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Optimal Stability Schemes in Transient Conjugate Heat Transfer Coupling

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Abstract **-** Transient flight cycles need accurate and stable solutions to the fluid flow/heat transfer problem. This is classically solved by coupling solvers for the fluid and thermal sub-problems, but this coupling is a new problem for which stable solutions are not easy to obtain in all cases. Here we show how Dirichlet-Robin and Neumann-Robin boundary conditions, with computed optimal parameter, have been successfully used for coupling fluid and thermal systems. These systems were respectively solved by the Fluent (ANSYS) and Z-set [4] (Onera/Northwest Numerics/Centre des Matériaux) codes**.**

Keywords**:** Conjugate Heat Transfer (CHT), Transient Coupling, Strong Coupling, Flight Cycle.

1. Introduction

Solutions to the fluid flow/heat transfer problem may be obtained using the adequate Dirichlet-Robin or Neumann-Robin boundary conditions. Here we are interested in the unsteady solution in the solid domain of this coupled problem, in the case of transient flight cycles, typically involving successive phases of lift-off, cruise and landing.

2. Optimal Stability Scheme

CHT (Conjugate Heat Transfer) is a mathematical model for the interaction between the two sides of a partitioned solution domain. The Robin boundary conditions (linear combinations of a Neumann and a Dirichlet condition) on the solid and fluid side can respectively be written as:

$$\left[\hat{q}_s^n + \alpha_f^{n-1}\hat{T}_s^n\right]_{t_c+\Delta t_c} = \left[q_f^{n-1} + \alpha_f^{n-1}T_f^{n-1}\right]_{t_c+\Delta t_c} \tag{1}$$

$$\left[\hat{q}_f^n - \alpha_s^n\hat{T}_f^n\right]_{t_c+\Delta t_c} = \left[q_s^n - \alpha_s^n T_s^n\right]_{t_c+\Delta t_c} \tag{2}$$

where q is the heat flux, T is the temperature, the hat marks are the unknowns, the n suffix is the time-step index, and α_s, α_s denote the coefficients for the solid and fluid sides respectively. Here we will use a Robin condition only on the solid side, whereas on the fluid side a degenerate Dirichlet ($\alpha=\infty$) or Neumann ($\alpha=0$) condition is used.

Solving for this interaction is not inherently stable [1] : the correct Dirichlet-Robin or Neumann-Robin boundary condition with the associated optimal parameter for the current Biot number [2] has to be used. See reference [3] for more details.

The widely used ``heat transfer coefficient" (h) is a crude approximation, approximately valid for a limited range of conditions; it is not adequate for configurations where high values of the Biot number appear, such as ceramic-coated turbine blades (with low conductivity), because the stability margin is insufficient. On the contrary, the optimal scheme provides ample stability margin, as the plot for the amplification factor (here named "gcof ") shows, Figure 1, with the optimal coefficient (here named "alpha_opt") and the "classical" h. This margin is essential for the initialization phase of computations, far from linear stability conditions, allowing for a good initial convergence.

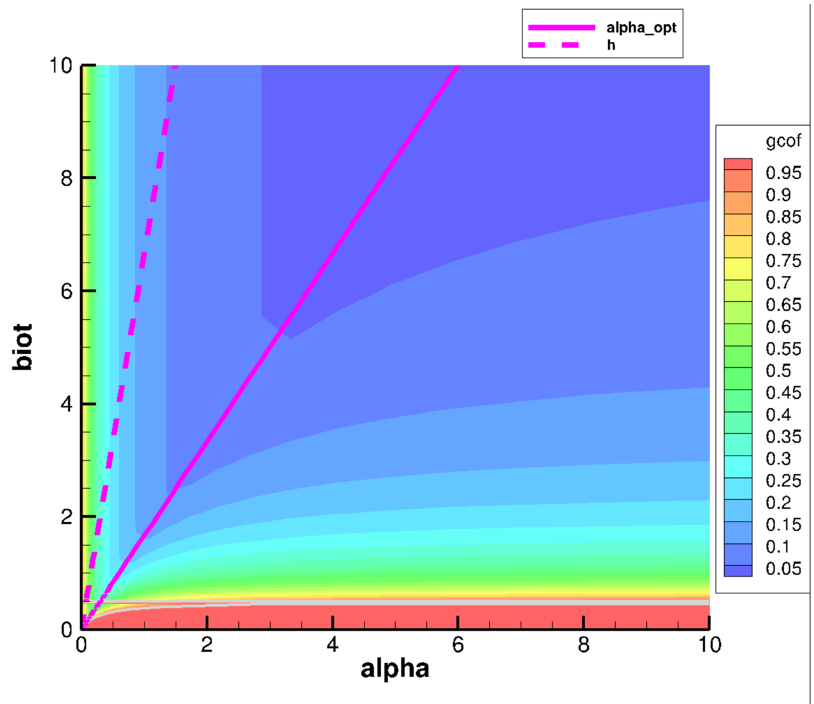

Fig. 1: Amplification factor “gcof” as a function of the α_f (“alpha”) stability coefficient and of the Biot number, for the Neumann-Robin boundary condition ($\alpha_s = 0$).

3. Coupling Strategy

As stated in the Introduction, the fluid and thermal systems were respectively solved by the Fluent and Z-set codes. Coupling two algorithms may lead to an unstable behavior, even when each one is perfectly stable. This is why we need to ensure that the coupled algorithm is stable.

