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Transport Solutions of Biot's Equations at Subsonic Loads

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Abstract - The investigation of transport movement in tunnels, transported cargo and fluids in pipelines leads to model problems on the action of moving loads in a continuous medium. A feature of such problems is that the transport speed significantly affects the type of differential equations, which parametrically depend on the ratio of the transport speed to the wave propagation speeds in the medium. Therefore, when solving transport problems, different motion modes arise, which affects the type of transport equations on which the properties of solutions and the choice of methods for constructing them depend. In this case, it becomes necessary to study the effect of transport speed on the surrounding massif depending on its physical and mechanical properties. This paper is devoted to studying the dynamics of a two-component Biot's model under transport loads whose speed does not exceed the speeds of wave propagation in media (subsonic loads). The Green's tensor of transport equations is constructed and, on its basis, their solutions are obtained for any type of transport loads from the class of generalized functions, both regular and singular, concentrated on moving surfaces and lines. Numerical calculations illustrating the wave dynamics of a two-component Biot's model at subsonic speeds of transport loads are carried out.

***Keywords*:** elasticity, two-component model, Biot's equations, Green's tensor, Fourier transformation

1. Introduction

In connection with the widespread construction of high-speed highways, there is an urgent need to study the influence of the speed of a vehicle on the surrounding massif depending on its physical and mechanical properties. Such processes have been studied best on the model of an isotropic elastic medium in the works [1-8] and others.

A real rock mass, in addition to elastic properties, has a number of other properties that have a significant effect on wave processes. Therefore, complication of the mathematical model for a more complete consideration of the factors in force in the study of wave, including seismic processes is absolutely necessary. Models that take into account the water saturation of the structures that make up the earth's crust, the presence of gas bubbles, etc., are multicomponent media. There are various mathematical models of such media. In paper to take into account the real properties of the rock mass, a two-component Biot's model is considered, containing two components: an elastic solid and a fluid component. The considered media allows modelling the dynamics of soils taking into account their porosity and water saturation and more accurately describing wave processes in it than the elastic medium model. For such a medium, dynamic processes are studied in [9-20] and others, to transport problems the works, for example, [21-24] are devoted.

Here transport solutions are constructed and the dynamics of a two-component Biot's model under subsonic transport loads are investigated. Note that the Biot media is characterized by three sound speeds c_1, c_2, c_3. Two of them describe the propagation speed of longitudinal (dilation) waves of the 1st and 2nd kind, and the third one describes the transverse (shear) wave. The construction of the Green's tensor of transport equations is given and, on its basis, their solutions are given for any type of transport loads from the class of generalized functions, both regular and singular, concentrated on moving surfaces and lines. To study the wave dynamics of the medium under consideration, numerical calculations of the obtained solutions are carried out, and the corresponding graphs of a two-component Biot's model at subsonic speeds of transport loads are pictured.

2. Biot's equations

The equations of motion of a homogeneous isotropic two-component Biot's model in the case of absence of fluid viscosity are described by the following system of second-order hyperbolic equations [9, 15]:

$$(\lambda+\mu)u^s_{j,ji}+\mu u^s_{i,jj}+Qu^f_{j,ji}+G^s_i=\rho_{11}\ddot{u}^s_i+\rho_{12}\ddot{u}^f_i \tag{1}$$

$$Qu^s_{j,ji}+Ru^f_{j,ji}+G^f_i=\rho_{12}\ddot{u}^s_i+\rho_{22}\ddot{u}^f_i,\ (x,t)\in R^3\times[0,\infty) \tag{2}$$

Where $u^s_i(x,t)$ are the components of the displacement vector of the elastic skeleton, $u^f_i(x,t)$ are the components of the displacement vector of the fluid, G^s_i, G^f_i are the volume forces acting on the solid and fluid components, respectively. The constants λ, μ, Q, R have the dimension of stresses, the constants ρ_{11}, ρ_{12}, ρ_{22} have the dimension of density and are related to the mass density of the particles that make up the skeleton ρ_s and the fluid ρ_f by relations:

$$\rho_{11}=(1-m)\rho_s-\rho_{12},\ \rho_{22}=m\rho_f-\rho_{12},\ \rho_{22}=m\rho_f-\rho_{12} \tag{3}$$

m is the porosity of the medium, $u_{i,j}=\partial_j u_i=\partial u_i/\partial x_j$, $\dot{u}_i=\partial_t u_i=\partial u_i/\partial t$. Everywhere over the same indices in the product, summation from 1 to 3 is performed (tensor convolution). For the medium under consideration, the relationship between stresses and strains has the form of a generalized Hooke's law:

$$\sigma_{ij}=\mu\left(u^s_{i,j}+u^f_{i,j}\right)+\left(\lambda u^s_{k,k}+Qu^f_{k,k}\right)\delta_{ij},\ \ \sigma=-mp=\left(Qu^s_{k,k}+Ru^f_{k,k}\right), \tag{4}$$

Where p is the fluid pressure in the pores, σ_{ij} are the components of the stress tensor in the elastic skeleton.

