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Thermal Stability Analysis of Toroidal Thermosyphon Models with Fuzzy Controllers

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Abstract **-** In this work, we develop a proportional-type fuzzy controller to investigate its ability to stabilize flow and temperature of a single-phase fluid in a natural convection loop. The convective loop has a toroidal shape and is filled with an incompressible fluid that exchanges energy along the torus. Known influx of heat occurs in some parts of the loop whereas heat efflux takes place in others. Normally, buoyancy forces – produced by temperature differences within the fluid – drive the flow inside the torus, generating three possible flow scenarios: stable, limit cycles and chaos. For the analysis, one-dimensional models are first developed from the momentum and energy equations on the basis of the Boussinesq approximation, and by assuming averaged values of velocity and temperature over the cross-section of the curved tube. The resulting integrodifferential equations are then converted to a nonlinear dynamical system and solved under different operating conditions. The controller is built upon the fuzzy logic technique, which has the ability to describe complex systems in terms of linguistic variables, following expert-based if-then rules to make inferences about their behavior. Quantification of the linguistic variables is done via triangular membership functions, and the rules are built from numerical data under different operating conditions from the mathematical model. Since the tilt angle for the loop and the heat flux are used as the parameters characterizing its dynamic behavior, these are the manipulated variables, whereas the control variables are average fluid velocity and temperatures inside the loop. MATLAB is used to implement the fuzzy controller, along with the corresponding control actions, while numerical experiments are conducted to assess its relative performance. Results demonstrate that the fuzzy controller can effectively stabilize the natural convection loop system.

Keywords**:** Fuzzy logic; Thermal control; Thermosyphon; Stability analysis.

1. Introduction

A toroidal thermosyphon, also known as a natural convection loop, is a thermal device with the shape of a torus that works on the basis of local changes in fluid temperature, leading to differences in fluid density. This difference in density causes buoyant forces thus promoting fluid flow which transports thermal energy from one region of the loop to another. Since thermosyphons do not need external pumps to drive the flow, they are important in a number of applications, including geothermal energy, energy storage, and electronic and nuclear reactor cooling, as well as in solar heaters, among others [1–3]. Because of their importance, understanding the behavior of these systems, particularly that which evolves with time, is necessary for both performance prediction and control. Thus, several experimental and numerical studies have been carried out and reported in the literature [4–8]. In all these investigations, the parameters chosen, e.g., heat input, wall temperature or tilt angle, would lead the system to have either a constant, a cyclic, or a chaotic behavior. Therefore, depending on the objective, often the system would need to be controlled in some fashion.

If the interest is in controlling the system for a specific application, then for a given design, a robust controller is necessary. The implementation of robust control schemes, however, is difficult to achieve due to complexities related to the dynamic nature of the fluid flow and the physics related to energy transfer. Thus, in practical applications, the most common scheme used is the proportional-integral-derivative (PID), since it is easy to implement [9]; however, its major drawback is the lack of robustness, since it requires constant re-tuning. Therefore, alternative control strategies for the control of natural convection loops may be necessary. Here we are interested in the application of control schemes based on fuzzy logic due to its ability for describing complex systems with linguistic variables and expert-based rules derived from human experience [10, 11]. These characteristics have permitted fuzzy controllers to be used effectively in a number of thermal systems, like heat exchangers, heat pumps, and photovoltaic systems, among others [12–15].

The objective of this work is to develop a robust fuzzy-logic-based controller for a toroidal thermosyphon system. To this end, the device and its mathematical model, based on a set of nonlinear first-order differential equations are described first. Next, a set of numerical tests are carried out for different conditions of the parameters corresponding to the three possible scenarios: stable, limit-cycles and chaos, which are then used as baseline to assess the controller. A brief introduction to fuzzy logic, along with details on the development of the controller, is presented next. Finally, numerical experiments are conducted to assess the relative performance and the results and corresponding analysis are discussed.

2. Problem Description and Mathematical Model

Consider a loop filled with a single-phase fluid, as depicted in Figure 1.