The strong coupling strategy used here involves iterating on the current time interval, Figure 2, until the specified and computed fluid temperatures at the current (forward) coupling point are close enough (see “Results” below). This strong coupling (i.e. using iterations) is required for precise computation of steep temperature profiles.

Notice that, although the coupling problem is unsteady, the fluid solver may be run here in steady mode. This is because the timescale of the fluid is much smaller than that of the solid, so that the fluid is always in equilibrium on the timescale of the coupling, here computed from the solid side. Only the thermal solver on the solid side must be run in unsteady mode. Also, the optimal coupling coefficients have to be computed from an unsteady analysis, as the results are quite different from those given by a steady analysis.

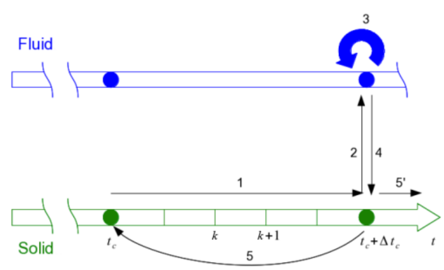

Fig. 2: Iterative procedure for strong coupling.

4. Test Configuration

We will limit ourselves here to an easily reproducible 2D case, see the schematic represented in Figure 3. The chosen test-case is a simple but stiff problem of a solid (e.g. metallic or ceramic) plate, with on the bottom side a prescribed temperature profile, represented in Figure 4, and along the other side an airflow with prescribed initial temperature. The leading-edge of the plate is a singularity, where the flux coefficient is theoretically infinite.

The fluid mesh is refined both normally to the plate for adequate boundary-layer representation, and longitudinally at the leading edge, to represent the maximum in heat transfer coefficient (smoothed singularity).

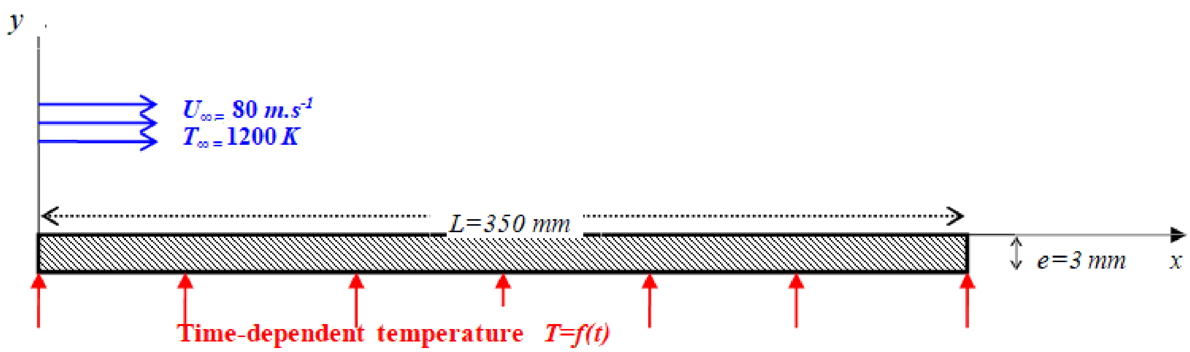

Fig. 3: Schematic of the flat-plate configuration.

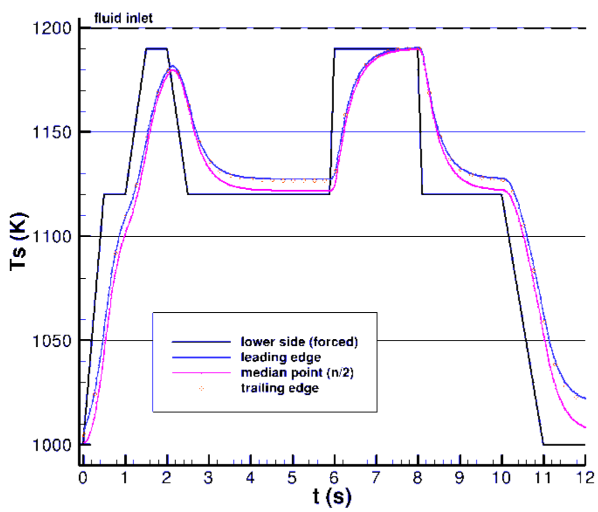

Fig. 4: Fluid response (blue, purple) to solid temperature profile (black) on the flat plate con-figuration, transient coupling using Dirichlet-Robin condition at low Biot number.

5. Results

With the optimal scheme, fast convergence is achieved for any Biot number, respectively using Dirichlet-Robin and Neumann-Robin conditions for low and high values of this parameter. Figure 4 corresponds to the Dirichlet-Robin condition, well adapted to a low Biot number.

For numerical validation purposes, smooth convergence on the fluid temperature has been achieved to 10^{-10} K in less then 30 coupling iterations, while the simple “heat transfer coefficient” (h) method gives here erratic pseudo-convergence around 0.1 K, even for 100 coupling iterations.

6. Conclusion

The presented coupling scheme for transient CHT coupling provides fast convergence for all values of the Biot number.

Results are included here for a flat plate in presence of an imposed temperature ramp on the lower surface, with constant-temperature “hot” fluid flow on the upper surface.

In the chosen low Biot number range, using Dirichlet-Robin condition, the temperature of the upper surface of the plate rapidly relaxes to the imposed lower temperature on the constant-valued intervals of the ramp. For a higher Biot number range, the Neumann-Robin condition would have been selected instead.

From a practical standpoint, using an example from airplane engines, this scheme thus covers all turbine blade materials, from bare metals (low Biot numbers) to pure ceramics (high Biot numbers), including thermal barriers.

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