remain highly active in this field, publishing fresh findings at a steady pace. In recent decades, new challenges have emerged, particularly in the balancing of robots designed for high-speed manipulation. The literature offers a variety of design concepts and methods to address these challenges, advancing the capabilities of robotic systems [1].

A review of methods for shaking force balancing in manipulators has highlighted the following principal approaches. Shaking force balancing can be achieved by adding counterweights to maintain the total centre of mass of the moving links stationary. For open-chain manipulators, the process begins with the outermost link, where a counterweight is added to align the link's centre of mass with the axis of the preceding joint. This balancing procedure is then repeated sequentially for each link, progressing inward, until the centre of mass for the entire chain is fixed at the base pivot [2]. Shaking force balancing can also be achieved by incorporating auxiliary structures. Various approaches have been developed to maintain the total centre of mass of the moving links stationary through the addition of these structures [3], [4]. Shaking force minimization can be achieved through the minimization of centre of mass acceleration. In [5], an innovative solution was proposed based on the optimal control of the robot's centre of mass. Unlike traditional methods that rely on end-effector trajectory control, this approach focuses on planning the displacements of the total centre of mass of the moving links to achieve effective shaking force reduction.

Regarding the shaking moment balancing of manipulators, several approaches have been developed. The concept of balancing through counter-rotation was initially explored for single-degree-of-freedom mechanisms and has since been

extended to manipulators, with further advancements documented in various studies [6]-[10]. Shaking moment balancing using modules based on dynamically balanced four-bar linkages enables complete balancing of both shaking force and shaking moment without the need for separate counter-rotation [11], [12]. This is made possible through the synthesis of fully balanced four-bar linkages. Studies have shown that a four-bar linkage can be fully balanced by optimally selecting the mass and inertia parameters of the moving links, provided the linkage has specific geometric characteristics and a certain relationship between the lengths of the links. Shaking moment balancing through the generation of optimal trajectories for moving links was explored in [13], where a redundant 3-DOF manipulator was designed with the system's centre of mass fixed by optimally redistributing the masses. Additionally, the system's dynamics were decoupled, which simplified the planning of optimal motions for balancing the manipulator's shaking moment. Shaking moment balancing through prescribed rotation of the end-effector was also developed. In [14], the shaking moment of a three-degrees-of-freedom planar parallel manipulator was eliminated using two approaches: selecting appropriate inertia and geometric parameters, and implementing suitable motion planning. Similarly, in [15], the shaking moment on the frame of a four-degrees-of-freedom SCARA-type robot was eliminated by prescribing the end-effector's velocity. Shaking moment balancing can also be achieved by adding an inertia flywheel to the frame that rotates with a prescribed angular velocity. It is well known that after shaking force balancing, the shaking moment applied to the base remains constant relative to any point. In other words, for a given position of the manipulator, the shaking moment has the same value at every point on the base. Leveraging this property, the shaking moment of any planar manipulator can be balanced by adding an inertia flywheel rotating at a prescribed angular velocity [7].

Presented below is a novel solution for shaking moment balancing in planar serial manipulators, combining two approaches: counter-rotation balancing and dynamic equation decoupling. This is demonstrated using the example of a two-degree-of-freedom manipulator to illustrate the proposed solution through simplified and more concise dynamic equations. However, a similar approach can be extended to other planar serial manipulators.

2. Shaking Force and Shaking Moment in Manipulators

The shaking force refers to the resultant dynamic force that acts on the base of the manipulator due to the acceleration and deceleration of its moving links. It arises because of the inertia of the manipulator's components and is caused by the imbalance of internal forces during motion.

It is the sum of all inertia forces and gravitational forces generated by the manipulator's links:

$$\mathbf{F}^{sh} = \sum_{i=1}^n m_i \mathbf{S}_i + \sum_{i=1}^n \mathbf{G}_i \quad (1)$$

where, m_i is the mass of the i -th link, $\mathbf{S}_i$ is the linear acceleration of the i -th link's centre of mass, $\mathbf{G}_i$ corresponds to the gravitational forces acting on the links of the manipulator, n is the number of moving links.

