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Structural Rationale of Hollow Tapered Cantilever Beam under a Uniformly Distributed Load

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Extended Abstract

Functionally graded materials (FGMs), a type of composite material, consist of different mixed materials and the properties are designed to vary spatially. Bamboo is considered as a ‘natural FGM’ as the vascular bundles or fibers increases radially from inner to the outer surface of the bamboo culm and the volume fraction of fibers increases with height. Bamboo can withstand against wind load despite its hollow tapered structure. To understand the structural rationale behind this, the purpose of this study is to investigate hollow tapered beam made from an axially FGM subjected to a uniformly distributed load, undergoing a large deflection. Non-linear governing equations are derived, and a parametric study is conducted to investigate the effect of the tapering and hollowing ratios, load magnitude, and inhomogeneous material on the deflection of the beam. A slender beam fixed at one end, with circular cross-section is investigated, and the axially FGM is considered by varying Young’s modulus with height. Nonlinear bending equations are derived for both solid and hollow tapered beams and are solved using Runge-Kutta method. A linear analysis is conducted to examine the bending stress of the beam. To do so, normalized bending stress, the location of the maximum stress, and the corresponding maximum stress equations are derived. Sets of deflection curves and angles along the beam are obtained and compared with values obtained from previous studies.

As a result, untapered beam was found to have the highest bending rigidity and deflection decreases with increasing Young’s modulus ratio. A Young’s modulus ratio greater than 1 indicates that Young's modulus increases with height. Therefore, axially FGM increases bending rigidity, and tapering structure decreases the rigidity. Furthermore, hollow structures are found to be more rigid than solid structures. This is because, if the volume of solid and hollow structures is assumed to be equal, then hollow structures would have higher second moment of area, hence higher flexural rigidity. This shows that the hollow structure and the axially FGM nature of bamboo aids in improving its rigidity against wind load. Sets of deflected angles were calculated from the base to the tip with varying dimensionless load and compared with the results obtained by using homotopy analysis method (HAM) [1]. The results are in good agreement; this then validate the results obtained from this study. In addition, by using linear analysis to derived bending stress equations of solid tapered beam under uniformly distributed load, it was found that when tapering ratio was less than 1/3, the location of maximum stress follows the equation derived in this study. Otherwise, the position of the maximum stress was at the fixed end. This differs from Horibe and Mori [2] which conducted similar research subjected to concentrated loads and found that the maximum stress shifted away from the fixed end when the tapering ratio was lesser than 2/3.

References

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