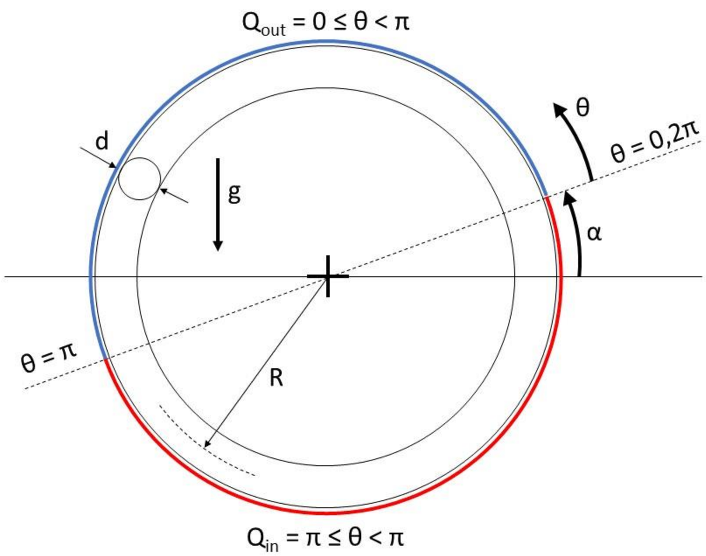


Fig 1: Toroidal thermosyphon.

The tube diameter is *d* and the length from the center of the loop to the midpoint of the tube is *R*, with *R*>>*d*. The angle θ describes the position along the circumference of the loop and the regions where heat enters and leaves the device. From $0 \leq \theta \leq \pi$ ($0° \leq \theta \leq 180°$), heat leaves the system whereas from $\pi \leq \theta \leq 2\pi$ ($180° \leq \theta \leq 360°$), heat enters the system. This creates a difference in temperature in the fluid and a difference in fluid density, thus causing its motion. There are three possible heating conditions: Known heat flux, known wall temperature and mixed conditions [8], whether the heat flux or the wall temperature are known over the entire loop, or whether these quantities are known for different parts of the loop. In the present study we will focus on the "known heat flux" heating condition.

Although a similar type of system has been modeled using two-dimensional versions of the conservation equations [16], one-dimensional versions have been very useful in studying the dynamics in these systems, and it is the type of model that will be used in this work. Mass conservation provides a velocity independent of the spatial coordinate; i.e., $u = u(t)$, while temperature is $T = T(t,\theta)$. Both, *u* and *T* are the dependent variables of the problem, whereas time *t* and the angle θ, as measured from the boundary separating heat input and output values, are the independent variables. Finally, the angle of inclination α (also known as tilt angle) is one of the parameters, while the heat flux is the other.

Thus, using the Boussinesq approximation for the buoyancy term, and neglecting axial conduction within the fluid, the integral of the momentum equation over the loop and the energy equation, both in nondimensional form, are [8]

$$\frac{du}{dt} + u = \frac{1}{\pi}\int_0^{2\pi} T\ \cos(\theta+\alpha)\,d\theta, \#(1)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial \theta} = Q;\ \text{with}\ \ Q = -\hat{Q}\sin\theta\,, \#(2)$$

where Q is a prescribed non-dimensional heat flux of strength $\hat{Q}$ either going in or out of the loop. It is to note that the thermal loop for the known heat flux, and other conditions, has been studied using Fourier [8] Karhunen-Loeve [17] expansions.

By following Pacheco-Vega et al., [8], the above equations can be transformed into a system of first-order ODEs, by expanding the temperature as

$$T(t,\theta) = T_0^c(t) + \sum_{m=1}^{\infty}[T_m^c(t)\cos m\theta + T_m^s(t)\sin m\theta]\,, \#(3)$$

so that, after using the orthogonality conditions, and integrating around the loop, Eqs. (1) and (2), become

$$\frac{du}{dt} = -u + T_1^c\cos\alpha - T_1^s\sin\alpha\,, \#(4)$$

$$\frac{dT_1^c}{dt} = -uT_1^s, \#(5)$$

$$\frac{dT_1^s}{dt} = -Q + uT_1^c, \#(6)$$

where *u(t)* is the fluid velocity, and $T_1^c(t)$ and $T_1^s(t)$ are the values of the coefficients – for the first mode – of the nondimensional temperature distribution in Eq. (3). By finding u, T_1^c and T_1^s, for a given Q and α, it enables computing T, and thus establishing the behavior of the thermosyphon.