The shaking moment refers to the resultant dynamic torque (moment) about the frame of the manipulator, caused by the rotational accelerations of the manipulator's links. It represents the angular counterpart to the shaking force:

$$\mathbf{M}^{sh} = \sum_{i=1}^n (\mathbf{r}_i \times m_i \mathbf{S}_i + \mathbf{I}_i \boldsymbol{\theta}_i) \quad (2)$$

where, $\mathbf{r}_i$ is the position vector of the i -th link's centre of mass relative to the frame, $\mathbf{I}_i$ is the moment of inertia tensor of the i -th link, $\boldsymbol{\theta}_i$ is the angular acceleration of the i -th link.

Shaking forces and moments are crucial considerations in the design and control of manipulators, particularly in high-speed and precision applications. Minimizing them can lead to reduced vibrations and noise, as well as improved accuracy and stability.

However, from a technical standpoint, this is a rather complex task. While force cancellation can be achieved by immobilizing the common centre of mass with the addition of counterweights, balancing the moment remains extremely challenging.

3. Dynamics of a Planar 2-DOF Serial Manipulator and Its Shaking Forces and Moments

Let us demonstrate that by performing dynamic decoupling of the manipulator's movements, the shaking forces and the shaking moments can be easily cancelled. For simplicity of explanation, let us consider a planar two-degree-of-freedom (2-DOF) serial manipulator.

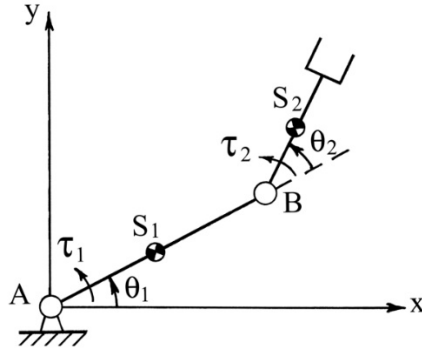


Fig. 1: Planar two-degree-of-freedom serial manipulator.

The dynamic equations of a planar two-degree-of-freedom serial manipulator can be rewritten as:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} D_{111} & D_{122} \\ D_{211} & D_{222} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} + \begin{bmatrix} D_{112} & D_{121} \\ D_{212} & D_{221} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \quad (3)$$

with

$$D_{11} = m_1 l_{AS1}^2 + m_2 l_1^2 + m_2 l_{BS2}^2 + 2m_2 l_1 l_{BS2} \cos \theta_2 + I_{S1} + I_{S2} \quad (4)$$

$$D_{12} = D_{21} = m_2 l_{BS2}^2 + m_2 l_1 l_{BS2} \cos \theta_2 + I_{S2} \quad (5)$$

$$D_{22} = m_2 l_{BS2}^2 + I_{S2} \quad (6)$$

$$D_{111} = 0 \quad (7)$$

$$D_{122} = -m_2 l_1 l_{BS2} \sin \theta_2 \quad (8)$$

$$D_{211} = m_2 l_1 l_{BS2} \sin \theta_2 \quad (9)$$

$$D_{222} = 0 \quad (10)$$

$$D_{112} = D_{121} = m_2 l_1 l_{BS2} \sin \theta_2 \quad (11)$$

$$D_{212} = D_{221} = 0 \quad (12)$$

$$D_1 = (m_1 l_{AS1} + m_2 l_1) g \cos \theta_1 + m_2 g l_{BS2} \cos(\theta_1 + \theta_2) \quad (13)$$

$$D_2 = m_2 g l_{BS2} \cos(\theta_1 + \theta_2) \quad (14)$$

where, m_1 and m_2 are the masses of links 1 and 2, l_1 is the distance between the joint centers A and B , θ_1 is the angular displacement of link 1 relative to the base, θ_2 is the angular displacement of link 2 relative to link 1, $\dot{\theta}_1$ is the angular velocity of link 1 relative to the base, $\dot{\theta}_2$ is the angular velocity of link 2 relative to link 1, $\ddot{\theta}_1$ is the angular acceleration of link 1 relative to the base, $\ddot{\theta}_2$ is the angular acceleration of link 2 relative to link 1, l_{AS1} is the distance between the center of mass S_1 of link 1 and joint center A , l_{AS2} is the distance between the center of mass S_2 of link 2 and joint center B , I_{S1} is the axial moment of inertia of link 1 relative to the centre of mass S_1 of link 1, I_{S2} is the axial moment of inertia of link 2 relative to the center of mass S_2 of link 2, g is the gravitational acceleration.