Biot's model is characterized by three sound speeds. Two of them describe the propagation speed of longitudinal (dilation) waves of the 1st and 2nd kind, and the third describes the transverse (shear) wave:

$$c^2_{1,2}=\frac{(\lambda+2\mu)\rho_{22}+R\rho_{11}-2Q\rho_{12}}{2\left(\rho_{11}\rho_{22}-\rho^2_{12}\right)}\pm$$

$$\pm\frac{\sqrt{\left((\lambda+2\mu)\rho_{22}-R\rho_{11}\right)^2+4\left((\lambda+2\mu)\rho_{12}-Q\rho_{11}\right)\left(R\rho_{12}-Q\rho_{22}\right)}}{2\left(\rho_{11}\rho_{22}-\rho^2_{12}\right)} \tag{5}$$

$$c_3=\sqrt{\frac{\mu\rho_{22}}{\rho_{11}\rho_{22}-\rho^2_{12}}} \tag{6}$$

where the upper sign in (5) corresponds to c_1 and the lower sign to c_2. For real porous media, the inequality $c_1>c_3>c_2$ holds, i.e., a dilation wave of the 2nd kind propagates in the medium more slowly than shear and dilation waves of the 1st kind. When the connection between the fluid and the elastic solid is absent

$$Q\to 0\ ,\ \ \rho_{12}\to 0 \tag{7}$$

we have

$$c_3\to\sqrt{\mu/\rho_{11}}\ ,\ \ c_1\to c_s=\sqrt{(\lambda+2\mu)/\rho_{11}},\quad c_2\to c_f=\sqrt{R/\rho_{22}} \tag{8}$$

where c_s is the speed of longitudinal waves in a solid skeleton, c_f is the speed of longitudinal waves in fluid.

A class of transport solutions of the system of equations (1) – (2) is considered under the assumption that the mass force acting in the medium moves with a constant speed c along the x_3 in the opposite direction and in the moving coordinate system $x' = (x_1', x_2', x_3') = (x_1, x_2, x_3 + ct)$ does not depend on time, i.e. $G_i = G_i(x_1, x_2, x_3 + ct)$. The sought displacements u_i have the same structure: $u_i = u_i(x_1, x_2, x_3 + ct)$. Then Eqs. (1) – (2) can be written as:

$$(\lambda + \mu)\frac{\partial^2}{\partial x_j' \partial x_i'} u_i + Q\frac{\partial^2}{\partial x_j' \partial x_i'} u_{i+3} + \mu\frac{\partial^2}{\partial x_j' \partial x_j'} u_i + G_i = c^2\frac{\partial^2}{\partial x_3^2}\left(\rho_{11}\ddot{u}_i + \rho_{12}\ddot{u}_{i+3}\right) \tag{9}$$

$$Q\frac{\partial^2}{\partial x_j' \partial x_i'} u_i + R\frac{\partial^2}{\partial x_j' \partial x_i'} u_{i+3} + G_{i+3} = c^2\frac{\partial^2}{\partial x_3^2}\left(\rho_{12}\ddot{u}_i + \rho_{22}\ddot{u}_{i+3}\right) \tag{10}$$

Here, for convenience, a six-dimensional displacement vector is introduced $u = \{u^s, u^f\} = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, assuming that u_i are the components of the solid phase displacement for $i = \overline{1,3}$ and the fluid for $i = \overline{4,6}$. Similarly, the vector of mass forces $G = \{G^s, G^f\} = \{G_1, G_2, G_3, G_4, G_5, G_6\}$. U_{21}, U_{22}, U_{23}

We will call Eqs. (9) - (10) the Biot's equations. The type of this system depends significantly on the speed of the transport load c. When $c < \min(c_1, c_2, c_3)$ the load is called subsonic, the type of equations is elliptic.