If we define x ≡ u, y ≡ T_1^c and x ≡ T_1^s for brevity, then the dynamical system becomes

$$\frac{dx}{dt} = -x + y\cos\alpha - z\sin\alpha\,, \#(7)$$

$$\frac{dy}{dt} = -xz, \#(8)$$

$$\frac{dz}{dt} = -Q + xy, \#(9)$$

where the variables are now $x(t)$, $y(t)$ and $z(t)$, and t is time.

3. Dynamic Behavior

In the case of a thermosyphon under known heat flux conditions, the heat flux is known over the entire loop. In this case, the heat influx and outflux are modeled as: $Q = \hat{Q}\sin\theta$. The parameters that determine the behavior of the system are the heat flux Q and the inclination angle α, so that, for a given value of these, the system can have a fluid flow that is stable, cyclic, or chaotic. The linear stability analysis and numerical solutions for this system, used to determine which of the three behaviors the system will have, were first reported by Sen et al. [7]. On the other hand, by following Pacheco-Vega at al. [8], the two critical points, P_1 and P_2, of this system [Eqs. (7)–(9)], are:

$$P_1 = (\bar{x}, \bar{y}, \bar{z}) = \left(\sqrt{Q\cos\alpha}, \sqrt{Q/\cos\alpha}, 0\right) \#(10)$$
$$P_2 = (\bar{x}, \bar{y}, \bar{z}) = \left(-\sqrt{Q\cos\alpha}, -\sqrt{Q/\cos\alpha}, 0\right) \#(11)$$

both of which exist if $-90° < \alpha < 90°$, with $\bar{x}, \bar{y}, \bar{z}$ representing the critical points (or steady state solutions). Furthermore, from a linear stability analysis, for instance, $P_1 = \left(\sqrt{Q\cos\alpha}, \sqrt{Q/\cos\alpha}, 0\right)$ is stable as long as $Q \leq \sin^2\alpha / \cos^3\alpha$, with $\alpha \geq 0$ [8]. The stability curve, shown in Figure 2, shows the stable and unstable regions in the parameter plane.

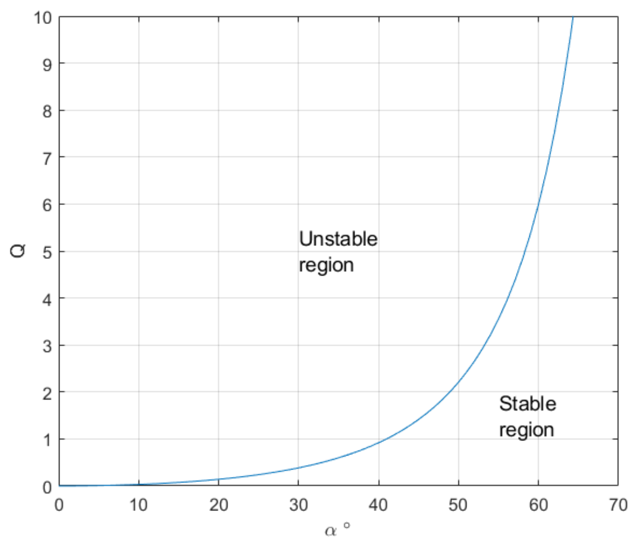


Fig. 2: Linear stability curve.

The relationship between α and Q lays the groundwork for developing the fuzzy controller. Examples of each type of behavior for the natural convection loop are shown in Figures 3, 4, and 5, respectively, for stable, periodic, and chaotic conditions for a constant value of $Q = 5$, and different tilt angles α = 60◦, α = 50◦, and α = 30◦, respectively.

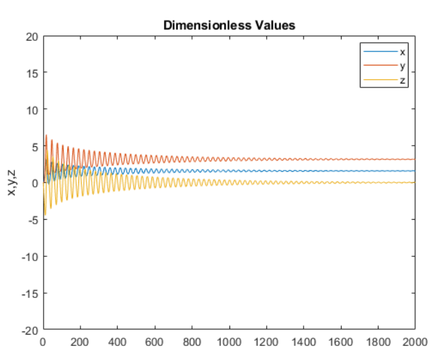


Fig 3: Stable behavior; Q = 5, α = 60°.

