*Proceedings of the 6th World Congress on Momentum, Heat and Mass Transfer (MHMT'21)*
*~~Lisbon, Portugal~~ Virtual Conference – June 17 – 19, 2021*
*Paper No. ICMFHT 304*
*DOI: 10.11159/icmfht21.lx.304*

# Thermo-Capillary Induced Motion in Multiphase System Using Smoothed Particles Hydrodynamics

**F. Almasi[1], M. S. Shadloo[1,2*], A. Hadjadj[1]**
[1]CORIA - UMR 6614, Normandie University, CNRS - INSA of Rouen, 76000 Rouen, France
[2]Institute of Chemical Process Engineering, University of Stuttgart, 70199 Stuttgart, Germany
[*]Corresponding author: msshadloo@coria.fr

## Objectives

The thermal gradient at the continuous phase causes inhomogeneities in the interfacial tensions in multiphase immiscible Newtonian fluids which can introduce thermocapillary instabilities. To balance the surface tension gradient, a shear stress starts to act tangential to the interface and induces a motion in the fluid. In this study, we assume that the surface tension linearly decreases with the increase in temperature. This phenomenon is known as the Marangoni effect and the surface tension gradient is called Marangoni force. The present study investigates the dynamics of a suspended bubble in a low-viscous stationary flow subject to Marangoni force. A numerical model will be presented and validated based on the semi-implicit Incompressible Smoothed Particle Hydrodynamics (ISPH) method.

## Scope

Incompressible multiphase fluids demonstrate strong migration and deformation subject to thermal fields. To simulate such system without phase change, ISPH method with its inherent ability in phase recognition and sharp interface capturing is applied in its non-conservative formulation of Continuum Surface Force (CSF) as described in [1]. In the context of the current work, we use the following SPH discretization

$$\rho_a = \sum_b m_b W_{ab} \tag{1}$$

$$\frac{D\vec{u}_a}{Dt} = -\sum_b \frac{m_b}{\rho_a \rho_b}\left(p_b + p_a\right)\nabla W_{ab} + \sum_b \frac{m_b}{\rho_a \rho_b}\left(\eta_a + \eta_b\right)\frac{\vec{r}_{ab}}{r_{ab}^2}\nabla_a W_{ab}\cdot\left(\vec{u}_a - \vec{u}_b\right) + \vec{f}_{body,a} + \frac{|\vec{n}_a|}{\rho_a}\left(\sigma_a \kappa_a \hat{\vec{n}}_a + \left(\nabla_S \sigma\right)_a\right) \tag{2}$$

$$\left(\frac{DT}{Dt}\right)_a = \sum_b \frac{m_b}{\rho_b}\left(k_a + k_b\right)\frac{\vec{r}_{ab}}{r_{ab}^2}\nabla_a W_{ab}\cdot\left(T_a - T_b\right) \tag{3}$$

for conservations of mass, momentum and energy, respectively. Here, indexes $a$ and $b$ represent, respectively, the particle of interest and its neighboring particles which are within the supported domain of the kernel function $W_{ab}$. Also, $\rho_i$ and $m_i$ are the density and the mass of each individual particle, respectively. Finally, $D/Dt$ represents the material time derivative, $\rho$ is the fluid density, $t$ is the physical time, $\vec{u}$ is the velocity vector, $p$ is the pressure, $\eta$ is the dynamic viscosity, $\vec{f}_{body}$ is the body force due to the gravitational acceleration, $c_p$ is the specific heat capacity at constant pressure, $T$ is the fluid temperature, and $\lambda$ is the thermal conductivity and $k = \lambda/\rho c_p$ is the thermal diffusivity. Additionally, we use the projection of the surface tension gradient into the tangential direction of the interface,

$$\left(\nabla_S \sigma\right)_a = \left(\nabla\sigma - \left(\nabla\sigma\cdot\hat{\vec{n}}\right)\hat{\vec{n}}\right)_a \tag{4}$$

with

$$(\nabla\sigma)_a = \sum_b \frac{m_b}{\rho_b}(\sigma_b - \sigma_a)\nabla_a W_{ab} \tag{5}$$

More details regarding the interface treatment and the evaluation of interface curvature and its normal unit function used in the current work can be found in [2]. Taking advantage of the semi-implicit nature of the solver, a relatively larger time increment Δt can be used along with a smaller number of time steps.

## Results

The capillary stress tangential to the interface under thermocapillary effect is verified at three SPH grid resolutions. To this end, a linear thermal profile is imposed on the two-phase square geometry consisting of two fluid equilaterally exposed to one another. The profile of the interfacial Marangoni force is obtained and in agreement in terms of force value and profile symmetry with the results of [3]. The surface tension relationship with temperature is linear as

$$\sigma = \sigma_{ref} + \sigma_t(T - T_{ref}) \tag{6}$$

where $\sigma_{ref}$ and $T_{ref}$ are the reference surface tension and the reference temperature, respectively for a given surface tension thermal coefficient $\sigma_t$ .

In another case study, a droplet with radius R is placed at the center of a square domain of size L (L/R=8) filled with another fluid. The convergence of thermocapillary dynamic forces is validated. First by the validation of the normal part of the surface tension illustrating that the pressure gradient at the interface is in complete agreement with Laplace analytical solution at ($\sigma$ = 0,0.25,0.5,1). Moreover, the pressure is also examined at four grid sizes per direction (120,200,240,480) by depicting the pressure difference along a linear profile that passes through the diameter of the bubble.

## Conclusion

In the first part of this work, we presented our ISPH method along with experimental and theoretical studies on the thermocapillary dynamics of a two-phase flow. The proposed model is first validated in case of a static multiphase system without exposure to the temperature field. We show that the at the steady-state the Young-Laplace relation and the analytical law are valid and therefore the model is correctly implemented. Next, for a system with two parallel liquids and flat interface, it is shown that for higher particle spacing (fully-refined grid), the range of the volumetric Marangoni force at the fluid-fluid interface is higher, which keep the total volumetric force constant across it. Finally, the migration of a suspended droplet subject to thermo-capilarity properties are presented where we observed a very well agreement between the results of our ISPH model with reference results by Tong [3]. In summary, both parallel and tangential components of interfacial forces are well captured using the CSF model presented in this work.

## Acknowledgment

The first author acknowledges the financial supports provided by Normandy region, France. The second author acknowledges the support provided by Alexander von Humboldt Foundation through the project FRA-1204799-HFST-E for the experienced researcher. The authors also acknowledge the access to French HPC resources provided by the French regional computing center of Normandy CRIANN (2017002).

## References

[1] Morris, Analysis of smoothed particle hydrodynamics with applications, Ph.D. thesis, Monash Univ., 1996.
[2] M. Hopp-Hirschler, M. S. Shadloo, U. Nieken, J. Computers and Fluids 176, pp. 1–19, 2018.
[3] M. Tong, D. J. Browne, Int. J. Heat and Mass Transfer, vol 73, Pp 284-292, 2014.