3. Green's tensor of Biot's transport equations

We construct the Green's tensor U_{ij} - the fundamental solutions under the action of concentrated transport forces, which are described by singular generalized functions of the form:

$$G_i = \delta_{ij}\delta(x_1)\delta(x_2)\delta(x_3 + ct) = \delta_{ij}\delta(x') \tag{11}$$

Here δ_{ij} is the Kronecker symbol, $\delta(x')$ is the generalized singular delta function. In this case, equations for Green's tensor U_{ij} (dimensions 6×6) will take the form:

$$(\lambda + \mu)U_{ik,kj} + \mu U_{ij,kk} + QU_{i(k+3),kj} - c^2\rho_{11}U_{ij,33} - c^2\rho_{12}U_{i(j+3),33} + \delta_{ij}\delta(x') = 0 \tag{12}$$

$$QU_{ik.kj} + RU_{i(k+3),kj} - c^2\rho_{12}U_{ij,33} - c^2\rho_{22}U_{i(j+3),33} + \delta_{i(j+3)}\delta(x') = 0 \tag{13}$$

The components of U_{ij} have the following physical meaning: for $1 \le j \le 3$ these are the j - th components of the solid phase displacements, for $4 \le j \le 6$ these are the $(j-3)$ - th components of the fluid displacements from the action of a concentrated force along the i - th coordinate axis on the solid phase (for $1 \le i \le 3$) or from the action of a concentrated force along the $(i-3)$ - th coordinate axis on the fluid (for $4 \le i \le 6$). It is necessary that the radiation conditions be satisfied:

$$\operatorname{supp} U_{ij}(x') \in R_+^3 = \{x' : x_3 \ge 0\} \tag{14}$$

$$U_{ij}(x') \to 0 \quad \text{for} \quad x_3' \to \infty \tag{15}$$

3.1. Green's tensor and its Fourier transform

In the construction of the Green's tensor is used apparatus of integral Fourier transforms allowing you to transfer from the differential equations for the tensor to linear algebraic equations for his image. Allowing the latter determine the transform of the tensor in the form of fractional rational function of the variables of integral Fourier transforms and then restore the original tensor, using the inverse transformation, it is often impossible. This leads to the attraction of the apparatus of the generalized Fourier transforms. In the space of Fourier transforms, we obtain a system of linear algebraic equations:

$$-(\lambda+\mu)\xi_k\xi_j U_{ik}-\mu\xi^2 U_{ij}-Q\xi_k\xi_j U_{i(k+3)}+c^2\xi_3^2\left(\rho_{11}U_{ij}-\rho_{12}U_{i(j+3)}\right)+\delta_{ij}=0 \tag{16}$$

$$-\xi_k\xi_j\left(QU_{ik}+RU_{i(k+3)}\right)+c^2\xi_3^2\left(\rho_{12}U_{ij}+\rho_{22}U_{i(j+3)}\right)+\delta_{i(3+j)}=0 \tag{17}$$

where ξ_k are the parameters of the Fourier transform in coordinates, $\xi^2=\xi_k\xi_k$, $k=\overline{1,3}$, $j=\overline{1,3}$, $i=\overline{1,6}$. Resolving this system, we obtain the Fourier transform of the Green's tensor, which we represent in a form convenient for constructing the original:

$$U_{kj}=\frac{c_3^2}{\mu}\left(\frac{b_{k3}\delta_{kj}}{c_3^2\left(\xi^2-M_3^2\xi_3^2\right)}-\frac{\xi_k\xi_j}{c^2\xi_3^2}\sum_{l=1}^{3}\frac{b_{kl}}{\xi^2-M_l^2\xi_l^2}\right),\quad k=\overline{1,3},\quad j=\overline{1,6}, \tag{16}$$

$$U_{kj}=U_{jk},\quad k=\overline{4,6},\quad j=\overline{1,3}, \tag{17}$$

$$U_{kj}=\frac{\delta_{kj}}{\rho_{22}c^2\xi_3^2}+\frac{c_3^2}{\mu}\left(\frac{d_3\delta_{kj}}{c_3^2\left(\xi^2-M_3^2\xi_3^2\right)}-\frac{\xi_{k-3}\xi_{j-3}}{c^2\xi_3^2}\sum_{l=1}^{3}\frac{d_l}{\xi^2-M_l^2\xi_l^2}\right),\quad k=\overline{4,6},\quad j=\overline{4,6}, \tag{18}$$