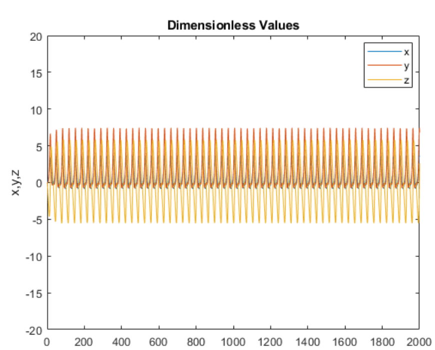


Fig 4: Cyclic behavior; Q = 5, α = 50°.

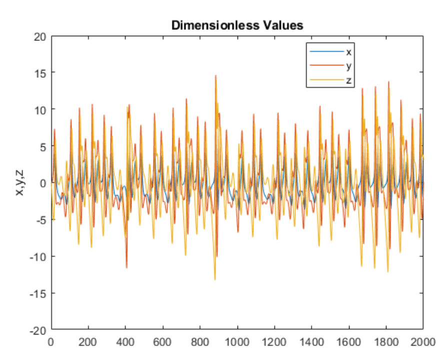


Fig. 5: Chaotic behavior; Q = 5, α = 30°.

4. Fuzzy Control

4. 1. Background on Fuzzy Logic

Fuzzy logic (FL) uses linguistic variables to develop rules (based on external 'expert' knowledge), and membership functions which form so-called fuzzy sets, which enable handling vagueness and imprecision in the data to solve a particular problem [18]. A key feature of FL is the concept of fuzzy sets, which include a sliding scale of membership of an element belonging to a set, as opposed to a strict binary crisp set. While in a crisp set an element can either belong to the set or cannot belong to it; there is only true or false statements. For example in the present case, either fluid temperature, T_f 'is' hot or it 'is not' hot. This is defined mathematically as

$$\mu_A(T_f) = \begin{cases} 1, & T_f \in A \\ 0, & T_f \notin A \end{cases}, \#(12)$$

and visually in Figure 6.

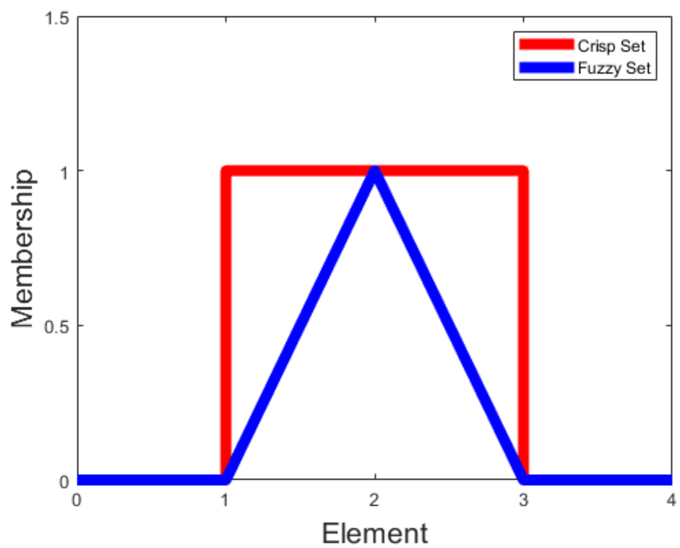


Fig. 6: Elements and membership in crisp and fuzzy sets

On the other hand, in fuzzy sets, an element can have a varying degree of membership to a specific set; so, in fact, it can partially belong to several sets. Thus, in the same context of fluid temperature, T_f, in a fuzzy set, a fluid can be described anywhere in between 'very hot', 'hot', 'warm', 'cold', or 'very cold'. This notion of degree of belonging allows for a smooth transition among membership functions of a specific variable. This is defined, mathematically, as

$$\mu_A(T_f) = \epsilon[0,1], \#(13)$$

with the fuzzy set A now being defined as

$$A(T_f) = \left\{ \sum_i \frac{\mu_A(T_{f,i})}{T_{f,i}} \right\}, \#(14)$$

with its visual correspondence also provided in Figure 6.

Although the majority of control applications use PID controllers, which are based upon crisp sets – where an element is either in or out – significant improvements in FL control strategies have been made recently. This is particularly the case of applications to thermal systems, showing that these types of controllers can be very useful in controlling complex devices [19–21], demonstrating their ability to act upon to control complex systems using expert if-then rules. Thus, it is the type of controller that will be used to manipulate the tilt angle, α, in controlling the thermosyphon.