The shaking forces and moments of such a manipulator can be represented as follows:

$$F^{sh} = \sqrt{F_{10x}^2 + F_{10y}^2} \quad (15)$$

$$M^{sh} = -\tau_1 \quad (16)$$

here F_{10x} and F_{10y} are the components of the reaction forces acting on the manipulator's frame.

These are nonlinear and highly coupled functions, making them challenging to cancel.

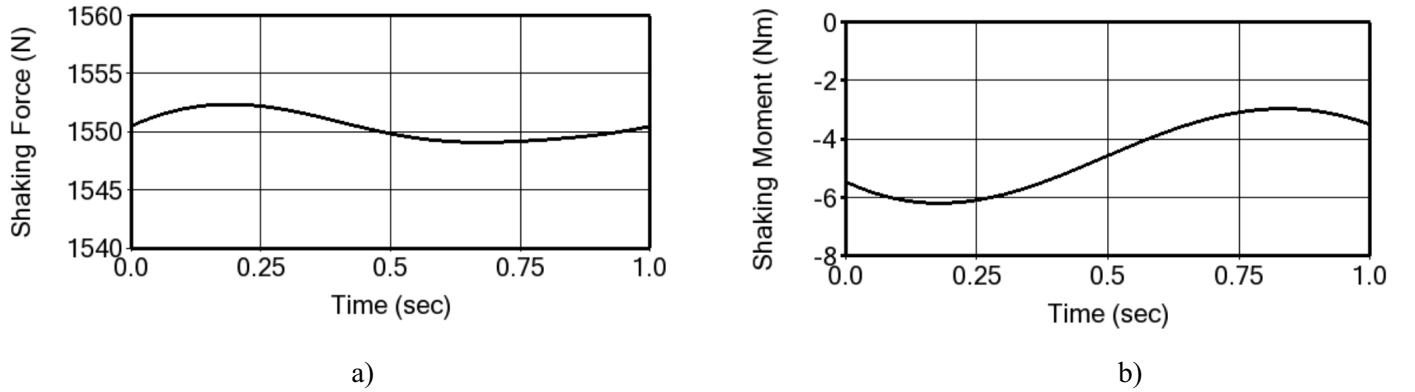


Fig. 2: Variations in the shaking force and shaking moment for unbalanced manipulator.

Figure 2 illustrates the variations in shaking force and moment for a manipulator with the following parameters: $l_1 = 0.3$ m, $l_{AS1} = l_{BS2} = 0.15$ m, $m_1 = 3$ kg, $m_2 = 0.5$ kg, $I_{S1} = 0.5$ kgm², $I_{S2} = 0.05$ kgm². The generation of motions between the initial and final positions of links ($\theta_{1i} = 0$, $\theta_{1f} = 30^\circ$, $\theta_{2i} = 60^\circ$, $\theta_{2f} = 90^\circ$) are carried out by fifth order polynomial laws.

4. Balancing of Shaking Forces and Moments

Let us first analyse the design of the manipulator, emphasizing its decoupled and linear dynamic equations. There are three main approaches developed for dynamic decoupling of manipulators via mechanical transformation: *i*) via mass redistribution [16], [17]; *ii*) via actuator relocation [18] and *iii*) via addition of auxiliary links [19], [20].