Where

$$b_{k1}=\frac{c_1^2-c_f^2}{c_2^2-c_f^2},\qquad b_{k2}=\frac{c_2^2-c_f^2}{c_2^2-c_f^2},\qquad b_{k3}=-1,\quad k=\overline{1,3} \tag{19}$$

$$b_{k1}=\zeta_1\frac{c_1^2-c_f^2}{c_2^2-c_f^2},\qquad b_{k2}=-\zeta_2\frac{\rho_{11}c_2^2-c_s^2}{\rho_{22}c_1^2-c_2^2},\qquad b_{k3}=-\zeta_3,\quad k=\overline{4,6} \tag{20}$$

$$d_1=\frac{\rho_{11}c_1^2-c_s^2}{\rho_{22}c_1^2-c_2^2},\qquad d_2=\frac{\rho_{11}c_2^2-c_s^2}{\rho_{22}c_1^2-c_2^2},\qquad d_3=-\zeta_3,\quad k=\overline{4,6} \tag{21}$$

Here $M_j=c/c_j$ is the Mach numbers which characterize the speed of the source relative to the speed of sound. The components of the tensor contain the following functions

$$\bar{f}_{kl}f(\xi_1,\xi_2,\xi_3)=\frac{1}{(-i\xi_3)^k}\left(\xi^2-M_l^2\xi_3^2\right)^{-k}=\frac{1}{(-i\xi_3)^k}\left(\xi_1^2+\xi_2^2+(1-M_l^2)\xi_3^2\right)^{-k},\quad k=0,1,2 \tag{22}$$

Note that the sign $(1-M_l^2)$ determines the type of the system and the form of the original of the functions (22).

3.2. Green's tensor for $M<1$

We will construct solutions of the Biot's transport equations for subsonic speeds of movement of the transport load. For $c<\min(c_1,c_2,c_3)$, $M_j<1$ $(j=\overline{1,3})$ we have elliptical system and

$$\bar{f}_{0j}(\xi)\leftrightarrow f_{0j}(r,x_3,m_j)=\frac{1}{4\pi\sqrt{m_j^2r^2+x_3^2}},\ r=\sqrt{x_kx_k},\ k=1,2 \tag{23}$$

$$\bar{f}_{2j}(\xi)\leftrightarrow f_{2j}(r,x_3,m_j)=\left(|x_3|\,ln\frac{\left(|x_3|+\sqrt{m_j^2r^2+x_3^2}\right)}{m_jr}-\sqrt{m_j^2r^2+x_3^2}\right)/4\pi \tag{24}$$

So, we have following fundamental solutions of Biot's transport equations at subsonic speeds:

$$U_{kj} = c_3^2\Big\{b_{k3}\delta_{kj}c_3^{-2}\left(m_3^2r^2 + x_3^2\right)^{-1/2} -$$
$$- c^{-2}\sum_{l=1}^{3} b_{kl}\Big[\left(x_3^2x_kx_j / r^4 - x_3(\delta_{k3}x_j + \delta_{j3}x_k) / r^2 + \delta_{k3}\delta_{j3}\right)\left(m_l^2r^2 + x_3^2\right)^{-1/2} - \tag{25}$$
$$- \left(\left(m_l^2r^2 + x_3^2\right)^{1/2} - m_lr\right)\left(\delta_{kj}r^2 - x_kx_j\right) / r^4\Big]\Big\} / 4\pi\mu, k = 1,2,3, j = \overline{1,6}$$

$$U_{jk} = U_{kj}, k = 4,5,6,\ j = 1,2,3 \tag{26}$$

$$U_{kj} = -\delta_{kj}\left|x_3\right| / 2\rho_{22}c^2 + c_3^2\Big\{d_3\delta_{kj}c_3^{-2}\left(m_3^2r^2 + x_3^2\right)^{-1/2} -$$
$$- c^{-2}\sum_{l=1}^{3} d_l\Big[\left(x_3^2x_kx_j / r^4 - x_3(\delta_{k3}x_j + \delta_{j3}x_k) / r^2 + \delta_{k3}\delta_{j3}\right)\left(m_l^2r^2 + x_3^2\right)^{-1/2} - \tag{27}$$
$$- \left(\left(m_l^2r^2 + x_3^2\right)^{1/2} - m_lr\right)\left(\delta_{kj}r^2 - x_kx_j\right) / r^4\Big]\Big\} / 4\pi\mu, k = 4,5,6, j = 4,5,6$$

For $\|x\| \to \infty \quad U_{kj} = O\left(|x|^{-1}\right)$. For the non-stationary case in $2D$ and $3D$ spaces, fundamental solutions of Biot's equations were constructed using the Fourier transform of generalized functions in [15, 19].