4. 2. Development of Controller

The objective of this controller is to achieve a specific fluid flow inside the thermosyphon device while, at the same time, maintaining its stability. This is done by controlling the change in the value of nondimensional velocity x, by adjusting the tilt angle α, in a closed feedback loop. As an example, if the change in x (i.e., Δx) was too much, that meant the system was not stable and the tilt angle Δα needed to be increased. If $x = 0$ (or better $\Delta x = 0$), the system was stable and had the capacity for the flow to be increased slightly to match the possible change in the heat flux Q.

It is to note that the three dependent variables, the non-dimensional velocity, and the two Fourier coefficients of fluid temperature, $x(t)$, $y(t)$ and $z(t)$, are interconnected by the physics of the process of heat transfer by convection. Therefore, the three variables will not separate into different behaviors; if one is stable then the others will also be. From this fact, only x will be used to determine how much the tilt angle α, would need to be changed. The error value Δx, will dictate the how much Δα will change until complete stability is achieved. During numerical simulations the ranges have been set to $\Delta x \in [-5,5]$ and $\Delta\alpha \in [-10,10]°$. Using these ranges, the fuzzy sets along the membership functions, shown in Figures 7 and 8, were developed for Δx and the Δα and the set of linguistic rules, illustrated in Table 1, were also established.

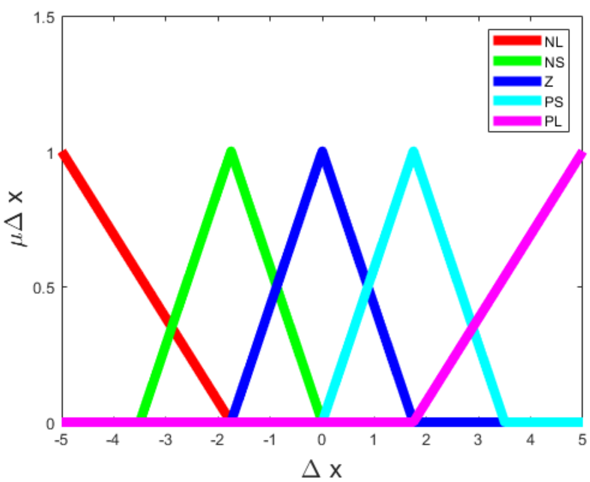


Fig. 7: Fuzzy sets & membership functions for Δx.

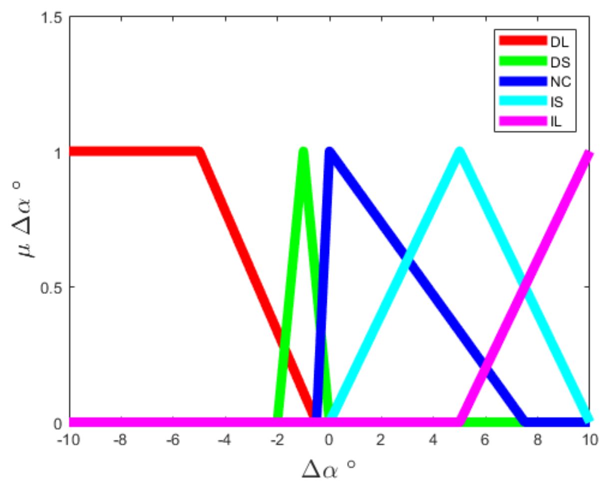


Fig. 8: Fuzzy sets & membership functions for Δα(°).

Table 1: Decision table for tilt angle adjustment.

Δx	Δα(°)
Negative large (NL)	Increase large (IL)
Negative small (NS)	Increase small (IS)
Zero (Z)	Decrease small (DS)
Positive small (PS)	Increase small (IS)
Positive large (PL)	Increase large (IL)

To control the thermosyphon system, a feedback loop – shown in Figure 9 – was designed to test the ability of the fuzzy control to manipulate the tilt angle to maintain stability of the system; i.e., x, y and z. The figure shows that the plant contains a Simulink block diagram that solves the system of non-linear ODEs [Eqs. (7)–(9)]. The reference for Δx is set to zero, which is perfect stability. A random value signal generator with an amplitude of 0.5 is used to induce perturbations in the Δx-value, simulating experimental data to test the controller. The cumulative block maintains the tilt angle value so a change in tilt angle can be added or subtracted after every iteration. To maintain an optimized value of x, the tilt angle was constrained to a maximum value of $\alpha = 60°$. The cumulative block also allows for initial conditions to be set. These conditions are used to determine if, when provided a value that should result in chaotic behavior, the system can be stabilized.