The solution to the present problem involves applying the method proposed in [19] and modifying the manipulator as shown in Fig. 3. Specifically, a pairs of identical gears (1' and 2') introduced: gear 2' rigidly connected with link 2, while gear 1' is mounted on link 1 via revolute joint C. Additionally, the masses of the links are redistributed to ensure that the manipulator's total centre of mass remains stationary and located at frame joint A [1].

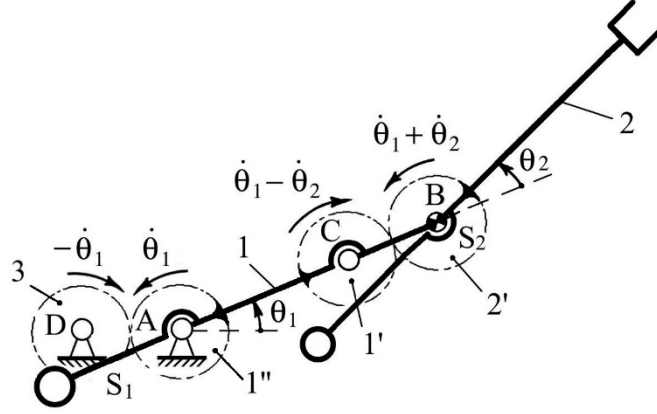


Fig. 2: The fully balanced manipulator.

After such a modification of the manipulator and taking into account that $I_{S2} + I_{S2'} = I_{S1'}$, the input torque become decoupled and linearized:

$$\tau_1 = [m_1 \rho_{AS1}^2 + (m_2 + m_{2'}) \rho_1^2 + m_1 \rho_{AC}^2 + I_{S1} + I_{S1'} + I_{S2} + I_{S2'}] \theta_1 \quad (17)$$

$$\tau_2 = (I_{S2} + I_{S2'} + I_{S1'}) \theta_2 \quad (18)$$

where m_1 , and m_2 , are the masses of added gears, $I_{S1'}$ and $I_{S2'}$ are their axial inertia moments of inertia.

Thus, for the obtained manipulator:

$$F^{sh} = const = \sum_{i=1}^2 G_i \quad (19)$$

$$M^{sh} = -\tau_1 = -[m_1 \rho_{AS1}^2 + (m_2 + m_{2'}) \rho_1^2 + m_1 \rho_{AC}^2 + I_{S1} + I_{S1'} + I_{S2} + I_{S2'}] \theta_1 \quad (20)$$

We now introduce gear 1'', rigidly attached to link 1 and meshing with gear 3 mounted on the frame. When the conditions $I_3 = m_1 \rho_{AS1}^2 + (m_2 + m_{2'}) \rho_1^2 + m_1 \rho_{AC}^2 + I_{S1} + I_{S1'} + I_{S2} + I_{S2'}$, and $\theta_3 = -\theta_1$, are satisfied, the shaking moment on the manipulator's frame is completely cancelled, achieving a fully balanced system.

For the manipulator illustrated in Section 3, the required parameters are as follows: $m_1 = 7$ kg, $m_1' = 2$ kg, $m_2 + m_2' = 1$ kg, $I_{AS1} = 0.1$ m, $I_{BS2} = 0$, $I_{AC} = 0.2$ m, $I_{S1} = 0.8$ kgm², $I_{S2} + I_{S2'} = 0.5$ kgm². With these parameters, the shaking forces and shaking moments will be fully balanced. This was validated through simulations conducted using ADAMS software.

5. Conclusion

The paper presents a comprehensive solution for balancing shaking forces and moments in planar serial manipulators. To achieve this, a combined strategy is employed. The first step involves decoupling and linearizing the motion equations. As a result, the shaking forces acting on the frame become constant and are equal to the total gravitational forces of the links, while the shaking moment is directly proportional to the acceleration of the first link connected to the frame. This approach allows for the introduction of a pair of identical gears mounted on both the manipulator's frame and the first link, effectively counteracting the shaking moment. The proposed method is demonstrated using a planar two-degree-of-freedom serial manipulator and validated through numerical simulations conducted with ADAMS software.

Furthermore, it is worth noting that this approach can also be adapted to achieve full balancing in other types of planar serial manipulators.

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