3.3. Solutions of Biot's equations for any type of mass forces

A solution of Biot's transport equations for any type of mass forces can be represented as a tensor-functional convolution

$$u_i = U_{ik} * G_k \tag{28}$$

Which for regular $G_k(x,t)$, has the following integral representation:

$$u_i(x) = U_{ij}(x) * G_j(x) = \int_{R^3} U_{ij}(x - y)G_j(y)dy_1dy_2dy_3, \quad i,j = 1,\dots,6 \tag{29}$$

For singular mass forces, the convolution should be taken according to the definition of the convolution of generalized functions. Substituting the obtained solution into Biot's law for stresses (4), we determine the stress state of the medium.

4. Computer implementation of Green's tensor in the case of subsonic speeds

The results of the computer implementation in the Mathcad are presented below. Figure 1 shows the displacements of the solid component under the action of a load in the solid component, and Figure 2 shows the displacements of the fluid component under the action of a load in the fluid.

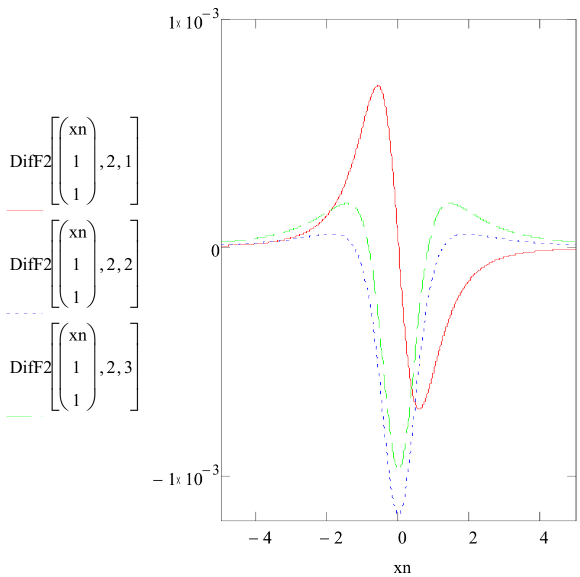

Fig. 1: The movement of the solid components U_{21}, U_{22}, U_{23} under the action of load in the solid component

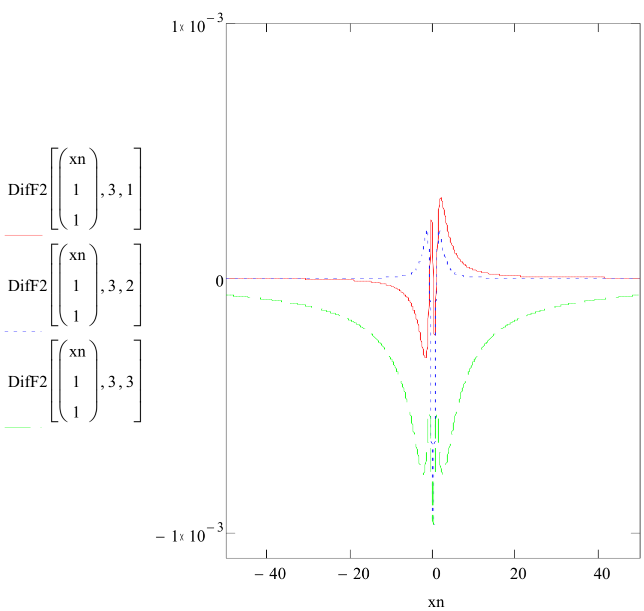

Fig 2: The fluid components moving U_{64}, U_{65}, U_{66} under the action of load in the fluid

The graphs of the medium displacements for different speeds of transport load, displacement vector fields of fluid and solid skeleton are obtained. The influence of the speeds of movement of the load on the fluid and solid components of the medium is investigated.

5. Conclusion

The Green's tensor of Biot's transport equations for subsonic speeds of loads and solutions for any type of mass forces have been constructed. The effect of transport load speed on the character of the stress-strain state of a two – component Biot model has been analysed. The results of the investigations allow us to take into account not only the elastic parameters of the underlying surface of vehicles, but also such important characteristics as porosity and water saturation.

Constructed Green's tensor can be used for solving on the basis of methods of boundary equations and boundary-element boundary-value problems in a Biot model with cylindrical boundaries, on which traffic load move with different speeds.

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