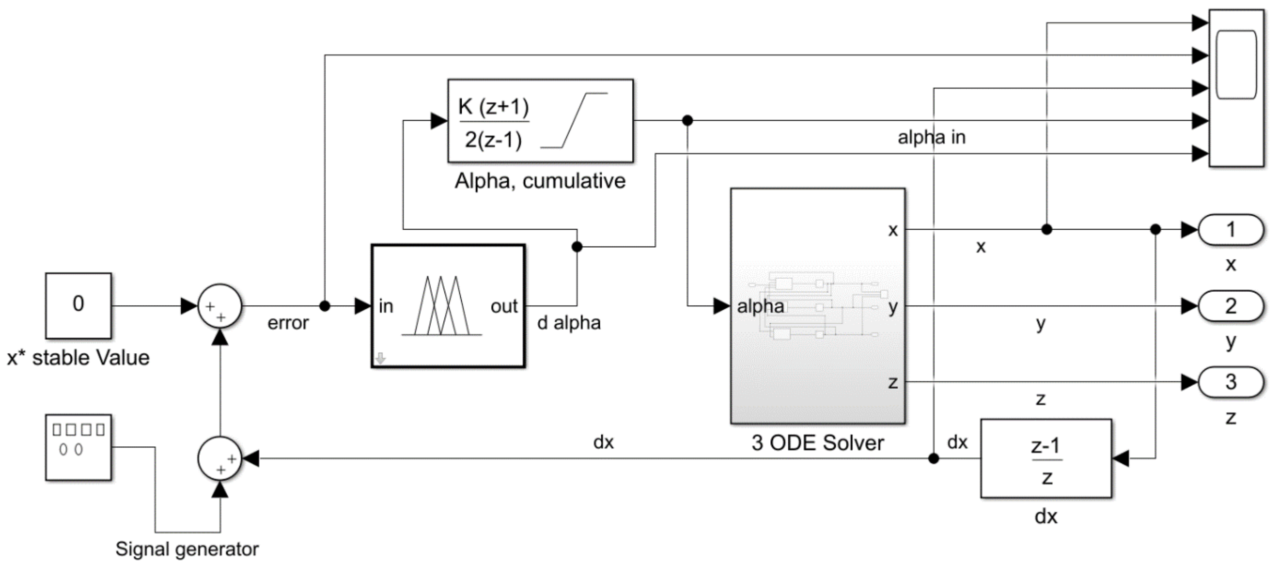


Fig. 9: Feedback loop.

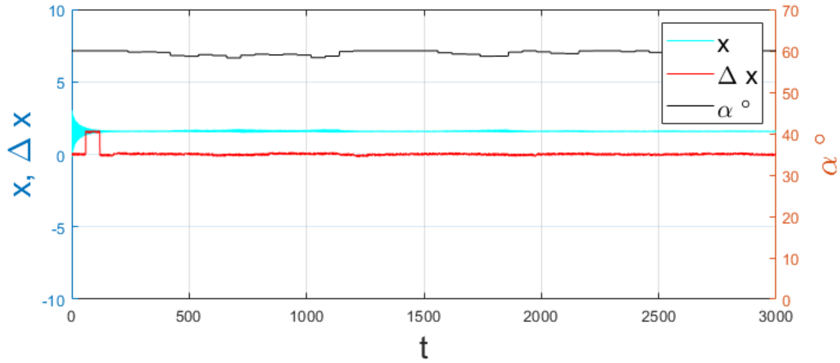
5. Thermal Stability Results

5.1 Numerical Simulation

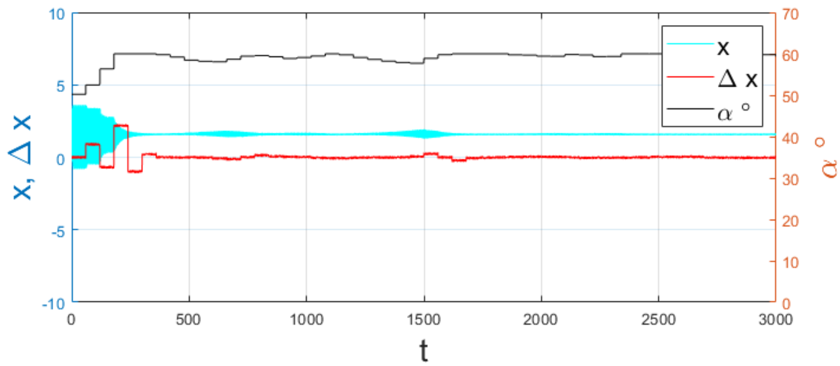
Numerical simulations were ran in MATLAB using a custom 4th order Runge-Kutta code to solve the system of ODEs (7)–(9). For heat input value of $Q = 5$ and values of $\alpha = 60°$, 50°, and 30°, respectively, the system behaves in a stable manner arriving at a constant value of either velocity and temperature as shown in Figure 3, or with periodic oscillations as illustrated in Figure 4, or in a chaotic manner as seen in Figure 5. These simulation results provide the ability to predict what behavior will result from various combinations of inputs for the controller.

5.2 Stability Using the Fuzzy Logic Controller

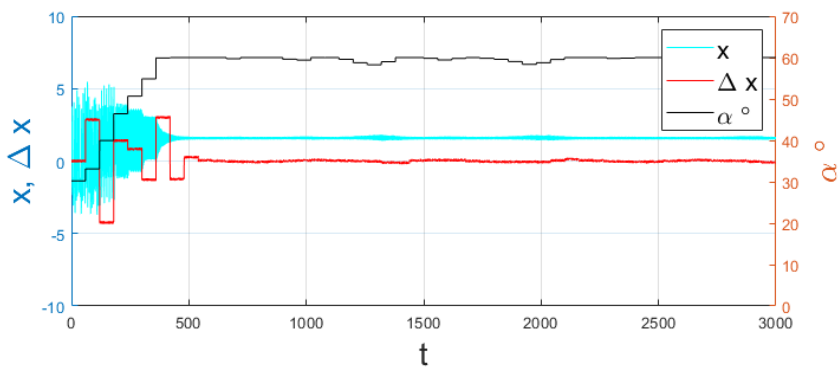
The objective of this FL controller was to maintain stability of this system. Various initial conditions were tested, including a signal generator that provided small random error signals. These were meant to test the robustness of the controller. As shown in Figure 2, a heat input value of $Q = 5$ and tilt angles α of 60°, 50°, and 30° should produce – respectively – stable, cyclic, and chaotic behavior. The results from those angles were used as initial conditions are shown in Figure 10. In all cases, the fuzzy controller is used to stabilize the system under these initial conditions and in the regions of stable and unstable conditions. The first figure, Fig. 10(a), shows an initial stable condition, which dampens within 100 iterations to a stable condition and continues to correct the tilt angle due to the induced perturbations. The second figure, Fig. 10(b), shows an initial cyclic condition, which dampens within 250 iterations. Finally, the third figure, Fig. 10(c), shows an initial chaotic condition, which dampens within 500 iterations due to the FL controller actions. All three simulations show the robustness of the fuzzy controller to stabilize x, y, and z, from any initial condition.



(a) Initial condition: α = 60°



(b) Initial condition: α = 50°



(c) Initial condition: α = 30°

Fig. 10: Thermal stability results of the fuzzy controller

6. Conclusion

Robust and efficient controllers are important to both ensure thermal stability of complex systems, like natural convection loops. Although PID controllers are common in industry, they lack robustness. In this work, we have developed a fuzzy-based controller that uses information on the velocity error to provide inputs on the tilt angle in order to stabilize the velocity and corresponding temperature. The numerical tests show that the fuzzy controller successfully performs the control actions and it is able to stabilize the system under different operating conditions. The controller has been tested against different initial conditions as well as induced perturbations to simulate experimental data in a simulation environment. This work has shown proof of concept that a fuzzy logic controller can be used for this application. Further testing will develop additional fuzzy controllers and to optimize the control of the tilt angle. These results will be presented in the future.

Acknowledgments

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